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The complexity of the T-coloring problem for graphs with small degree

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Abstract

In the paper we consider a generalized vertex coloring model, namely T-coloring. For a given finite set T of nonnegative integers including 0, a proper vertex coloring is called a T-coloring if the distance of the colors of adjacent vertices is not an element of T. This problem is a generalization of the classic vertex coloring and appeared as a model of the frequency assignment problem. We present new results concerning the complexity of T-coloring with the smallest span on graphs with small degree Δ. We distinguish between the cases that appear to be polynomial or NP-complete. More specifically, we show that our problem is polynomial on graphs with $\Delta \leqslant 2$ and in the case of k-regular graphs it becomes NP-hard even for every fixed T and every $k > 3$. Also, the case of graphs with $\Delta = 3$ is under consideration. Our results are based on the complexity properties of the homomorphism of graphs.

Keywords: Vertex coloring; T-coloring; T-span; Homomorphism; NP-completeness

1. Introduction

We consider the T-coloring problem, as a generalized classical vertex coloring problem, which is one of the variants of the channel assignment problem in broadcast networks [8,16]. In this problem one wishes to assign to each transmitter $x_i \in \{x_1, \ldots, x_n\}$, located in a region, a frequency $f(x_i)$ avoiding interference between transmitters, i.e.

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two interfering transmitters (because of proximity, meteorological or other reasons) must be assigned frequencies so that the distance between them does not belong to the forbidden set T of nonnegative integers including 0. The most common objective is to minimize the span of a frequency band. For more about applications of this problem the reader is referred to [2,3,14,15].

Let $G=(V,E)$ be a simple loopless graph with vertex set $V=V(G)$ and edge set $E=E(G)$. By $\Delta(G)$ we mean the maximum degree $\rho(v)$ over all vertices v of graph G, by $\chi(G)$ and $\omega(G)$ we denote the chromatic number and the clique number of graph G, respectively. Let $G(W)$ denote the subgraph of graph G induced by $W \subset V$.

Definition 1. Let T be a finite set of nonnegative integers satisfying $0 \in T$. By a T-coloring of graph G we mean a vertex coloring $c: V \to \mathbb{N}$ satisfying $|c(v)-c(w)| \notin T$, whenever $\{v,w\} \in E$. The T-span is defined as $\mathrm{sp}_T(G) = \min_c \mathrm{sp}_T(G,c)$, where $\mathrm{sp}_T(G,c) = \max c(V) - \min c(V)$ and c is a proper vertex T-coloring of graph G. A T-coloring c is said to be optimal if $\mathrm{sp}_T(G,c) = \mathrm{sp}_T(G)$.

Following [13] we introduce the notion of T-graphs.

Definition 2. For a given set T, we define an infinite T-graph G_T, with vertex set $V(G_T) = \mathbb{N} \cup \{0\}$ and edge set $E(G_T) = \{\{x,y\}: |x-y| \notin T\}$. By G_T^{d+1} we mean the subgraph of G_T induced by $\{0,\ldots,d\}$.

Given a graph G, set T and positive integer k, the problem of verifying the inequality $\mathrm{sp}_T(G) \leqslant k$ we call the T-Span Problem. This differs from the T-Coloring Problem, which requires an optimal T-coloring as its output. The notion of a T-coloring was introduced in [8]. The problem has been studied extensively (see [3,4,12,13–18]). The majority of results concern lower and upper bounds on $\mathrm{sp}_T(G)$, see [3,11,17]. The first complexity result comes independently from [6,12], where the authors showed NP-completeness in the strong sense of the T-Span Problem on complete graphs (so even a pseudopolynomial algorithm for the T-Span Problem cannot exist unless P=NP). We call the above problems Fixed T-Span Problem and Fixed T-Coloring Problem if set T is fixed. Furthermore, in [7] the authors have developed a linear algorithm for solving the Fixed T-Coloring Problem on complete graphs (but exponential with respect to $\max T$). So far, the problem on graphs with “small” degree has been still open. Therefore, in Sections 2 and 3 we deal with some new properties of homomorphisms and in Section 5 we show NP-completeness of the Fixed T-Span Problem on subcubic graphs (i.e. with $\Delta \leqslant 3$), and r-regular graphs (i.e. with all vertices of degree r) with $r \geqslant 3$. In Section 4 we show a polynomial time algorithm for the T-Coloring Problem on graphs with $\Delta \leqslant 2$.

