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Dyscalculia: What is its prevalence? Research evidence from case studies

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Abstract

This paper presents an examination of two case studies conducted by the author who has, over the past two decades conducted numerous such studies. One, whom we will call Len, is typical of a person with a "learned difficulty'' in mathematics and as such is representative of the vast majority of clients diagnosed by the author. The other, whom we will call Sophie, is the most severe example of a very small group of clients who, the author believes, suffers from dyscalculia. The author contends that the prevalence of dyscalculia is much lower than that reported in some of the literature (Geary, 2000; Munro, 200). Wadlington & Wadlington, (2008) for example claim it is the most common type of learning difficulty in mathematics with an incidence of up to 8%. The author contends that these figures generally include a significant proportion of pupils with what are better called "learned difficulties" and that the incidence of dyscalculia, a permanent neurological disorder, is less than 2%. The author recommends that further research is clearly needed to determine the incidence of genuine dyscalculia, as opposed to learned difficulties, in the population. Furthermore, much more research is needed to determine whether or not or to what extent suffers can benefit from regular instruction.

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1. Introduction

Learning difficulties in mathematics have been a focus of attention in mathematics education and much research has been carried out over the last several decades. During this time the focus has shifted from "the slow learner in mathematics" (see, for example, Schulz, 1972) to current research on "dyscalculia" (Munro, 2003; Wadlington & Wadlington, 2008; Westwood, 2008). The author is a lecturer in Mathematics Education at Queensland University of Technology (QUT) and has lectured to both graduate and undergraduate students in the diagnosis and remediation of learning difficulties in mathematics over the last 20 years. As such the author is aware of the importance of accurate diagnosis and identification of the causes and type of learning difficulty a pupil is experiencing if any remediation is to be successful. The author recently became concerned with reports in this literature relating to the prevalence of dyscalculia. This prompted the author to present this paper which analyses data from two case studies as evidence that these figures generally include a significant proportion of pupils with what are better called

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"learned difficulties" and that the true incidence of genuine dyscalculia most likely considerably lower, perhaps 1-2%.

The author believes that there is danger that incorrect diagnosis and classification can lead to "learned helplessness" in which the learner believes that the difficulties are too serious as to benefit from remediation.
This study was conducted to achieve the following objectives;

1. To illustrate the difference between dyscalculia and a learned disability in mathematics
2. To estimate the prevalence of dyscalculia in the population

Types of Learning Difficulties

Learning difficulties in mathematics generally fall into 3 categories:

1. *The slow learner*. An individual classified as a slow learner is, in a word, consistent. Performance in all academic and in many non-academic areas is slow and achievement is generally below average. The slow learner thus defined has a mental age below that of the chronological age for the grade equivalent. While many pupils who experience difficulties in mathematics belong to this category, the fact that a pupil is not achieving satisfactorily does **not** imply that he/she is in this category. The slow learner needs to be in a special program or simply given more time to learn in a regular environment. The slow learner is not necessarily in need of extensive diagnosis nor requiring remediation but needs to be taught using strategies and materials suited to his/her learning style and is generally capable of learning given adequate time, the right learning environment and instruction in a suitable curriculum. Without these, however, the problems are compounded (Peard, 1997). In research up to about the mid seventies this term was often used to describe all those with learning difficulties in mathematics (See, for example, Pikaart & Wilson, 1972). More recently attention has been focused on other causes of difficulties (Geary, 2000; Tall & Razali, 1993; Peard, 1995).

