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Throughflow and non-uniform heating effects on double diffusive oscillatory convection in a porous medium

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Abstract A weak nonlinear oscillatory mode of thermal instability is investigated while deriving a non autonomous complex Ginzburg–Landau equation. Darcy porous medium is considered in the presence of vertical throughflow and time periodic thermal boundaries. Only infinitesimal disturbances are considered. The disturbances in velocity, temperature and solutal fields are treated by a perturbation expansion in powers of amplitude of applied temperature field. The effect of throughflow has either to stabilize or to destabilize the system for stress free and isothermal boundary conditions. Nusselt and Sherwood numbers are obtained numerically and presented the results on heat and mass transfer. It is found that, throughflow and thermal modulation can be used alternatively to control the heat and mass transfer. Further, it is also found that oscillatory flow enhances the heat and mass transfer than stationary flow. Effect of modulation frequency and phase angle on mean Nusselt number is also discussed.

1. Introduction

The problem of thermal instability in a porous medium is well documented by Vafai [1–6]. The study of thermal instability in porous media is an important concept in thermal and engineering sciences, geothermal energy utilization, oil reservoir modeling, building of thermal insulations, and nuclear waste disposals to mention a few. There is a growing interest in externally modulated hydrodynamic systems, both theoretically and experimentally. These systems show a novel behavior in response to parametric forcing near a point of instability. Depending on the relative strength and rate of forcing, predictions exist for a variety of responses to the modulation. Davis [7] pointed that, the dynamic of stabilization and destabilization may lead to dramatic changes of behavior depending on the proper tuning of the amplitude and frequency of the modulation. If an imposed modulation can destabilize an otherwise stable state, then there is a major enhancement of heat/mass/momentum transport. If an imposed modulation can stabilize or otherwise in unstable state, then higher efficiencies can be attained in various processing techniques. The convective

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phenomenon can be controlled by applying a time periodic modulation to the driving force provided by some mechanical source, rotation, magnetic field, buoyancy, temperature-gradient. One of the effective mechanisms that considered in this paper is a time periodic heating of wall temperature at the boundaries, where in many practical applications the steady state temperature field is a function of both space and time. This nonuniform temperature gradient can be used as a mechanism to stabilize or destabilize the convective flow. The related problems are investigated by [8–18] for various physical models and boundary conditions.

Convection concerns the process of combined heat and mass transfer which are driven by buoyancy forces are usually referred as double diffusive convection. In this case the mass friction gradient and the temperature gradient are independent. In some practical problems, such as seawater flow, mantle flow in the earth's crust, in devicing an effective method [19] of disposing waste material and extraction of energy and engineering applications the double diffusive convection plays an important role. The linear and nonlinear stability of double diffusive convection in porous media has been studied extensively in the presence of uniform temperature and concentration gradients [6], and [20]. Siddheshwar et al. [21] investigated temperature and gravity modulation effects on double diffusive convection in porous medium. They found that, both modulations can be used simultaneously to enhance or diminish heat and mass transfer in the system while considering a weakly non-linear theory for stationary mode. Bhadauria [22] also analyzed the additional effects of internal heating and anisotropy. He found that, internal heat and anisotropy also can be used to enhance or diminish heat and mass transfer in the system. Malashetty et al. [23] studied the effect of rotation on double diffusive convection while considering linear theory for onset of convection and nonlinear theory for finite amplitude convection. Kiran and Bhadauria [24] investigated double diffusive magnetoconvection under the effects of gravity modulation and chaotic, oscillatory mode of convection. They found that, gravity modulation can be used to control thermal instability and dynamic of the problem with suitable ranges of modulation parameters.

Throughflow effect on double-diffusive convection in a porous medium is important concept due to its applications in engineering, geophysics and seabed hydrodynamics. Throughflow plays an important role in the directional solidification of concentrated alloys, in which mushy zone exists and it is regarded as a porous layer with double diffusive origin. The basic state temperature profile of throughflow changes from linear to nonlinear with layer height, which in turn affects the stability of the system significantly. The effect of throughflow on the onset of convection in a horizontal porous medium has also been given in [25–28]. Nield [29] and Shivakumara [30] have shown that a small amount of throughflow can have a destabilizing effect if the boundaries are of different types and a physical explanation for the same has been given. They also found that, the effect of throughflow is not invariably stabilizing and depends on the nature of the boundaries. Khalili and Shivakumara [31] have investigated the effect of throughflow and internal heat generation on the onset of convection in a porous medium. They have shown that throughflow destabilizes the system, even if the boundaries are of the same type, a result which is not true in the absence of an internal heat source. The non-Darcian effects on convective instability in a porous medium with throughflow have been investigated in order to account for inertia and boundary effects by Shivakumara [32]. The effect of throughflow on the stability of double diffusive convection in a porous layer is investigated by Shivakumara and Khalili [33] for different types of hydrodynamic boundary conditions. They found that throughflow is destabilizing even if the lower and upper boundaries are of the same type and stabilizing as well as destabilizing, irrespective of its direction, when the boundaries are of different types. Khalili and Shivakumara [34] investigated throughflow in the porous layer is governed by Darcy–Forchheimer equation and the Beavers–Joseph condition is applied at the interface of fluid and the porous layer. They found that destabilization arises due to throughflow, and the ratio of fluid layer thickness to porous layer thickness, plays an important role in deciding the stability of the system depending on the Prandtl number. Hill [35] investigated linear and nonlinear thermal instability of vertical throughflow in a fluid-saturated porous layer, while Hill et al. [36] have extended the problem for penetrative convection by considering density is quadratic in temperature. Brevdo and Ruderman [37,38] have analyzed convective instability in a porous medium with inclined temperature gradient and vertical throughflow. Later on many researchers have investigated throughflow effects considering different physical models, some of them are given in [39–46].

