# Flavor and CP violating $Z$ exchange and the rate asymmetry in $B \rightarrow \phi K_S$

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**Abstract**

Recent measurements of time dependent CP asymmetry in $B \rightarrow \phi K_S$, if confirmed, would indicate a new source of CP violation. We examine flavor violating tree-level $Z$ currents in models with extra down-type quark singlets that arise naturally in string compactified gauge groups like $E_6$. We evaluate the new operators at the scale $\mu \approx \mathcal{O}(m_b)$ in NLO, and using QCD improved factorization to describe $B \rightarrow \phi K_S$, find the allowed range of parameters for $\rho$ and $\psi$, the magnitude and phase of the flavor violating parameter $U_{bs}$. This allowed range does satisfy the constraint from flavor changing process $b \rightarrow s \ell^+ \ell^-$. However, further improvement in measurement of these rates could severely constrain the model.
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## 1. Introduction

The ongoing $B$ physics experiments by BaBar and Belle Collaborations [1,2] provide a unique opportunity to study the flavor structure of the Standard Model quark sector and also the origin of CP violation. In addition to this, any new physics effects in $B$ physics can also be tested in these experiments. Recent time dependent asymmetries measured in the decay $B \rightarrow \phi K_S$ both by BaBar and Belle Collaborations [1–4] show significant deviation from the Standard Model and this has generated much theoretical speculation

regarding physics beyond the Standard Model [5]. In the standard model, the process $B \rightarrow \phi K_S$ is purely penguin dominated and the leading contribution has no weak phase. The coefficient of $\sin(\Delta m_B t)$ in the asymmetry therefore should measure $\sin 2\beta$, the same quantity that is involved in $B \rightarrow \psi K_S$ in the Standard Model. The most recent measured average values of asymmetries are [4,6] $S_{\psi K_S} = 0.734 \pm 0.055$ and $S_{\phi K_S} = -0.15 \pm 0.33$. The value for $S_{\psi K_S}$ agrees with the Standard Model expectation. The deviation in the $\phi K_S$ is intriguing because a penguin process being a loop induced process is particularly sensitive to new physics which can manifest itself through exchange of heavy particles. In this Letter we will consider an extension of the Standard Model, with extra down type singlet quarks. These extra down type sin-

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glet quarks appear naturally in each 27-plet fermion generation of $E_6$ Grand Unification Theories (GUTs) [7–10]. The mixing of these singlet quarks with the three SM down type quarks, provides a framework to study the deviations from the unitarity constraints of $3 \times 3$ CKM matrix. This model has been previously studied in connection with $R_b$ and F–B asymmetry at the $Z$ pole as it provides a framework for violation of the unitarity of the CKM matrix [10–12]. This mixing also induces tree-level flavor changing neutral currents (FCNC). These tree-level FCNC couplings can have a significant effects on different CP conserving as well as CP violating $B$ processes [11,13–24].

In this Letter we study the FCNC effect arising from the $Z - b - \bar{s}$ coupling $U_{bs}$ to the $B \to \phi K_S$ process. This new FCNC coupling $U_{bs}$ can have a phase, which can generate the additional source of CP violation in the $B \to \phi K_S$ process, and thus affect measured values of $S_{\phi K_S}$ and $C_{\phi K_S}$. We parameterize this coupling by $U_{bs} = \rho e^{i\psi}$. We then study $B \to \phi K_S$ taking into account the new interactions in the QCD improved factorization scheme (BBNS approach) [25]. This method incorporates elements of naive factorization approach (as its leading term) and perturbative QCD corrections (as subleading contributions) and allows one to compute systematic radiative corrections to the naive factorization for the hadronic $B$ decays. Recently, several studies of $B \to PV$, and specifically $B \to \phi K_S$ have been performed within the frame work of QCD improved factorization scheme [26–30]. In our analysis of $B \to \phi K_S$, we follow [30] which is based on the original paper [25]. In our analysis, we only consider the contribution of the leading twist meson wave functions, and also neglect the weak annihilation contribution which is expected to be small. Inclusion of these would introduce more model dependence in the calculation through the parameterization of an integral, which is otherwise infrared divergent.