2. *Learned disability in mathematics*. This individual is generally inconsistent showing specific weaknesses and often some strengths in mathematics with average or above average achievement in other subjects. These difficulties may range from mild to severe. The nature and causes of these disabilities are many and often complex but are generally extrinsic to the learner. "Learned disabilities" include misconceptions, mental blocks, and attitudes that require careful diagnosis to discover. The vast majority of pupils who are experiencing problems with school mathematics fall into this category (Peard, 1995). Pupils with learned disabilities are not all alike. Each has a unique set of strengths and weaknesses and each shares in the universal, though highly variable, attributes, concerns and needs of other human beings. Unfortunately, much literature on learning difficulties fail to distinguish between these two categories and groups both categories together. Both the slow learner and the learned disabled may exhibit very similar types of performance in mathematics. The primary school teacher who teaches the pupil for several subjects is in a better position to distinguish between the two categories than the secondary mathematics teacher who sees only the pupil's mathematical performance. Both categories are often simply labelled as "failures". There is also a large number of pupils who are "under achieving" in mathematics. The causes of this under achieving may be similar to the causes of learned disabilities but the effects not as pronounced. The pupil is "able to cope" and often compensates for low achievement in mathematics with high achievement in other subjects (Peard, 1995).

3. *Learning disability in mathematics. Dyscalculia* is specific learning disability in mathematics which is generally believed to be a permanent neurological condition that affects the processing of

numerical and quantitative concepts (Munro, 2003). It results in severe learning difficulties in mathematics. Unfortunately, there is no general agreement on the precise meaning of this term. Levy, Reis & Grafman, (1999) note that researchers have yet to understand the causes of dyscalculia and that the neurological cause may be only one of several. According to the dyscalculia forum "Dyscalculia is a word you use to describe when people have significant problems with numbers - but still have a normal or above normal IQ. It seems that no dyscalculic has problems with math alone, but also struggle with problems being able to learn to tell time, left/right orientation, rules in games and much more". They note that dyscalculia is a "vital but unresearched field" (dyscalculiaforum.com). One identifying characteristic is a more fundamental inability to conceptualize numbers as abstract concepts of comparative quantities and specific processing difficulties of mathematical concepts. These may have resulted from organic brain dysfunction (Dehaene, 1997) or other neurological impairments (Levy, Reis & Grafman, 1999; Munro, 2003). Although the difficulties these people experience are not directly caused by an inappropriate learning environment, if left in such an environment the difficulties compound. Munro (2003) defines acquired dyscalculia as "low performance due to cerebral trauma" and developmental dyscalculia as "mathematical learning difficulties with similar features but without evidence of cerebral trauma" (p. 25). Munro states that "underachievement due to developmental dyscalculia has a neuropsychological foundation … lacking particular information or processing strategies in mathematics… but can learn in (other) contexts" (p.25). Adams and Hitch (1997) argue that working memory is a major factor and Geary (1993) suggested there was a working memory deficit for those who suffered from dyscalculia. However, as shown in this case study of Len, working memory problems are confounded with general learning difficulties. It is the opinion of the author that Geary and other researchers are including characteristics that are not specific to dyscalculia but are representative of other learning difficulties and thus greatly over estimating the occurrence of dyscalculia.

Not all educators agree on this classification and some more recent researchers break down these three categories into further subdivisions (Geary, 2000). On the other hand earlier researchers such as Schulz (1972) group all as "slow learners" noting that "there is little evidence that human beings can be categorised with any degree of precision" (Schulz, 1972, p. 1). For the purpose of this paper no further subdivisions are employed.

2. Methodology

Diagnostic assessment is data gathering in which the teacher or researcher tries to gather as much useful data about the pupil or client as is possible in the time available. In this it is important to determine what the client knows and can do as well as well as what is not known and cannot be done. While the most common form is the written diagnostic test, either commercial or constructed, this type of data gathering is not appropriate for people with severe difficulties as they often have experienced failure with written tests and tend either not to respond or to respond in a limited manner. Consequently, limited and inaccurate data are obtained. The diagnosis in this study proceeded via the structured individual interview. This methodology has been used in studying mathematical knowledge for some time now (See, Romberg and Uprichard, 1977). In this a predetermined set of questions is asked. Written and verbal responses to these questions are then recorded and written and verbal protocols are formulated for analysis. This study proceeded in this manner using a *structured clinical interview* in which digressions from the predetermined questions were permitted and encouraged. The major research instrument then is the set of interview questions and questions arising from digressions according to responses. Questions and responses were recorded on audio for later analysis. No standardised tests or other such measurement instruments were employed.