From the literature no study has been found which considers modulation along with vertical throughflow for nonlinear mode of thermal instability. Throughflow has been investigated for various boundary conditions with linear stability analysis. It is to be noted that, for understanding heat and mass transfer in the system one must study the interaction of streamline flow with temperature, and solutal concentrations through nonlinear analysis. The objective of the present article was, therefore, to investigate weakly nonlinear stability characteristics of a porous layer with simultaneous temperature and solute concentration gradients for constant vertical throughflow. Analytic expressions for both Nusselt and Sherwood numbers were derived from the complex non-autonomous Ginzburg–Landau equation [15,16,47–49] to calculate finite amplitude.

2. Mathematical formulation

An infinitely extended horizontal binary fluid saturated porous medium of depth d has been considered. The porous layer is homogeneous and isotropic and it is heated and salted from below. The physical configuration of the problem is given in Fig. 1. Using the modified Darcy's law and employing the Boussinesq approximation for density variations, the governing equations of the present problem are given by Bhadauria [22] for isotropic porous medium:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial \vec{q}}{\partial t} = -\nabla p + \rho \vec{g} - \frac{\mu}{K}\vec{q}, \tag{2}$$

$$\gamma \frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T = \kappa_T \nabla^2 T, \tag{3}$$

$$\frac{\partial S}{\partial t}+(\vec{q}\cdot\nabla)S=\kappa_S\nabla^2 S, \quad (4)$$

$$\rho=\rho_0[1-\alpha_T(T-T_0)+\beta_S(S-S_0)], \quad (5)$$

where $\vec{q}$ is the fluid velocity, $\vec{g}$ is acceleration due to gravity, ρ is density, ε porosity of the porous medium, K is the permeability of porous material, κ_T and κ_S are the effective thermal, solutal diffusivity in vertical direction, α_T is the coefficient of thermal expansion, β_S is the coefficient of solute expansion and γ is the heat capacity ration taken to be 1 for simplicity. The externally imposed thermal and solutal boundary conditions are given by Venezian [8]

$$\begin{aligned}T&=T_0+\frac{\Delta T}{2}[1+\chi^2\delta\ \cos(\Omega t)]\ \text{at}\ z=0\\&=T_0-\frac{\Delta T}{2}[1-\chi^2\delta\cos(\Omega t+\phi)]\quad\text{at}\ z=d\end{aligned} \quad (6)$$

$$\begin{aligned}S&=S_0+\Delta S\quad\text{at}\ z=0\\&=S_0\quad\text{at}\ z=d,\end{aligned} \quad (7)$$

where ΔT and ΔS are the temperature and solute difference across the porous medium, χ is the smallness of amplitude of modulation, ϕ is the phase angle, δ, and Ω are amplitude and frequency of temperature modulation.

3. Conduction state temperature and solutal fields

The basic state is assumed to be quiescent and the quantities in this state are given by

$$\begin{aligned}&q_b=(0,0,w_0),\ \rho=\rho_b(z,t),\ p=p_b(z,t),\\&T=T_b(z,t)\ \text{and}\ S=S_b(z,t).\end{aligned} \quad (8)$$

Substituting Eq. (8) into Eqs. (1)–(5), we can obtain the following relation which helps us to define hydrostatic pressure, temperature and solutal fields:

$$\frac{\partial p_b}{\partial z}=\frac{\mu}{K}w_0-\rho_b g, \quad (9)$$

$$w_0\frac{\partial T_b}{\partial z}=\kappa_T\frac{\partial^2 T_b}{\partial z^2}, \quad (10)$$

$$w_0\frac{\partial S_b}{\partial z}=\kappa_S\frac{\partial^2 S_b}{\partial z^2}, \quad (11)$$

$$\rho_b=\rho_0[1-\alpha_T(T_b-T_0)+\beta_S(S_b-S_0)]. \quad (12)$$

The solution of Eqs. (10) and (11) subject to the boundary conditions Eqs. (6) and (7), is given by

$$T_b(z,t)=f(z)+\epsilon^2\delta\mathrm{Re}[f_1(z,t)], \quad (13)$$

$$S_b=S_0+\Delta S\frac{e^{(Pe\Gamma^{-1})z}-e^{(Pe\Gamma^{-1})}}{1-e^{(Pe\Gamma^{-1})}}. \quad (14)$$

Here $f(z)$ is the steady part, while $f_1(z,t)$ is the oscillatory part of the basic temperature field which will be defined in the next section, where $Pe=\frac{w_0 d^2}{\kappa_T}$ is the Péclet number.

4. Dimensionless governing equations

The finite amplitude perturbations on the basic state are superposed in the form,

$$\begin{aligned}&\vec{q}=\vec{q}_b+\vec{q}\,',\ \rho=\rho_b+\rho',\ p=p_b+p',\ T=T_b+T'\ S\\&=S_b+S'.\end{aligned} \quad (15)$$

Now let us introduce the Eqs. (13)–(15) into the system of Eqs. (1)–(5), and then using the stream function ψ as $u'=\frac{\partial\psi}{\partial z},\ w'=-\frac{\partial\psi}{\partial x}$, for two dimensional flow, the equations are then non-dimensionalized using the physical variables: $(x,y,z)=d(x^*,y^*,z^*),\ t=\frac{d^2}{\kappa_T}t^*,\ \psi=\kappa_T\psi^*,\ T'=\Delta T\ T^*,\ S'=\Delta S\ S^*$ and $\Omega=\frac{\kappa_T}{d^2}\Omega^*$. The resulting non-dimensionalized system of equations can be expressed as (dropping the asterisk)

$$\frac{1}{Pr_D}\frac{\partial\psi}{\partial t}+\nabla^2\psi=Rs\frac{\partial S}{\partial x}-Ra\frac{\partial T}{\partial x}, \quad (16)$$

$$-\frac{\partial T_b}{\partial z}\frac{\partial\psi}{\partial x}-\left(\nabla^2-Pe\frac{\partial}{\partial z}\right)T=-\frac{\partial T}{\partial t}+\frac{\partial(\psi,T)}{\partial(x,z)}. \quad (17)$$

$$-\frac{dS_b}{dz}\frac{\partial\psi}{\partial x}-\left(\Gamma\nabla^2-Pe\Gamma^{-1}\frac{\partial}{\partial z}\right)S=-\frac{\partial S}{\partial t}+\frac{\partial(\psi,S)}{\partial(x,z)}, \quad (18)$$

