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Hyperfine splitting in muonic hydrogen constrains new pseudoscalar interactions

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ABSTRACT

We constrain the possibility of a new pseudoscalar coupling between the muon and proton using a recent measurement of the 2S hyperfine splitting in muonic hydrogen.

Recent measurements of $2S$–$2P$ transition frequencies in the exotic atom constituted by a proton orbited by a muon [1,2] find the proton charge radius to be 7σ smaller than the 2010-CODATA [3] value obtained using ordinary hydrogen and e–p scattering. The 2S hyperfine splitting deduced from the same measurements shows excellent agreement with predictions [1]. The discrepancy in the proton radius has generated a lot of interest, including the invocation of new fundamental interactions as an explanation. Here, we focus on the implications of the hyperfine splitting for new interactions between the muon and proton. Specifically, we consider the possibility of a new pseudoscalar particle that couples to the muon and proton. Such an interaction is spin and velocity dependent and has a negligible effect on the Lamb shift (which is used to extract the proton radius) in the nonrelativistic limit [4], but has a significant effect on the hyperfine splitting.

The measured value of the 2S hyperfine splitting (HFS) [1]

$$\Delta E_{HFS} = 22.8089 \pm 0.0051 \text{ meV} \tag{1}$$

is to be compared with the theoretical prediction [5]

$$\Delta E^{th}_{HFS} = (22.9843 \pm 0.0030) - (0.1621 \pm 0.0010) r_Z + \delta E_a \tag{2}$$

in meV, where the Zemach radius [6]

$$r_Z = 1.045 \pm 0.004 \text{ fm} \tag{3}$$

is obtained from e–p scattering.[1] δE_a is the contribution to HFS from the new pseudoscalar interaction. Taking the experimental and theoretical uncertainties in quadrature, the best-fit to the experimentally measured ΔE_{HFS} and r_Z occurs for $r_Z = 1.045$ fm and $\delta E_a = -0.006$ meV, and

$$-0.018 \text{ meV} \le \delta E_a \le 0.006 \text{ meV at } 2\sigma\,. \tag{4}$$

We now compute δE_a, and subject it to the above 2σ constraint. In the nonrelativistic (NR) limit, the pseudoscalar vertex becomes

$$J_5 = \bar{u}(\boldsymbol{p}')i\gamma_5 u(\boldsymbol{p}) \xrightarrow{\text{NR}} i\chi'^{\dagger}\tfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{2m}\chi - i\chi'^{\dagger}\tfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}'}{2m}\chi\,,$$

where χ and χ' are 2-component Pauli spinors. The μ–p interaction in terms of the muon line (given by $\chi_\mu, \boldsymbol{\sigma}_\mu$) and the proton line (given by $\chi_p, \boldsymbol{\sigma}_p$) is then (see Fig. 1)

$$J_{5,\mu} J_{5,p} = \tfrac{i}{2m_\mu}\chi'^{\dagger}_{\mu}\boldsymbol{\sigma}_\mu\cdot(\boldsymbol{p}-\boldsymbol{p}')\chi_\mu \; \tfrac{i}{2m_p}\chi'^{\dagger}_{p}\boldsymbol{\sigma}_p\cdot(\boldsymbol{P}-\boldsymbol{P}')\chi_p\,,$$

and the NR scattering amplitude for $\boldsymbol{p}+\boldsymbol{P}\to\boldsymbol{p}'+\boldsymbol{P}'$ is

$$i\mathcal{M} = if_\mu J_{5,\mu}\,\tfrac{i}{q^2-m_a^2}\,if_p J_{5,p}\,, \text{ with } q = p - p' = P' - p\,.$$

The couplings of the light pseudoscalar a of mass m_a to the muon and to the proton are f_μ and f_p, respectively. Then,

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[1] The use of the value of r_Z obtained from e–p scattering is appropriate here because the correction to r_Z from using the new μ–p interaction arises at loop order.