The time dependent CP asymmetry of $B \to \phi K_S$ is described by:

$$\mathcal{A}_{\phi K_S}(t) = \frac{\Gamma(\overline{B^0}(t) \to \phi K_S) - \Gamma(B^0(t) \to \phi K_S)}{\Gamma(\overline{B^0}(t) \to \phi K_S) + \Gamma(B^0(t) \to \phi K_S)} \tag{1}$$

$$= -C_{\phi K_S} \cos(\Delta m_B t) + S_{\phi K_S} \sin(\Delta m_B t), \tag{2}$$

where $S_{\phi K_S}$ and $C_{\phi K_S}$ are given by

$$S_{\phi K_S} = \frac{2 \, \mathrm{Im} \, \lambda_{\phi K_S}}{1 + |\lambda_{\phi K_S}|^2}, \qquad C_{\phi K_S} = \frac{1 - |\lambda_{\phi K_S}|^2}{1 + |\lambda_{\phi K_S}|^2} \tag{3}$$

and $\lambda_{\phi K_S}$ can be expressed in terms of decay amplitudes:

$$\lambda_{\phi K_S} = -e^{-2i\beta} \frac{\bar{\mathcal{M}}(\overline{B^0} \to \phi K_S)}{\mathcal{M}(B^0 \to \phi K_S)}. \tag{4}$$

The branching ratio and the direct CP asymmetries of both the charged and neutral modes of $B \to \phi K_S$ have been measured [1–4,6,31][1]

$$\mathcal{B}(B^0 \to \phi K_S) = (8.0 \pm 1.3) \times 10^{-6}, \tag{5}$$

$$\mathcal{B}(B^+ \to \phi K^+) = (9.4 \pm 0.9) \times 10^{-6}, \tag{6}$$

$$S_{\phi K_S} = +0.45 \pm 0.43 \pm 0.07 \quad (\text{BaBar}), \tag{7}$$

$$= -0.96 \pm 0.50^{+0.09}_{-0.11} \quad (\text{Belle}), \tag{8}$$

$$= -0.15 \pm 0.33 \quad (\text{world average}), \tag{9}$$

$$C_{\phi K_S} = -0.19 \pm 0.30, \tag{10}$$

$$\mathcal{A}_{CP}(B^+ \to \phi K^+) = (3.9 \pm 8.8 \pm 1.1)\%. \tag{11}$$

## 2. $B \to \phi K_S$ in the QCDF approach

In the Standard Model, the effective Hamiltonian for charmless $B \to \phi K_S$ decay is given by [25]

$$\mathcal{H}_{\mathrm{eff}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \bigg[ C_1(\mu)\mathcal{O}_1(\mu) + C_2(\mu)\mathcal{O}_2(\mu)$$
$$+ \sum_{i=3}^{10} C_i(\mu)\mathcal{O}_i(\mu) + C_{7\gamma}\mathcal{O}_{7\gamma} + C_{8g}\mathcal{O}_{8g} \bigg], \tag{12}$$

where the Wilson coefficients $C_i(\mu)$ are obtained from the weak scale down to scale $\mu$ by running the renormalization group equations. The definitions of the operators can be found in Ref. [25]. The Wilson coefficients $C_i$ can be computed using different schemes [35]. In this Letter we will use the NDR scheme. The NLO values of $C_i$ ($i = 1$–10) and LO values of

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[1] Latest results were reported at XXXIX Rencontres de Moriond, Electroweak Interactions and Unified Theories, Italy, March 2004. See talks [32–34]. The Belle result on $S_{\phi K}$ is unchanged [32] while BaBar finds $S_{\phi K} = 0.47 \pm 0.34^{+0.08}_{-0.06}$ [33] which is very close to the result in Eq. (7). Hence, our observations remain unchanged.