The non-dimensionalizing parameters in the above equations are as follows: $Pr_D=\frac{\varepsilon\nu d^2}{K\kappa_T}$ is the Prandtl Darcy number, $Ra=\frac{\beta_T g\Delta T d K}{\nu\kappa_T}$ is thermal Rayleigh number, $Rs=\frac{\beta_S g\Delta S d K}{\nu\kappa_S}$ is the solutal Rayleigh number, $\Gamma=\frac{\kappa_S}{\kappa_T}$ is diffusivity ratio and $\nu=\frac{\mu}{\rho_0}$ is kinematic viscosity. It is clear from Eqs. (17) and (18) that, throughflow and basic state profile of temperature, and solutal fields affect the stability problem. The above system will be solved by considering stress free and isothermal boundary conditions as given below (Bhadauria and Kiran [13,16,21]):

$$\psi=T=S=0\quad\text{at}\ z=0\ z=1. \quad (19)$$

5. Derivation of complex Ginzburg–Landau equation

Introduce a small perturbation parameter χ that shows deviation from the critical state of onset of convection, and the variables for a weak nonlinear state may be expanded as power series of χ as [8,50]

$$\begin{aligned}&Ra=R_0+\chi^2R_2+\chi^4R_4+\dots,\\&\psi=\chi\psi_1+\chi^2\psi_2+\chi^3\psi_3+\dots,\\&T=\chi T_1+\chi^2T_2+\chi^3T_3+\dots,\\&S=\chi S_1+\chi^2S_2+\chi^3S_3+\dots,\end{aligned} \quad (20)$$

where R_0 is the critical value of the Darcy–Rayleigh number at which the onset of convection takes place in the absence of temperature modulation. According to the studies of [15,16,47–49] and Kim et al. [51] the fast time scale of time τ and the slow time scale of time s as $\frac{\partial}{\partial t}=\frac{\partial}{\partial\tau}+\chi^2\frac{\partial}{\partial s}$ were introduced.

5.1. Lowest order system

At lowest order, the nonlinear terms in governing equations vanish therefore, the lowest order problem reduces to the linear stability case for oscillatory mode of convection then arriving at

$$\begin{bmatrix} \nabla^2 & R_0\frac{\partial}{\partial x} & -Rs\frac{\partial}{\partial x} \\ -\frac{dT_b}{dz}\frac{\partial}{\partial x} & \left(\frac{\partial}{\partial \tau}-\nabla^2+Pe\frac{\partial}{\partial z}\right) & 0 \\ -\frac{dS_b}{dz}\frac{\partial}{\partial x} & 0 & \left(\frac{\partial}{\partial \tau}-\Gamma\nabla^2+Pe\Gamma^{-1}\frac{\partial}{\partial z}\right) \end{bmatrix}\begin{bmatrix}\psi_1\\ T_1\\ S_1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} \tag{21}$$

The solution of the lowest order system subject to the boundary conditions given in Eq. (19), is considered to be

$$\psi_1 = \left(\mathbb{A}(s)e^{i\omega\tau}+\overline{\mathbb{A}}(s)e^{-i\omega\tau}\right)\sin ax \sin \pi z, \tag{22}$$

$$T_1 = \left(\mathbb{B}(s)e^{i\omega\tau}+\overline{\mathbb{B}}(s)e^{-i\omega\tau}\right)\cos ax \sin \pi z. \tag{23}$$

$$S_1 = \left(\mathbb{C}(s)e^{i\omega\tau}+\overline{\mathbb{C}}(s)e^{-i\omega\tau}\right)\cos ax \sin \pi z. \tag{24}$$

The undetermined amplitudes are functions of slow time scale, and are related by the following relation:

$$\mathbb{B}(s) = -\frac{4\pi^2 a}{(4\pi^2+Pe^2)(c+i\omega)}\mathbb{A}(s), \tag{25}$$

$$\mathbb{C}(s) = -\frac{4\pi^2 a}{(4\pi^2+(Pe\Gamma^{-1})^2)(\Gamma c+i\omega)}\mathbb{A}(s), \tag{26}$$

where $c = a^2+\pi^2$. The critical Rayleigh number for oscillatory mode of convection is given by

$$R_0 = \frac{(4\pi^2+Pe^2)(c-\omega^2 Pr_D)}{4a^2\pi^2} + \frac{Rs(4\pi^2+Pe^2)(\Gamma c^2+\omega^2)}{(4\pi^2+(Pe\Gamma^{-1})^2)(\Gamma^2c^2+\omega^2)}. \tag{27}$$

The corresponding critical wave number will be calculated while minimizing the critical Rayleigh number with respect to the wave number. The growth rate ω^2 can be defined as

$$\omega^2 = \frac{4\pi^2 a^2 Rs(1-\Gamma)}{(4\pi^2+Pe^2)(1+cPr_D^{-1})} - c^2\Gamma^2. \tag{28}$$

It is to be noted that for existing an oscillatory mode of convection ($\omega > 0$) the values of Pe, Pr_D, diffusivity ratio Γ must be considered to satisfy Eq. (28).

5.2. Second order system

In this case one can observe that, from the energy and solutal equations the Jacobian terms give second order profile on the basis of first order solutions, then the following relations can be obtained:

$$\psi_2 = 0, \tag{29}$$

$$\left(\frac{\partial}{\partial \tau}-\nabla^2\right)T_2 = \frac{\partial(\psi_1, T_1)}{\partial(x,z)}, \tag{30}$$

$$\left(\frac{\partial}{\partial \tau}-\Gamma\nabla^2\right)S_2 = \frac{\partial(\psi_1, S_1)}{\partial(x,z)}. \tag{31}$$