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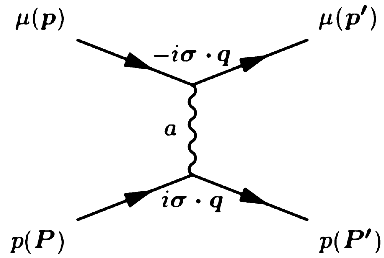

Fig. 1. Pseudoscalar exchange in muonic hydrogen.

$$\mathcal{M} = -\frac{f_\mu f_p}{4m_\mu m_p}\ \chi_\mu'^\dagger \boldsymbol{\sigma}_\mu\cdot\boldsymbol{q}\chi_\mu\ \chi_p'^\dagger\boldsymbol{\sigma}_p\cdot\boldsymbol{q}\chi_p\ \frac{1}{\boldsymbol{q}^2+m_a^2}$$
$$= -\frac{f_\mu f_p}{4m_\mu m_p}\frac{1}{3}\boldsymbol{q}^2\ \chi_\mu'^\dagger\boldsymbol{\sigma}_\mu\chi_\mu\ \cdot\ \chi_p'^\dagger\boldsymbol{\sigma}_p\chi_p\ \frac{1}{\boldsymbol{q}^2+m_a^2}\ ,$$

with the relative angle averaged for the s wave. The effective Hamiltonian is

$$\delta H_a = \frac{1}{3}\frac{f_\mu f_p}{4m_\mu m_p}\left[\delta^3(\boldsymbol{r}) - \frac{m_a^2 e^{-m_a r}}{4\pi r}\right]\boldsymbol{\sigma}_\mu\cdot\boldsymbol{\sigma}_p\ ,$$

so that

$$\delta E_a = \frac{f_\mu f_p}{3m_\mu m_p}\left[|\psi(0)|^2 - m_a^2\int|\psi(\boldsymbol{r})|^2\frac{e^{-m_a r}}{4\pi r}d^3\boldsymbol{r}\right],$$

where ψ is the wave function of the 2S state:

$$\psi(\boldsymbol{r}) = \frac{1}{2\sqrt{2\pi a_B^3}}\left(1-\frac{r}{2a_B}\right)e^{-\frac{r}{2a_B}}\ .$$

Here, $a_B = \frac{1}{\alpha m_r}$ is the Bohr radius for muonic hydrogen with $m_r = m_\mu m_p/(m_\mu + m_p)$, the reduced mass of the system. On convolving, we obtain

$$\delta E_a = \frac{f_\mu f_p \alpha^3 m_r^3}{3m_\mu m_p}\frac{1}{8\pi}F\left(\frac{m_a}{m_r}\right)\ , \tag{5}$$

where

$$F(x) = 1 - x^2\frac{\alpha^2+2x^2}{2(\alpha+x)^4}\ . \tag{6}$$

It is important to distinguish between m_r and m_μ in the equations above. The m_r dependence comes from the Bohr radius a_B, and m_μ from the NR reduction. The function $F(x)$ interpolates between 1 and 0 for $x = 0$ and $x \to \infty$ which is consistent with decoupling behavior.

In Fig. 2, we show the 2σ allowed values of $f_\mu f_p$ as a function of m_a. The region between the solid curves is allowed. We restrict $m_a \le 100$ MeV so that $f_\mu f_p$ remains comfortably in the perturbative regime.

Note that in the potential model of the proton with nonrelativistic quarks, the proton pseudoscalar coupling f_p arises from the pseudoscalar couplings f_u, f_d of the up and down quarks, which are of the same order of magnitude. In this simplified picture, we have $f_p = \frac{4}{3}f_d - \frac{1}{3}f_u$ (as for the magnetic moments). If $f_u = f_d$, we have the simple result, $f_p = f_u = f_d$.

In principle, the anomalous magnetic moment of the muon places a stringent independent constraint on f_μ since for the m_a of interest, pseudoscalar couplings yield a negative contribution to a_μ [7,8],[2] while the measured value is *higher* than the standard model expectation: $\Delta a_\mu = a_\mu^{exp} - a_\mu^{th} = (29\pm 9)\times 10^{-10}$ [9]. However, the scalar sector may be more intricate than envisioned here, and may offer a fine-tuned (and perhaps unnatural) cancellation of the pseudoscalar contribution.