Table 1
Standard Model Wilson coefficients in NDR scheme

| Scale | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ |
|---|---|---|---|---|---|---|
| $\mu = m_b/2$ | 1.137 | −0.295 | 0.021 | −0.051 | 0.010 | −0.065 |
| $\mu = m_b$ | 1.081 | −0.190 | 0.014 | −0.036 | 0.009 | −0.042 |
| | $C_7/\alpha_{em}$ | $C_8/\alpha_{em}$ | $C_9/\alpha_{em}$ | $C_{10}/\alpha_{em}$ | $C_{7\gamma}$ | $C_{8g}$ |
| $\mu = m_b/2$ | −0.024 | 0.096 | −1.325 | 0.331 | −0.364 | −0.169 |
| $\mu = m_b$ | −0.011 | 0.060 | −1.254 | 0.223 | −0.318 | −0.151 |

$C_{7\gamma}$, $C_{8g}$, respectively, at $\mu = m_b/2$ and $m_b$ used by us based on Ref. [25] are shown in Table 1.

In the QCD improved factorization scheme, the $B \rightarrow \phi K_S$ decay amplitude due to a particular operator can be represented in following form:

$$\langle \phi K | \mathcal{O} | B \rangle$$
$$= \langle \phi K | \mathcal{O} | B \rangle_{\text{fact}} \left[ 1 + \sum r_n \alpha_s^n + \mathrm{O}(\Lambda_{\text{QCD}}/m_b) \right], \tag{13}$$

where $\langle \phi K | \mathcal{O} | B \rangle_{\text{fact}}$ denotes the naive factorization result. The second and third term in the bracket represent higher order $\alpha_s$ and $\Lambda_{\text{QCD}}/m_b$ correction to the hadronic transition amplitude. Following the scheme and notations presented in Ref. [30], we write down the total $B \rightarrow \phi K_S$ amplitude, which is the sum of the Standard Model as well as $Z$ exchange tree-level contribution from extra down-type quark singlets (EDQS) model in the heavy quark limit

$$\mathcal{M}(B^+ \rightarrow \phi K^+)$$
$$= \mathcal{M}(B^0 \rightarrow \phi K^0)$$
$$= \frac{G_F}{\sqrt{2}} m_B^2 f_\phi F_1^{B \rightarrow K}(m_\phi^2) V_{pb} V_{ps}^* \left[ a_3^p + a_4^p + a_5^p \right.$$
$$\left. - \frac{(a_7^p + a_9^p + a_{10}^p)}{2} + a_{10a}^p \right], \tag{14}$$

where $p$ is summed over $u$ and $c$. The coefficients $a_i^p$ are given by

$$a_3^u = a_3^c = C_3' + \frac{C_4'}{N_c} \left[ 1 + \frac{C_F \alpha_s}{4\pi} (V_\phi + H_\phi) \right],$$

$$a_4^p = C_4' + \frac{C_3'}{N_c} \left[ 1 + \frac{C_F \alpha_s}{4\pi} (V_\phi + H_\phi) \right] + \frac{C_F \alpha_s}{4\pi N_c} P_\phi^p,$$

$$a_5^u = a_5^c = C_5' + \frac{C_6'}{N_c} \left[ 1 + \frac{C_F \alpha_s}{4\pi} (-12 - V_\phi) \right],$$

$$a_7^u = a_7^c = C_7' + \frac{C_8'}{N_c} \left[ 1 + \frac{C_F \alpha_s}{4\pi} (-12 - V_\phi - H_\phi) \right],$$

$$a_9^u = a_9^c = C_9' + \frac{C_{10}'}{N_c} \left[ 1 + \frac{C_F \alpha_s}{4\pi} (V_\phi + H_\phi) \right],$$

$$a_{10}^u = a_{10}^c = C_{10}' + \frac{C_9'}{N_c} \left[ 1 + \frac{C_F \alpha_s}{4\pi} (V_\phi + H_\phi) \right],$$

$$a_{10}^p = \frac{C_F \alpha_s}{4\pi N_c} Q_\phi^p, \qquad a_{10a}^u = a_{10a}^c = \frac{C_F \alpha_s}{4\pi N_c} Q_\phi \tag{15}$$

with $C_F = (N_c^2 - 1)/2N_c$ and $N_c = 3$. The effective Wilson coefficients $C_i' = C_i + \tilde{C}_i$, $(i = 3\text{–}10)$ is the sum of the Standard Model and the EDQS model Wilson coefficients. The quantities $V_\phi$, $H_\phi$, $P_\phi^p$ and $Q_\phi^p$ are hadronic parameters that contain all nonperturbative dynamics, are given in Refs. [25,36].