From the above relations, according to the studies of [15,16,47,48] and Kim et al. [51], one can deduce that, the velocity, temperature and solutal fields have the terms having frequency 2ω and independent of fast time scale. Thus, at this order the temperature and solutal expressions may be considered as [15]:

$$T_2 = \{T_{20}+T_{22}e^{2i\omega\tau}+\overline{T}_{22}e^{-2i\omega\tau}\}\sin 2\pi z, \tag{32}$$

$$S_2 = \{S_{20}+S_{22}e^{2i\omega\tau}+\overline{S}_{22}e^{-2i\omega\tau}\}\sin 2\pi z, \tag{33}$$

where (T_{20}, T_{22}) and (S_{20}, S_{22}) are temperature and solutal fields having the terms having the frequency 2ω and independent of fast time scale, respectively. The second order solutions can be defined using T_2, $andS_2$ in Eqs. (30) and (31). The horizontally averaged Nusselt and Sherwood numbers, for the oscillatory mode of convection are given by

$$\mathrm{Nu} = 1+\frac{\left[\frac{a_c}{2\pi}\int_0^{\frac{2\pi}{a_c}}\left(\frac{\partial T_2}{\partial z}\right)dx\right]_{z=0}}{\left[\frac{a_c}{2\pi}\int_0^{\frac{2\pi}{a_c}}\left(\frac{\partial T_b}{\partial z}\right)dx\right]_{z=0}} \tag{34}$$

$$\mathrm{Sh} = 1+\frac{\left[\frac{a_c}{2\pi}\int_0^{\frac{2\pi}{a_c}}\left(\frac{\partial S_2}{\partial z}\right)dx\right]_{z=0}}{\left[\frac{a_c}{2\pi}\int_0^{\frac{2\pi}{a_c}}\left(\frac{\partial S_b}{\partial z}\right)dx\right]_{z=0}}. \tag{35}$$

5.3. Third order system

For third order system the following system is obtained:

$$\begin{bmatrix} \nabla^2 & R_0\frac{\partial}{\partial x} & -Rs\frac{\partial}{\partial x} \\ -\frac{dT_b}{dz}\frac{\partial}{\partial x} & \left(\frac{\partial}{\partial \tau}-\nabla^2+Pe\frac{\partial}{\partial z}\right) & 0 \\ -\frac{dS_b}{dz}\frac{\partial}{\partial x} & 0 & \left(\frac{\partial}{\partial \tau}-\Gamma\nabla^2+Pe\Gamma^{-1}\frac{\partial}{\partial z}\right) \end{bmatrix}\begin{bmatrix}\psi_3\\ T_3\\ S_3\end{bmatrix} = \begin{bmatrix}R_{31}\\R_{32}\\R_{33}\end{bmatrix}, \tag{36}$$

where the expressions for R_{31}, R_{32} and R_{33} are given in the appendix. Now under the solvability condition for the existence of third order solution, the following complex non-autonomous Ginzburg–Landau equation (CGLE) is obtained, which describes the temporal variation of the amplitude $\mathbb{A}(s)$ of the convection cell [15,16,47–49] and Vadász [52]

$$\frac{d\mathbb{A}(s)}{ds} - \gamma_1^{-1}F(s)\mathbb{A}(s) + \gamma_1^{-1}k_1|\mathbb{A}(s)|^2\mathbb{A}(s) = 0. \tag{37}$$

where the coefficients γ_1, $F(s)$ and k_1 are given in the appendix. $\mathbb{A}(s)$ in the phase-amplitude form can be written as

$$\mathbb{A}(s) = |\mathbb{A}(s)|e^{i\theta}. \tag{38}$$

Now substituting the above expression Eq. (38) in Eq. (37), we can get the following equations for the amplitude $|\mathbb{A}(s)|$:

$$\frac{d|\mathbb{A}(s)|^2}{ds} - 2p_r|\mathbb{A}(s)|^2 + 2l_r|\mathbb{A}(s)|^4 = 0, \tag{39}$$

$$\frac{d(ph(\mathbb{A}(s)))}{ds} = p_i - l_i|\mathbb{A}(s)|^2, \tag{40}$$

where $\gamma_1^{-1}F(s) = p_r + ip_i$, $\gamma_1^{-1}k_1 = l_r + il_i$ and $ph(\cdot)$ represents the phase shift. The mean value of Nusselt number $\overline{\mathrm{Nu}}$ is defined over an appropriate interval $(0, 2\pi)$ for temperature modulation on heat transport,

$$\overline{\mathrm{Nu}} = \frac{1}{2\pi}\int_0^{2\pi}\mathrm{Nu}\,ds. \tag{41}$$

The amplitude $\mathbb{A}(s)$ is obtained numerically and hence $\overline{\mathrm{Nu}}$ is also to be numerically evaluated. The factor I_1 determines whether the modulation amplifies or diminishes the amplitude of convection. Similarly the mean Sherwood number $\overline{\mathrm{Sh}}$ also defined.

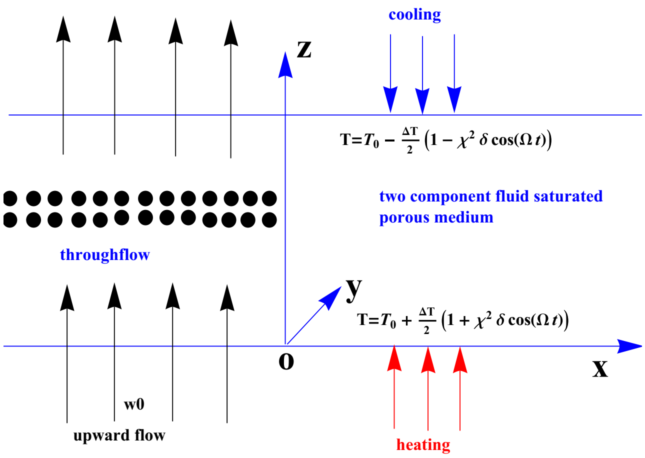

Figure 1 A sketch of the physical problem.