2. Simple properties of graph homomorphisms

The idea of graph homomorphism is a generalization of vertex coloring. Moreover, it generalizes the T-coloring problem as well.

Definition 3. For two simple graphs G and H a *graph homomorphism* is a function $h: V(G) \to V(H)$ such that $\{h(v), h(w)\} \in E(H)$, whenever, $\{v, w\} \in E(G)$ for all $v, w \in V(G)$.

We write $G \to H$ if there exists a homomorphism from G to H. Furthermore, if the homomorphism is onto, then it is called an *epimorphism*. In addition, if there exists h^{-1} and it is a homomorphism from H to G, then we call it an *isomorphism* and graphs G and H are said to be isomorphic, in symbols $G \simeq H$. We write $H \tilde{\subset} G$ if H is isomorphic to any subgraph of G.

There is a straightforward equivalence between the properties of T-span and the existence of homomorphism from G to G_T^{d+1} (see [13]).

Proposition 4. *Given a graph G, any set T and a nonnegative integer d we have $\mathrm{sp}_T(G) \leqslant d$ if and only if $G \to G_T^{d+1}$.*

Let us note that if $T = \{0\}$, then the T-coloring problem reduces to the well-known vertex coloring problem, and moreover $G_T^{d+1} \simeq K_{d+1}$. Thus we get

Corollary 5. *Given a graph G and a positive integer d we have $\chi(G) \leqslant d$ if and only if $G \to K_d$.*

The composition of graph homomorphisms is still a graph homomorphism. Moreover, an image of a complete graph under a homomorphism is a complete graph with the same number of vertices so

Corollary 6. *If $K_n \to G$ then $K_n \tilde{\subset} G$.*

And

Proposition 7. *If $h: V(G) \to V(H)$ is a homomorphism then $\psi(G) \leqslant \psi(H(h(V(G))))$, where ψ is any of the functions from the list $\{\chi, \omega, \mathrm{sp}_T\}$.*

From the above is easy to see that if $G \to H$ and H is bipartite, then graph G is bipartite. Concluding this section note an important upper bound proved in [17].

Theorem 8 (Tesman [17]). *For any given graph G and set T the following inequality holds*

$$\mathrm{sp}_T(G) \leqslant |T| \cdot (\chi(G) - 1).$$

Let us also recall that

Theorem 9 (Brooks). *If G is a connected graph that is neither a complete graph nor an odd cycle, then $\chi(G) \leqslant \Delta(G)$.*

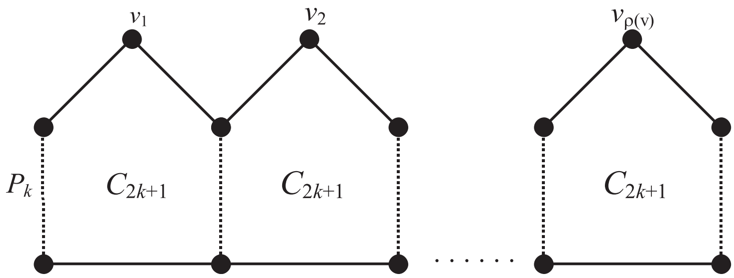

Fig. 1. Graph A_v^k replacing the vertex v.

3. Homomorphisms into odd cycles

The problem of graph homomorphism is considered in [1,5]. Let H be a fixed graph, the decision problem of the existence of a homomorphism from G to H will be denoted Hom(H), where G is any graph from the specified family. The most important result comes from [9].