For the sake of completeness, we give the branching ratio for $B \rightarrow \phi K_S$ decay channel in the rest frame of the $B$ meson.

$$\mathcal{BR}(B \rightarrow \phi K_S) = \frac{\tau_B}{8\pi} \frac{|P_{cm}|}{m_B^2} |\mathcal{M}(B \rightarrow \phi K_S)|^2, \tag{16}$$

where, $\tau_B$ represents the $B$ meson lifetime and the kinematical factor $|P_{cm}|$ is written as

$$|P_{cm}|$$
$$= \frac{1}{2m_B}$$
$$\times \sqrt{\left[ m_B^2 - (m_K + m_\phi)^2 \right] \left[ m_B^2 - (m_K - m_\phi)^2 \right]}. \tag{17}$$

## 3. FCNC $Z$ couplings in EDQS model

Models with extra down-type quarks (EDQS) have a long history. The earliest consideration of such models was in the context of the grand unification group $E_6$ which arises from string compactification. The quarks and leptons of each generation belong to

the 27 representation [7–10]. Each generation has one extra quark singlet of the down type, and also one extra lepton of the electron type. The group also has extra $Z$ bosons, which we will assume to be too heavy to have any effect on $B \to \phi K_S$ process. The down type mass matrix is then a $6 \times 6$ by-unitary matrix, and in general when we rotate the quarks to their mass basis, off-diagonal couplings arise. In EDQS model, the $Z$ mediated FCNC interactions are given by

$$\mathcal{L} = \frac{g}{2\cos\theta_W}\big[\bar{d}_{L\alpha}U_{\alpha\beta}\gamma^\mu d_{L\beta}\big]Z_\mu. \quad (18)$$

In general for $n$ copies of extra down-type quark singlet model, $U_{\alpha\beta}$ is:

$$U_{\alpha\beta} = \sum_{i=1}^{3} V_{\alpha i}^\dagger V_{i\beta} = \delta_{\alpha\beta} - \sum_{i=4}^{N_d} V_{\alpha i}^\dagger V_{i\beta}$$
$$(\alpha, \beta = d, s, b, B_1, B_2, \ldots), \quad (19)$$

where $N_d = 3 + n$ represents the number of down type quark states, and $U$ is the neutral current mixing matrix for the down quark sector. The nonvanishing components of $U_{\alpha\beta}$ will lead to FCNC process at tree level, generating new physics contribution to the measured CP asymmetries. The new tree level FCNC $Z$ mediated contribution to the $b \to sq\bar{q}$ process is shown in Fig. 1. The new operators arising from this tree-level FCNC process have been shown to lead to the following effective Hamiltonian for $b \to sq\bar{q}$ process in this model [22]:

$$\mathcal{H}_Z^{\text{new}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* [\tilde{C}_3 \mathcal{O}_3 + \tilde{C}_7 \mathcal{O}_7 + \tilde{C}_9 \mathcal{O}_9], \quad (20)$$

where, the new Wilson coefficients $\tilde{C}_3$, $\tilde{C}_7$ and $\tilde{C}_9$ at the scale $M_Z$ are given by:

$$\tilde{C}_3(M_Z) = \frac{\kappa}{6}, \quad (21)$$

$$\tilde{C}_7(M_Z) = \kappa\frac{2}{3}\sin^2\theta_W, \quad (22)$$

$$\tilde{C}_9(M_Z) = -\kappa\frac{2}{3}\big(1 - \sin^2\theta_W\big), \quad (23)$$

where, $\kappa = \frac{U_{bs}}{(V_{tb}V_{ts}^*)}$, and operators $\mathcal{O}_i$ in Eq. (20) are given in Ref. [25]. We now evolve these new Wilson coefficients from the scale $M_Z$ to the scale $\mu \approx \mathcal{O}(m_b)$ using the renormalization group equation. While doing this we have considered NLO QCD correction [37], neglecting the order $\alpha$ electroweak
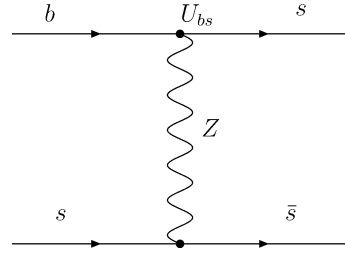