6. Results and discussions

When a horizontal porous layer is heated uniformly from below and cooled from above, a cellular regime of steady convection is set up at values of the Rayleigh number exceeding a critical value. To determine this amplitude of convection one has to develop a nonlinear theory to analyze the nonlinear interactions of fluid motion with temperature and concentration. A method is presented here to determine an amplitude of convection and analyze the heat and mass transfer. The combined effect of temperature modulation and vertical throughflow on Bénard–Darcy convection is studied. Using CGLE a weakly nonlinear thermal instability has been performed to investigate the effect of both temperature modulation and vertical throughflow on heat and mass transports. Since the porous medium is assumed to be closely packed, the Darcy law was employed. It is observed that, for existing the oscillatory mode of convection the oscillatory frequency (ω) must be positive, hence the values of Pe, Pr_D, and diffusivity ratio Γ are considered according to Eq. (28). Also, the values of δ and Ω are considered small, for small values of these parameters the maximum the heat and mass transfer. A small amount of throughflow in a particular direction is either to destabilize or to stabilize the system, hence, consider Pe values around 0.1. The numerical results for Nu and Sh obtained from the expressions given in Eqs. (34) and (35) by solving the amplitude Eq. (39) have been presented in Figs. 2–5. The effect of each parameter on heat and mass transport is shown in Figs. 2–5 wherein the plots of Nu and Sh versus s are presented. It is found from the figures that, the values of Nu

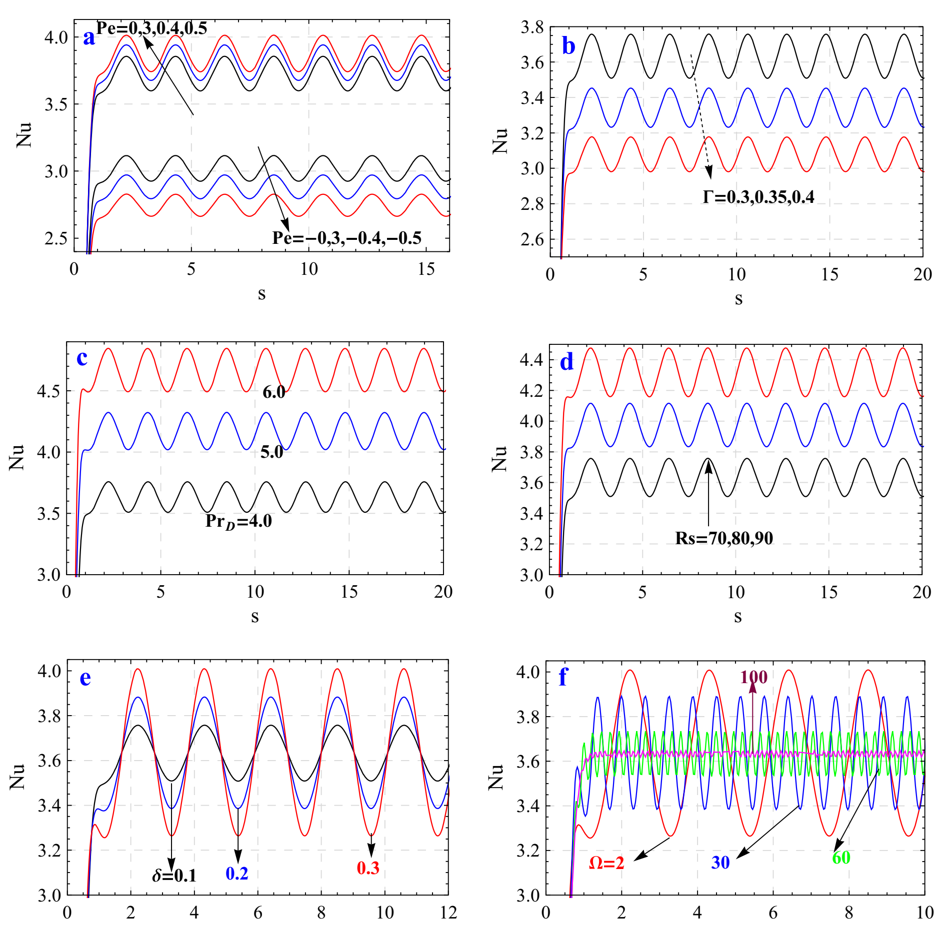

Figure 2 Effect of various parameters on heat transport for Out of Phase Modulation.