Prior to the predetermined mathematical questions, initial questions relating to the background and interests of the client are asked, sometimes described as a "sweep". Questions are then selected from the set to "probe" more deeply into the client's understanding. Determining the client's interests allows questions to be put in contexts that will be better understood. Thus any difficulties identified will be conceptual and not the result of unfamiliar context. This "sweep" should also put the client at ease and provide the starting point for the more formal predetermined questions which will "probe" into those areas of difficulty that have been identified in the "sweep". The diagnosis should determine the client's position on the hierarchy of mathematical skills. It is of little value to simply determine what difficulties the client is experiencing at a particular point in time. The difficulty with a written test is that to do this, too many items are needed. Thus it is often better to first administer a general "overview" or "sweep" and then select subsequent specific predetermine follow-up items to "probe" specific difficulties. The author has conducted many such diagnostic interviews and has also trained both graduate and undergraduate students in the conduction of such interviews in university Units at QUT.

3. Findings

The findings of this study were mainly based on the qualitative data gathered from the two clients using a structured clinical interview.

3.1 *Background to the clients: The "sweep"*

Len

Len came to the author as a private client seeking help with mathematics. Len was in his early twenties, single. He had left school after completing Grade 10 and had worked in mostly sales and clerical type jobs. He had begun a TAFE course in business and was experiencing difficulties with the basic mathematics. He recognized that he was in need of remedial help in mathematics. The author agreed to perform a clinical diagnosis and provide a written report that he could use for remediation on the understanding that the data gathered could be used for this research.

Len had done Year 10 mathematics but said he could not remember much about it and did not "do too well". He said he had been an "average" student passing all other subjects. Since leaving school his interests were fairly typical of young men. He surfed, played rugby and followed most sports.

Sophie

Sophie was referred to the author by SPELD (Specific Learning Disabilities Association, Queensland), Brisbane. She was in her early forties, married with two children. She had successfully completed Grade 12 at school though had not done any mathematics since year 10 and reported that she had never passed any mathematics courses at school. Sophie worked in her husband's real estate business doing mostly clerical work. She was musical playing both the violin and piano, widely read, and led an apparently normal and happy family life. She was however extremely frustrated by her inability in mathematics; an inability she reported having had all her life. Sophie reported being constantly frustrated with mathematics at school. Neither she nor her teachers could understand why she achieved highly in most other subjects, but could not make any achievement at all in mathematics. In her post school life she reported that she avoided doing any computations or quantitative work.

She reported that she can't easily read a traditional analogue clock, and consequently always arrives at meetings sometime early for fear of being late. When it comes to paying in shops or restaurants, she either hands over large bills and collects the change or gives her wallet to a friend and asks them to make the payment. She said that she could not remember ATM PIN numbers or numerical passwords for which she always kept a written record. She approached SPELD in order to determine the causes of this after reading some of their publications. The author agreed to perform a clinical diagnosis and provide a written report on the understanding that the data gathered could be used for this research. Sophie later (post diagnosis) published a book outlining her frustrations.

3.2 *Responses to selected questions*

The following is a summary of some of the key item responses given by each client to the questions asked in the structured clinical interview. It is not the intention of this paper to present a complete profile of each (though this was done for the clients). Rather the objective is to illustrate the different *types* of learning difficulties illustrated by the responses.