Fig. 1. Feynman diagram for $Z$ exchange tree-level contribution to $b \to ss\bar{s}$ process.

contributions to the RG evolution equation which are tiny. At the low energy, after the RG evolution the above three Wilson coefficients ($\tilde{C}_3, \tilde{C}_7, \tilde{C}_9$) generate new set of Wilson coefficients ($\tilde{C}_i$, $i = 3$–10) in this model. The values of Wilson coefficients (without taking the overall factor $\kappa$) at scales $\mu = (m_b/2, m_b)$ are shown in Table 2.

## 4. *B* physics constraints on $U_{bs}$

In this section, we review the constraints on the flavor violating parameter $U_{bs}$ from different flavor changing $B$ processes. These processes can be classified into two classes, CP conserving and CP violating. Among the CP conserving processes, $\mathcal{B}(B \to X_s\ell^+\ell^-)$, and $\Delta M_{B_s}$ can put constraints on $U_{bs}$ [13,15,16]. Using recent Belle [38] measurement of $\mathcal{B}(B \to X_s\ell^+\ell^-) = (6.1 \pm 1.4^{+1.4}_{-1.1}) \times 10^{-6}$ the authors in Ref. [21] had shown that $|U_{bs}| \leqslant 1 \times 10^{-3}$. However, this bound has recently been updated in Ref. [22] to

$$|U_{bs} - 4.0 \times 10^{-4}| \leqslant 8 \times 10^{-4}, \quad (24)$$

which also updates their previous bounds in Refs. [17, 39]. The bound in Eq. (24) is based on inclusive $B \to X_s e^+ e^-$ decays at NNLO [40]. We shall adopt this bound in our analysis. This bound is valid in both general $n$ extra down-type singlet quark model and in the model with a single extra down-type quark singlet [16]. Similarly, the $b \to s\gamma$ branching ratio provides comparable limits [14,16].

It has been shown in Refs. [20,41] that in the presence of tree-level FCNC coupling $U_{bd}$ and/or $U_{bs}$, the standard box diagram for $B_{d/s}$–$\bar{B}_{d/s}$ is not gauge invariant by itself, but requires $Z$ exchange penguin diagram as well as tree level $Z$ FCNC diagram. The

Table 2
Wilson coefficients of EDQS model in NDR scheme, without the overall multiplicative factor $\kappa$

| Scale | $\tilde{C}_3$ | $\tilde{C}_4$ | $\tilde{C}_5$ | $\tilde{C}_6$ | $\tilde{C}_7$ | $\tilde{C}_8$ | $\tilde{C}_9$ | $\tilde{C}_{10}$ |
|---|---|---|---|---|---|---|---|---|
| $\mu = m_b/2$ | 0.195 | −0.088 | 0.0180 | −0.053 | 0.133 | 0.108 | −0.604 | 0.174 |
| $\mu = mb$ | 0.182 | −0.0629 | 0.0157 | −0.0370 | 0.136 | 0.0732 | −0.574 | 0.122 |

additional Feynman diagrams are given in Ref. [41]. Following the paper [20], with slight change in the notations, we write down the expression for the $B_q$–$\bar{B}_q$ mixing:

$$\Delta M_{B_q} = \frac{G_F^2 M_W^2 f_{B_q}^2 \hat{B}_{B_q} m_{B_q^0}}{6\pi^2}$$
$$\times \left[ (\lambda_{qb}^t)^2 \eta_{tt}^{B_s} S_0(x_t) + \Delta_{\text{new}} \right], \tag{25}$$

where $(q = d, s)$ and

$$\Delta_{\text{new}} = -8 U_{bq} \lambda_{qb}^t \eta_{tt}^{B_q} Y_0(x_t) + \frac{4\pi \sin^2 \theta_W}{\alpha} \eta_Z^{B_q} U_{bq}^2, \tag{26}$$

where the definitions of different parameters used above can be found in Ref. [20].