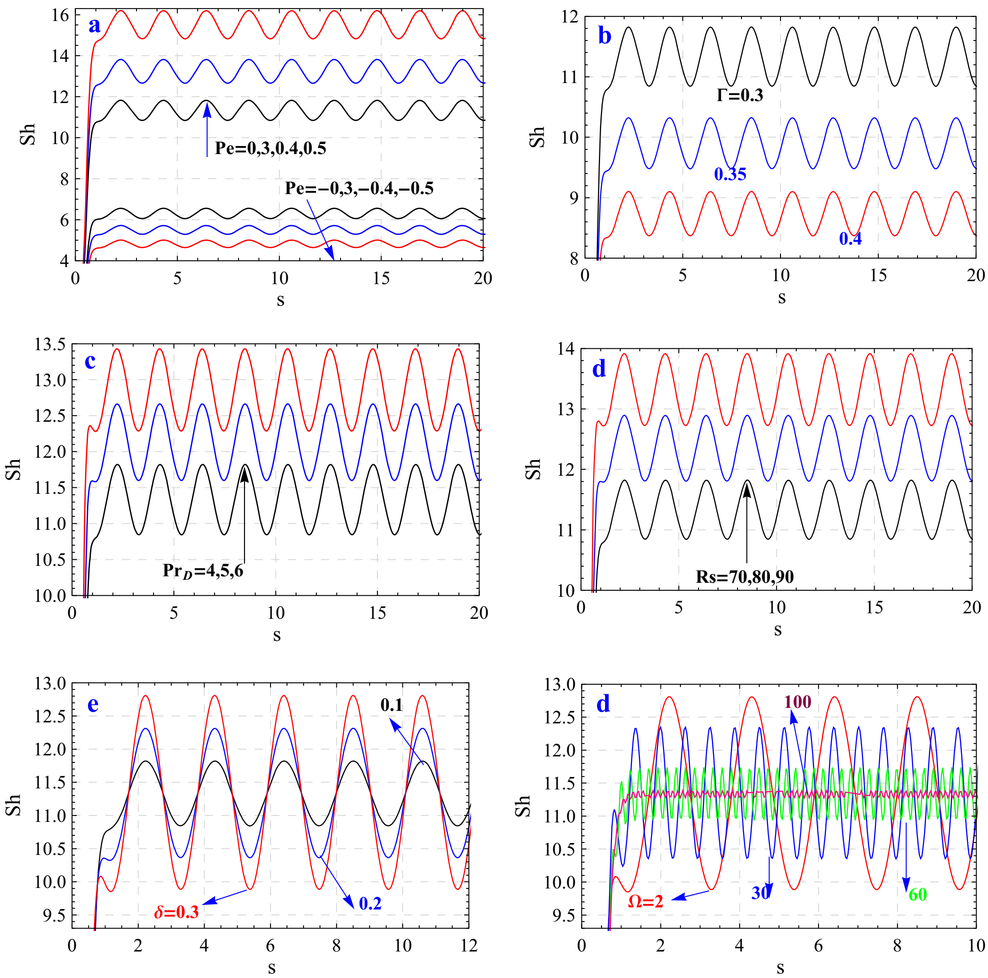

Figure 3 Effect of various parameters on mass transport for Out of Phase Modulation.

and Sh start with one and remain constant for quite some time, thus showing the conduction state initially. Then the values of Nu and Sh increase with time, thus showing the convection state and finally become constant on further increasing s thus achieving the steady state. In order to see the effect of temperature modulation on the system the following three types of temperature profiles are considered:

1. In-phase modulation (IPM) ($\phi = 0$).
2. Out-phase modulation (OPM) ($\phi = \pi$).
3. Only Lower boundary modulated (LBMO) ($\phi = -i\infty$), which means that the modulation effect will not be considered in upper boundary but only in lower boundary.

The Rayleigh number increases upon increasing Pe, and it is independent of throughflow direction. This may be due to the fact that, throughflow is to confine significant thermal gradients to a thermal boundary layer at the boundary towards which the throughflow is directed. The effective length scale is thus smaller than the thickness of the porous medium. Hence the Rayleigh number will be much less than the actual value of Rayleigh number. Therefore, large values of Rayleigh number are needed for the onset of convection when the throughflow strength increases and conforms the results of Khalili and Shivakumar [34] for free-free boundaries. The opposite results were obtained by Nield [29] in the case of a fluid layer for small amount of throughflow. Shivakumara and Sureshkumar [40] defined the reason for opposite effect may be due to the distortion of steady-state basic temperature distribution from linear to nonlinear by the throughflow. A measure of throughflow is given by the basic state temperature and this can be interpreted as a rate of energy transfer into the disturbance by interaction of the perturbation convective motion with basic temperature gradient. The maximum temperature occurs at a place where the perturbed vertical velocity is high, and this leads to an increase in energy supply for destabilization.

Now let us discuss the results related to out-of-phase modulation (OPM). Fix the values of parameters as $Pr_D = 4.0$, $Rs = 70$, $Pe = 0.2$, $\Gamma = 0.3$, $\delta = 0.1$, and $\Omega = 3.0$, to see the individual effect of each parameter on the system. In figures only the corresponding parameter value has been given. The basic state thermal and solutal concentration distributions are obtained for representative values of Pe and Γ, and are presented graphically. For throughflow, the basic state distributions become nonlinear, and deviate from each other with

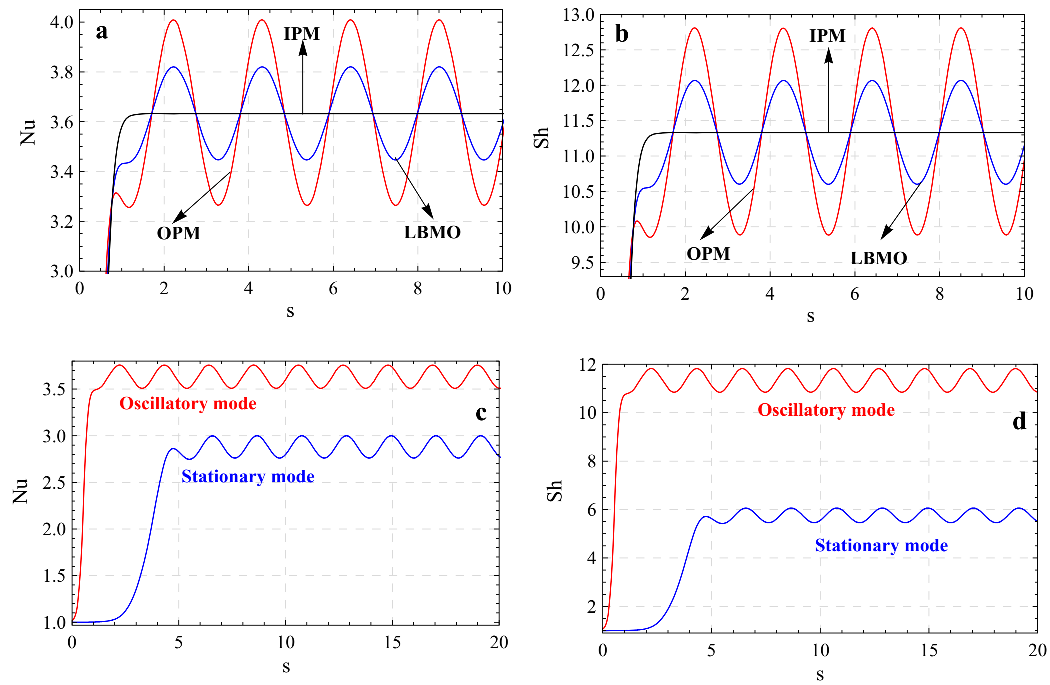

Figure 4 Comparison (a and b) for 3 temperature profiles (c and d) for stationary and oscillatory mode of convection.