TQ: Test question from predetermined set
D: Digression
L: Len
S: Sophie

Topic: Conceptual understanding of operations and Number fact recall

TQ: How would you determine the cost of 3 CDs if each one costs $9?
Len

L: ..9… 18.. nine more… 27, $27
D: What if there were 8 CDs?
L: Oh, 9 times 8 would be?? I'm not sure. I'd use the calculator.
D: Could you make a guess? How much would it be less than?
L: Less than $90
D: Less than $80?
L: Yes, less than $80

Sophie

S: I don't know
D: If they were $10 each?
S: $30
D: How did you get that?
S: 3 x 10 would be 30
D: So at $9 each how much less?
S: a bit
D: What if there were 8 CDs?
S: A lot more
D: If you wanted to buy these, what would you do? Could you make a guess? How much would it be less than?
S: Less than $100. I'd ask how much or just hand over $100 and wait for the change
D: Less than $80?
S: Maybe

Topic: Volume
Len
TQ: A swimming pool is 10m long, 3m wide and 2m deep. Can you tell me what volume of water it holds?

L: Um, …no, there's a formula for volume but I can't remember what it is.
TQ: Could you find the area of this rectangle (shows diagram) 10m x 3m?
L: Yeah. I think that would be 30? Right?
D: (taking 3 long MAB) If I put these like this how many do I have?
L: Yeah, 30 like I said.
D: Now if I put a second 10 x 3 layer on top, how many do I have?
L: 60
D: So if the blocks were each cubic metres, how many cubic metres are there?
L: 60
D: and the measurements of the pool?
L: Yeah (pointing) 10, 3, 2
This procedure was then repeated with another example.
D: So in general, could you use the same process to find the volume of any pool this shape?
L: Sure
RP: Which is?
L: Multiple them.

Sophie

S: No. No idea
TQ: Could you find the area of a rectangle (drawing) 10m x 3m?
S: Not really
D: (taking 3 long MAB) If I put these like this how many do I have?
S: (counting) Thirty ?
D: Now if I put a second 10 x 3 layer on top, how many do I have?
S: (shaking head) Don't know

3.3 *Diagnostic Profiles: a comparison of selected topics*

Topic: Whole number place value

Len had a reasonably good understanding

Sophie had some reasonable number and place value concepts up to three digits. She can count forwards by 1s, 2s, 5s, and 10s, but counting in backwards was inconsistent. Beyond 3 digits her place value concepts were limited and she could not group in threes to read multi-digits correctly.

Topic: Decimal place value

Len had a good understanding of tenths and hundredths in situations of money and measurement but limited beyond hundredths.

Sophie showed some knowledge of tenths and hundredths in situations involving money, but was limited to this and lacking in other situations such as measurement.

Topic: Operation concepts and number facts

Len could identify addition, subtraction, multiplication and division in most contexts. However he had limited number fact recall and strategies, though he did have reasonable estimation strategies. He could do some multi-digit algorithms using school taught procedures but was limited by number fact recall and showed some error patterns in use.

Sophie on the other hand had very little understanding of the operational concepts. She could identify addition, subtraction in only limited contexts; while she could recognize when to add and subtract she did so inconsistently. Multiplication is viewed as repeated addition and recognised in very few situations. She has no real understanding of the division concept. She had very few number facts and very limited strategies. She could add and subtract by counting forward and back, recognized some doubles, some 5x by counting in fives, and 10x of single digits. She showed no knowledge of estimation strategies and did not attempt any multi-digit operations. She had some number fact skills for addition and subtraction although she relied largely on the "count on" strategy and did not appear to use other strategies such as "near 10", "double" etc. Her number fact skills for multiplication and division are limited due to her lack of conceptual understanding of the operations.

Topic: Fraction concepts and operations

Len: Identified simple fractions, understands concept of part of whole, operations limited to simple fraction, concept of equivalence deficient.

Sophie: Limited to the concept of part of whole. Unable to apply in situations such as "how many ½ cups in 3 cups?", or to recognize ½ way between 1 and 2, or to order even simple fractions.

Topic: Measurement

Len: Reasonable skills with distance, quantity and time. Limited by decimal place value beyond hundredths.