We have found that to satisfy the measured $\Delta M_{B_d}$ [2] within one sigma, where we consider both the theory error of 20% arising from the value of $f_{B_q}^2 \hat{B}_{B_q}$ and the experimental error taken in quadrature, the FCNC coupling $|U_{bd}|$ should be less than $\sim (2\text{–}3) \times 10^{-4}$. This is a very stringent limit.

The $\Delta M_{B_s}$ has not been measured yet, and so only lower limit on the mass difference is available. We have found that the new contribution to $\Delta M_{B_s}$ from EDQS model is less than 3% when compared to the Standard Model. It can be shown that for similar values of $U_{bd}$ and $U_{bs}$, the FCNC effects on $\Delta M_{B_s}$ will be suppressed by a factor $\sim \lambda^2$ when compared with the effects on $\Delta M_{B_d}$. This implies that in EDQS model, FCNC effects are hard to detect in $B_s$–$\bar{B}_s$ mixing [41].

## 5. $B \to \phi K_S$ analysis

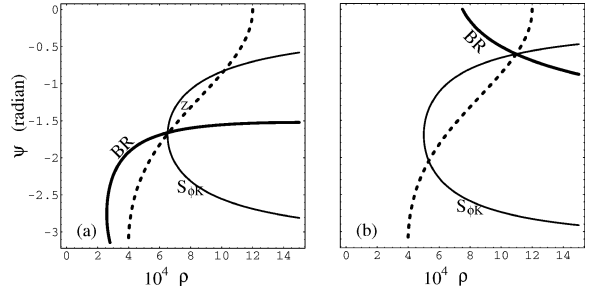In the last section we have discussed the allowed range of the FCNC parameter $U_{bs}$ from different $B$



Fig. 2. Contour plots of $S_{\phi K_S}$, $\mathcal{B}(B \to \phi K_S)$ and $|U_{bs} - 4 \times 10^{-4}| = 8 \times 10^{-4}$ in $\rho$–$\psi$ plane for two values of $\mu = m_b/2$ (a) and $m_b$ (b), respectively. The area left side of the dotted contour is the allowed region of $U_{bs}$ from the inclusive $B \to X_s \ell^+ \ell^-$ process. In figure (a), the area marked by $Z$ is the $2\sigma$ allowed regions from the measurement of $S_{\phi K}$ and $\mathcal{B}(B \to \phi K_S)$. In figure (b), such $2\sigma$ allowed region is a point where the three curves intersect.

processes. In this section we will study the effect of this FCNC parameter ($U_{bs}$) in the $B \to \phi K_S$ process. For this we express $U_{bs}$ in the following form: $U_{bs} = \rho e^{i\psi}$. We will then vary $\rho$ and $\psi$ [3] in range such that Eq. (24) is satisfied. We then study the allowed region of parameters in the $\rho$–$\psi$ plane from the three measured quantities (a) $\mathcal{B}(B \to \phi K_S)$, (b) $S_{\phi K_S}$ and (c) $C_{\phi K_S}$. To get the allowed parameter space, from the $\mathcal{B}(B \to \phi K_S)$ branching ratio, we allow it to vary by $2\sigma$, respectively, from its central value. This $2\sigma$ band contains both experimental and theoretical errors. The main source of theoretical error is the form factor $F_1^{B \to K}$. In our analysis we have considered 20% error on this parameter. Similarly, we vary $C_{\phi K_S}$ and $S_{\phi K_S}$ by $1\sigma$ and $2\sigma$ from their central value to get the allowed region in the $\rho$–$\psi$ plane.

In Fig. 2(a) we show such allowed region in $\rho$–$\psi$ plane for the scale $\mu = m_b/2$. The whole area left of the dotted contour is allowed by saturating Eq. (24). The area outside the thick contour labeled by BR is $2\sigma$ allowed region from the branching ratio measurement. The parameter space enclosed by the thin contour

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[2] The experimental value for $|\Delta M_{B_d}| = 0.489 \pm 0.008$ ps$^{-1}$ [20].