an increase in Pe. In fact, the nonlinearity in base-state solute concentration stratification becomes more dominant as compared to temperature stratification with a decrease in Γ. It is found that Nu and Sh start with one, increase with time s and then become oscillatory showing modulation effect. The effect of upward throughflow ($Pe > 0$) is to enhance and downward ($Pe < 0$) throughflow and is to diminish the heat and mass transfer in the system as in Figs. 2 and 3a. The effect of diffusivity ratio Γ is to diminish heat and mass transfer in the system as in Figs. 2 and 3b, and these results conform the results of Bhadauria [22] and Bhadauria and Kiran [14]. The corresponding results of Pr_D are presented in Figs. 2 and 3c and it was observed that, Nu and Sh increase upon increasing Pr_D and these results conform the results of Bhadauria [22] and Bhadauria and Kiran [13]. The effect of solutal Rayleigh number Rs in Figs. 2 and 3d is to increase the heat and mass transfer. Though the stabilizing factor solutal gradient increases the stability and decreases heat and mass transfer in the system, due to the presence of the strong finite amplitude flows Bhadauria and Kiran [14], for large values of Rayleigh number, tend to mix the solute and redistribute it so that the interior layers of the fluid are more neutrally stratified. As a result the enhancing effect of the solute concentration is greatly decreased; hence, fluid will convect more, due to this heat and mass transfer increases when Rs is increased. Further, in Figs. 2 and 3e the effect of amplitude of modulation is to increase the magnitudes of Nu and Sh, thus increasing the heat and mass transport and advancing the convection. Also, from Figs. 2 and 3f, it is observed that, increasing the value of Ω decreases the magnitude of Nu and Sh, and so the effect of frequency of modulation on heat and mass transport diminishes. At high frequency the effect of temperature modulation on thermal instability disappears altogether. This result agrees quite well with the linear theory results of Venezian [8], where the correction in the critical value of Rayleigh number due to thermal modulations becomes almost zero at high frequencies. Fig. 4a and b shows the comparison between stationary ($\omega = 0$) and oscillatory mode of convection, and found that, the oscillatory mode of convection increases heat and mass transfer rather than stationary mode of convection. These results were obtained by Bhadauria and Kiran [16,24,47,48]. In order to avoid the repetition of the figures, the results for IPM and LBMO have not been presented graphically. But, in Fig. 4c and d the rate of heat and mass transfer for three types of temperature profile has been presented and found that, OPM case enhances heat and mass transfer than LBMO. It is also found that, IPM case is similar to unmodulated system which conforms the results of Bhadauria and Kiran [13–16].

Siddheshwar et al. [53] show the effect of temperature modulation on mean Nusselt number Eq. (41) depends on both the phase difference ϕ and frequency Ω of modulation than only on the choice of the small amplitude (δ) modulation. From Fig. 5 it is clear that, for a given frequency of modulation there is a certain range of ϕ in which $\overline{\mathrm{Nu}}$ increases with increasing ϕ and another range in which $\overline{\mathrm{Nu}}$ decreases. In Fig. 5a–d, for various values of Rs and ϕ and for lower values of Ω heat transfer effect is effective. Similarly in Fig. 5e and f, upon increasing Ω the modulation effect disappears. Thus, suitable combination of choices of Ω and ϕ can be made depending on the demands on heat transport applications. Heat transfer can be regulated (enhanced or reduced) with the external mechanism of temperature modulation effectively. These results are similar to Siddheshwar et al. [53] and Bhadauria and Kiran [15,16]. It is clear that for temperature modulation the boundary temperatures should not be in in-phase modulation (synchronized), where the effect of modulation is negligible on heat transport. The similar results can be obtained for $\overline{\mathrm{Sh}}$ case.