Sophie: Limited by deficient decimal and fraction concepts.

Topic: Space and shape

Len: Adequate visualization skills, area and volume concepts.

Sophie: Sophie showed very good visualization skills; well above average. She was able to recognize reflected and rotated shapes, describe shapes and visualize projections of simple 3D shapes into 2D, missing jig-saw pieces and other spatial and visual concepts. In all of this she was well above average. This is consistent with reports in the literature for some suffers of dyscalculia. However, abilities in geometric situations involving computations such as area and volume were very limited.

3.4 *Recommendations for remediation*

Sophie had very limited strengths on which to build any remediation program. When asked if she would like to attempt a remedial program with the author she declined stating that she was worried that it might result in anxiety and that she was coping adequately mainly by avoidance. She preferred to continue using her coping strategies including avoidance. Whether or not she would benefit from a regular program of remediation is contentious and in need of further research. Sophie would appear to be representative of the type of learner described in literature as having developmental dyscalculia as an underachiever due to a neuropsychological foundation who is lacking particular information or processing strategies in mathematics, but who can learn and achieve in other contexts (Munro, 2003).

Len on the other hand had clearly identifiable strengths and specific areas of weakness. These weaknesses were a result, at least in part, of poor pedagogy in which he attempted to rely heavily on memory without any conceptual understanding. When his memory fails he has no conceptual base on which to fall back. For example, at school he had been "taught" the formulae for volume without any reasoning or development. Consequently, he forgets or confuses formulae. Data from the interview show that he is quite capable of learning and he was pleased that these strengths and weaknesses were identified and was anxious to begin a remedial program so that he could continue with his studies.

4. Conclusion

Len is typical of the many people who experience learned difficulties in mathematics and is capable of learning given appropriate experiences and pedagogy. Thus, these difficulties, though severe may be temporary. Estimates of the proportion of pupils and people exhibiting such difficulties vary. Jordan (2010) claims that up to 10% of students are diagnosed with a learning disability in mathematics at some point in their school careers and that many more learners struggle in mathematics without a formal diagnosis, and that the total could be much higher. This is consistent with estimates of the author of up to 20% (Peard, 1995).

Sophie is an example of the most severe case of a very small group of clients who, the author believes, suffers from a genuine disability, dyscalculia. Whether or not she would be capable of benefiting from any program of remediation is contentious due to both her lack of motivation and the permanent neurological nature of the disability, (Munro, 2003). In the experience of the author only a small proportion, perhaps a few percent *of those from all categories of difficulties* are in this group. Even if we accept the higher overall figure of 20% of the population with some form of difficulty and that a maximum of 10% of these suffer from permanent neurological dyscalculia, this figure will be less than 2 %. This is in contrast with what is reported in much of the literature.

In referring to their Inaugural National Conference 2009, the dyscalculia forum stated, without support or reference, "4-6% of the world's population are dyscalculic". They later report it "affecting about 5 per cent of people - roughly the same proportion as are dyslexic". (http://www.dyscalculiaforum.com/news.php). They later reported "studies done by Gross-Tsur, Manor and Shalev in 1996, claim 6.5% are dyscalculic". And that "Lewis, Hitch and Walker in

1994, … 3.6% of the world's population are dyscalculic". Wadlington & Waddlington (2008) claim it is the most common type of learning difficulty in mathematics with an incidence of up to 8%. Munro (2003) cites several studies claiming 3-6.5% of the population suffering from developmental dyscalculia. Westwood (2008) estimates about 3% and this would be more in line with (though still above) the conclusion of the present study which is that these figures includes a significant proportion of people such as Len with learned disabilities.

The author recommends that further research is clearly needed to determine the incidence of genuine dyscalculia, as opposed to learned difficulties, in the population. Furthermore, much more research is needed to determine whether or not or to what extent suffers can benefit from regular instruction.

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