[3] $\psi$ in units of radian.

marked by $S_{\phi K}$ is allowed by $2\sigma$ from data on the $S_{\phi K_S}$. This whole parameter space is allowed by $1\sigma$ from $C_{\phi K_S}$ measurement. The regions (marked by $Z$) is the only allowed parameter space in $\rho$–$\psi$ plane with $6.5 \times 10^{-4} \leqslant \rho \leqslant 10 \times 10^{-4}$ and $-1.7 \leqslant \psi \leqslant -0.85$ which satisfy the experimentally measured $C_{\phi K_S}$, $S_{\phi K_S}$ and $\mathcal{B}(B \to \phi K_S)$ within the errors described above. We note that only negative values of $\psi$ give acceptable range of $S_{\phi K}$. The Fig. 2(b) corresponds to the scale $\mu = m_b$. In this case though we have larger allowed area from the $S_{\phi K}$ measurement, but the $2\sigma$ branching ratio contour pushes the allowed range towards higher values of $\rho$ and somewhat lower range of the phase $\psi$. This particular behavior of the branching ratio contour can be understood from that fact that for $\mu = m_b$, the SM branching ratio is $3.8 \times 10^{-6}$, which is much smaller than the lower end of the $2\sigma$ band of the experimental number. Hence, one needs larger values of $\rho$ to push the total branching ratio within the $2\sigma$ limit. For this reason the allowed region shrinks to a point in this case.

## 6. Conclusions

In this Letter, we have studied the tree-level flavor violating $Z$ contribution to $B \to \phi K_S$ process in models with extra down-type quark singlets which arise naturally in the context of the grand unification group $E_6$. In the presence of such flavor violating interactions, $B \to \phi K_S$ process receives additional contributions, governed by a set of new operators which can be expressed in terms of the standard operators $\mathcal{O}_i$, $(i = 3$–$10)$. We then evolved these new Wilson coefficients from the scale $M_Z$ to the scale $\mu = \mathcal{O}(m_b)$ relevant for our process using the renormalization group equation. We have found that, at the lower scale these Wilson coefficients significantly modified from their initial values at the scale $\mu = M_Z$. We have found that this new flavor violating interaction can modify the Standard Model Wilson coefficients $C_i$, $(i = 3$–$10)$ significantly. We have used following experimentally measured quantities: $S_{\phi K_S}$, $C_{\phi K_S}$ and $\mathcal{B}(B \to \phi K_S)$ to constrain the flavor violating parameter $U_{bs}$. We have shown that in the model with an arbitrary number of down type singlet quarks, the value of $S_{\phi K_S}$ and $C_{\phi K_S}$ can be well explained by the values of $\rho$ and $\psi$ in the region marked by $Z$ in Fig. 2(a). Improvements in

measurements of $B \to X_s \ell^+ \ell^-$ can tighten the constraints in Eq. (24) and either rule in or rule out this model.

## Appendix A. Input parameters and different form factors

In this appendix we list all the input parameters, decay constants and form factors used for the calculation of $B \to \phi K_S$.

(1) Coupling constants and masses (in units of GeV):

$$\alpha_{em} = 1/129, \qquad \alpha_s(M_Z) = 0.118,$$
$$G_F = 1.16639 \times 10^{-5} \ (\text{GeV})^{-2},$$
$$M_Z = 91.19, \qquad m_b = 4.88, \qquad m_B = 5.2787,$$
$$m_\phi = 1.019, \qquad m_K = 0.493.$$

(2) Wolfenstein parameters:

$$\lambda = 0.2205, \qquad A = 0.815,$$
$$\eta = 0.324, \qquad \rho = 0.224.$$

(3) Constituent quark masses $m_i$ $(i = u, d, s, c, d)$ (in units of GeV):

$$m_u = 0.2, \qquad m_d = 0.2, \qquad m_s = 0.5,$$
$$m_c = 1.5, \qquad m_b = 4.88.$$

(4) The decay constants (in units of GeV):

$$f_B = 0.19, \qquad f_\phi = 0.237, \qquad f_K = 0.16.$$

(5) The form factors at zero momentum transfer:

$$F_1^{B \to K} = 0.33.$$

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