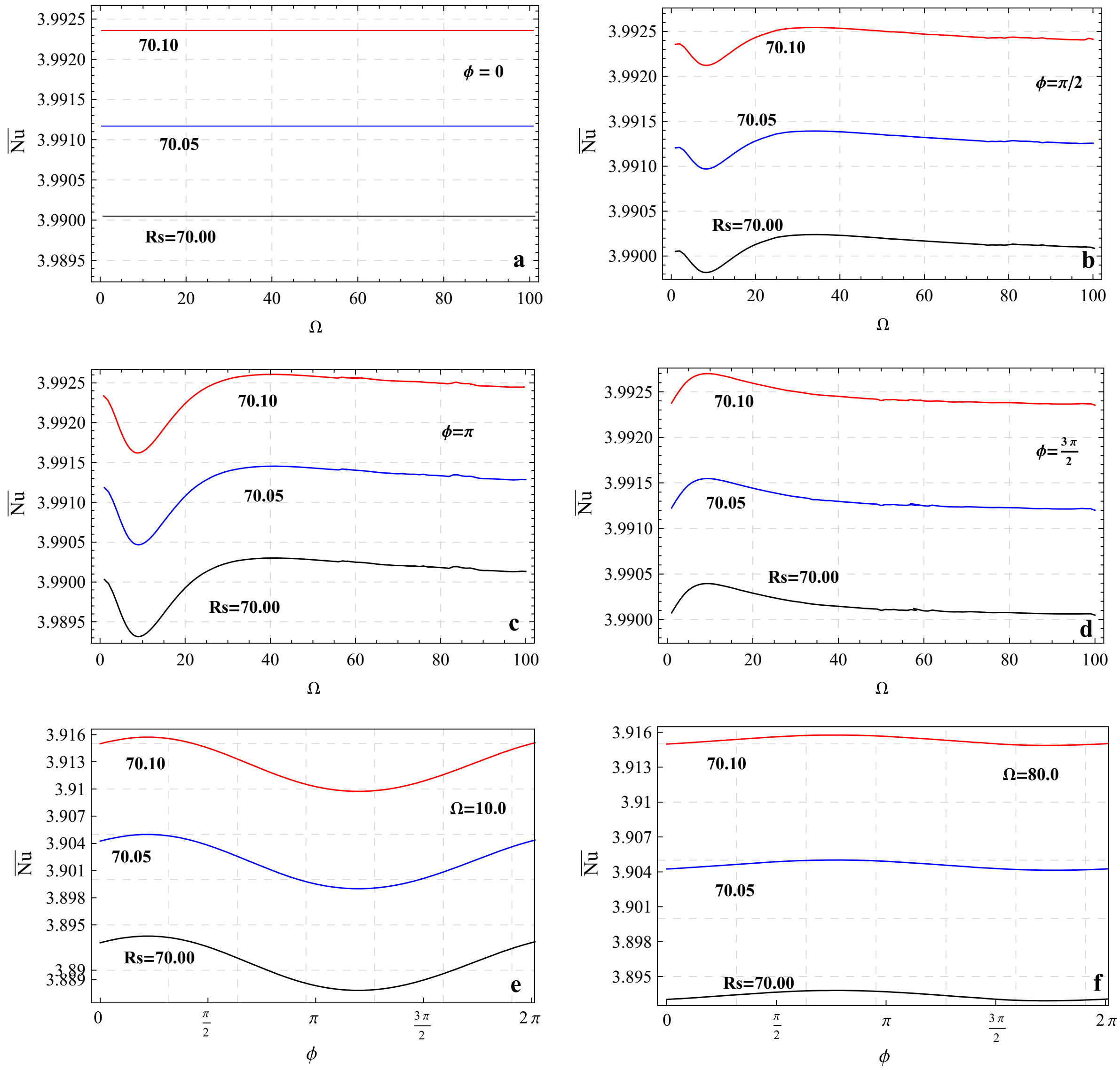

Figure 5 Effect of ϕ and Ω on mean Nusselt number $\overline{\mathrm{Nu}}$ for different values of Rs.

7. Conclusions

The effect of throughflow and temperature modulation on binary fluid saturated porous medium is investigated for oscillatory mode of convection, performing a weak nonlinear stability analysis resulting in the complex Ginzburg–Landau amplitude equation. The following conclusions are drawn from the above study:

1. The effect of upward throughflow enhances or downward throughflow diminishes heat and mass transfer.
2. Upon increasing the value of δ is to enhance the heat and mass transfer.
3. The frequency Ω of modulation decreases the heat and mass transfer as its value increases.
4. Oscillatory mode of convection is effective than stationary mode of convection.
5. Throughflow and temperature modulation can be used to regulate heat and mass transfer in the system effectively.
6. OPM and LBMO cases are suitable cases for heat and mass transfer.
7. IPM case is negligible on heat and mass transfer.

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Appendix A. The factor $\frac{\partial T_b}{\partial z}$, which is in Eq. (17) is given by

$$\frac{\partial T_b}{\partial z} = f'(z) + \epsilon^2 \delta \mathrm{Re}[f'_1(z,t)], \tag{42}$$

where $f' = \frac{Pe e^{Pez}}{1-e^{Pe}}$, $f'_1(z,t) = [B(\theta_2)e^{\theta_2 z} + B(-\theta_2)e^{-\theta_2 z}]e^{-i\Omega t}$, $B(\theta_2) = \frac{\theta_1+\theta_2}{2} \frac{(e^{-i\phi} - e^{\theta_1 - \theta_2})}{e^{\theta_1}(e} \theta_2 - e^{-\theta_2})$, $\theta_1 = \frac{Pe}{2}$, $\theta_2 = \frac{\sqrt{Pe^2+4\lambda_3^2}}{2}$ and $\lambda_3 = (1-i)\sqrt{\frac{\Omega}{2}}$.

The expressions given in Eqs. (34) and (35), can be simplified as

$$\mathrm{Nu}(s) = 1 + \frac{e^{Pe}-1}{Pe(4\pi^2+Pe^2)} \times \left(\frac{2\pi^2 a^2 c}{(c^2+\omega^2)} + \frac{2\pi^4 a^2}{\sqrt{4\pi^4+\omega^2}\sqrt{c^2+\omega^2}}\right)|\mathbb{A}(s)|^2,$$

$$\mathrm{Sh}(s) = 1 + \frac{e^{(Pe\Gamma^{-1})}-1}{Pe\Gamma^{-1}(4\pi^2+(Pe\Gamma^{-1})^2)} \times \left(\frac{2\pi^2 a^2 c}{(\Gamma^2 c^2+\omega^2)} + \frac{2\pi^4 a^2}{\sqrt{4\pi^4\Gamma^2+\omega^2}\sqrt{\Gamma^2 c^2+\omega^2}}\right)|\mathbb{A}(s)|^2.$$

The expressions given in Eq. (36) are

$$R_{31} = -\frac{1}{Pr_D}\frac{\partial}{\partial s}(\nabla^2\psi_1) - R_2\frac{\partial T_1}{\partial x} - R_0\frac{\partial T_2}{\partial x},$$

$$R_{32} = \frac{\partial \psi_1}{\partial x}\frac{\partial T_2}{\partial z} - \frac{\partial T_1}{\partial s} + \delta f_1(z,\tau)\frac{\partial \psi_1}{\partial x},$$

$$R_{33} = \frac{\partial \psi_1}{\partial x}\frac{\partial S_2}{\partial z} - \frac{\partial S_1}{\partial s}.$$

The coefficients given in Eq. (37) are

$$\gamma_1 = \frac{c}{Pr_D} + \frac{4\pi^2 R_0 a^2}{(4\pi^2+Pe^2)(c+i\omega)^2} - \frac{4\pi^2 Rs a^2}{(4\pi^2+(Pe\Gamma^{-1})^2)(\Gamma c+i\omega)^2},$$

$$F(s) = \frac{4\pi^2 R_2 a^2}{(4\pi^2+Pe^2)(c+i\omega)} - \frac{2R_0 a^2}{(c+i\omega)}\delta I_1 \text{ where}$$

$$I_1 = \int_0^1 f_2 \sin^2(\pi z)dz.$$

$$k_1 = \frac{a^4\pi^2 c R_0}{(4\pi^2+Pe^2)(c^2+\omega^2)(c+i\omega)} + \frac{a^4\pi^4 R_0}{(4\pi^2+Pe^2)(2\pi^2+i\omega)(c+i\omega)^2}$$

$$- \frac{a^4 c\pi^2 Rs}{(4\pi^2+(Pe\Gamma^{-1})^2)(\Gamma^2c^2+\omega^2)(\Gamma c+i\omega)} - \frac{a^4\pi^4 Rs}{(4\pi^2+(Pe\Gamma^{-1})^2)(2\pi^2\Gamma+i\omega)(\Gamma c+i\omega)^2}.$$

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