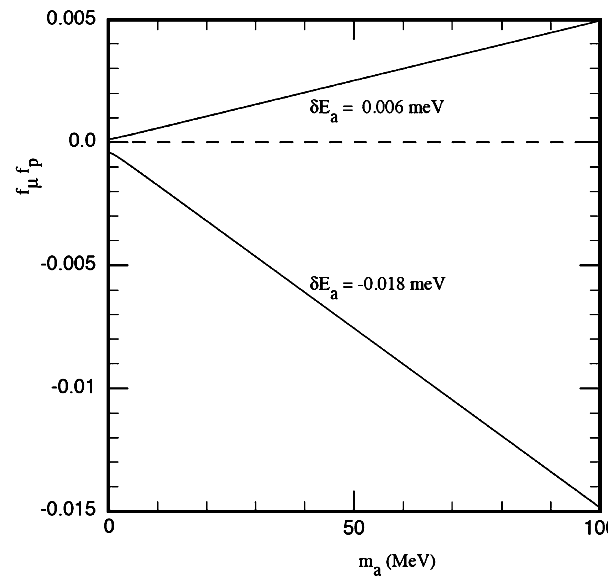

Fig. 2. The values of $f_\mu f_p$ allowed at 2σ lie between the solid curves.

For the sake of comparison, the QED contribution at leading order is

$$\delta H_{\rm QED} = \frac{e^2}{6}\frac{g_\mu g_p}{4m_\mu m_p}\ \delta^3(\boldsymbol{r})\ \boldsymbol{\sigma}_\mu\cdot\boldsymbol{\sigma}_p\ .$$

Here, g_μ (≈ 2), and g_p (≈ 5.5857) are the gyromagnetic ratios for the muon and proton. Correspondingly,

$$\delta E_{\rm QED} = \frac{\alpha^4 m_r^3}{12m_\mu m_p}g_\mu g_p\ .$$

The above QED result, though simple, represents the first three significant digits of the dedicated theoretical calculation, and is consistent with the recent measurement of Ref. [1].

The ratio of the pseudoscalar contribution to the leading QED contribution is

$$\frac{\delta E_a}{\delta E_{\rm QED}} = \frac{2}{4\pi\alpha}\frac{f_\mu f_p}{g_\mu g_p}F\left(\frac{m_a}{m_r}\right)$$
$$= \frac{2}{4\pi\alpha}\frac{f_\mu f_p}{g_\mu g_p}\left[1 - \frac{m_a^2}{m_r^2}\frac{\alpha^2+2(m_a/m_r)^2}{2(\alpha+m_a/m_r)^4}\right].$$

In sum, the 2S hyperfine splitting in muonic hydrogen constrains the product of the pseudoscalar couplings of the muon and proton $f_\mu f_p$ to lie in the 2σ ranges $[-0.00040, 0.00013]$, $[-0.00173, 0.00058]$ and $[-0.015, 0.005]$ for $m_a = 0$, 10 MeV and 100 MeV, respectively. As the pseudoscalar mass is further increased, the constraint is weakened. The couplings have no impact on the discrepant measurements of the proton radius.

For $m_a < 100$ MeV, no direct limits on $f_\mu f_p$ exist from colliders, although limits for higher m_a were obtained by the CMS experiment using the dimuon channel in pp collisions [10]. The CMS upper limits on the cross section times branching fraction, $\sigma\cdot B(pp\to a\to\mu^+\mu^-)$, directly constrain $f_\mu f_p$ in the mass ranges, 5.5–8.8 GeV and 11.5–14 GeV. CLEO's nonobservation of the decay $J/\psi\to\gamma a$ with a invisible [11] gives the 90% C.L. constraint $|f_p| < 0.029$ (assuming the J/ψ–a coupling to be f_p) for $m_a < 100$ MeV [4].

[2] In Eq. (11) of Ref. [7], C_P^2 should be replaced by $|C_P|^2$, since C_P, as defined in Eq. (9) therein, is complex.

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