Theorem 10 (Hell and Nesetril [9]). *The problem* Hom(H) *on arbitrary graphs is polynomial, whenever H is bipartite, otherwise it is NP-complete.*

In this section we prove that the problem Hom(C_{2k+1}) on subcubic graphs is NP-complete for every positive integer $k \geqslant 2$, in contrast to the problem Hom(C_3), which is polynomial. Moreover, we prove analogous result for 3-regular graphs and NP-completeness of the problem Hom(C_{2k+1}) on r-regular graphs, for every $r \geqslant 4$ and $k \geqslant 1$.

We start with a general construction. Let G be an arbitrary graph and k be any positive integer greater than 1. We replace each vertex $v \in V(G)$ of degree $\rho(v)$ with the graph A_v^k shown in Fig. 1 (the dotted vertical lines in Fig. 1 mean path P_k). We replace also every edge $\{v,w\} \in E(G)$ with the edge $\{v_i, w_j\}$ such that no two inserted edges are incident. Let G'_k be the graph constructed from G as above. It is easy to see that G'_k is always a subcubic graph.

Theorem 11. *The problem* Hom(C_{2k+1}), $k \geqslant 2$ *is NP-complete on subcubic graphs.*

Proof. By Theorem 10 it suffices to show $G \to C_{2k+1}$ iff $G'_k \to C_{2k+1}$. First, observe that $A_v^k \to C_{2k+1}$ and moreover for every homomorphism $h_v : V(A_v^k) \to V(C_{2k+1})$ we have $|h_v(\{v_1,\ldots,v_{\rho(v)}\})| = 1$. Otherwise, we have $h_v(v_i) \neq h_v(v_{i+1})$ for some $i \in \{1,\ldots,\rho(v)-1\}$, hence $h_v(v_i) = h_v(x)$, where $\{v_i,s\},\{v_{i+1},s\},\{s,x\} \in E(A_v^k)$ and $x \notin \{v_1,\ldots,v_{\rho(v)}\}$. Thus C_{2l-1} is subgraph of $C_{2k+1}(h(V(A_v^k)))$ for some $l < k$, which is impossible. So, constructing a homomorphism $g : V(G) \to V(C_{2k+1})$ from a homomorphism $g' : V(G'_k) \to V(C_{2k+1})$ is straightforward.

Conversely, let $g : V(G) \to V(C_{2k+1})$ be a homomorphism, then we let $g'(v_i) = g(v)$ and for $w \in V(A_v^k) \setminus \{v_1,\ldots,v_{\rho(v)}\}$ $g'(w) = \tau_v \circ h_v(w)$, where $h_v : V(A_v^k) \to V(C_{2k+1})$ is a

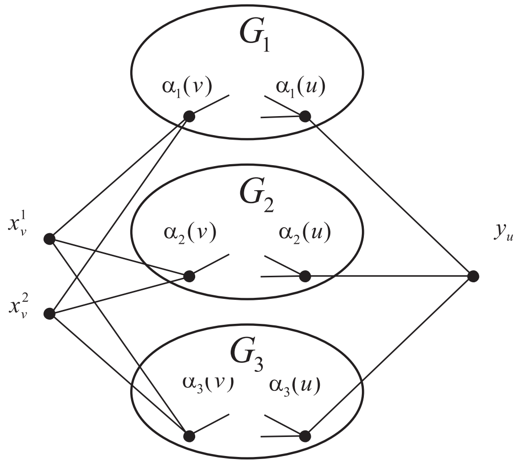

Fig. 2. A graph G'.

homomorphism and τ_v is any automorphism of C_{2k+1} such that $\tau_v(h_v(v_i)) = g(v)$. One can check that $g' : V(G'_k) \to V(C_{2k+1})$ is a homomorphism. □

Theorem 12. *The problem* Hom(C_{2k+1}), $k \geqslant 2$ *is NP-complete on 3-regular graphs.*

Proof. It suffices to show the equivalence $G \to C_{2k+1}$ iff $G' \to C_{2k+1}$ for any subcubic connected graph G, where $k \geqslant 2$ and G' is a cubic graph defined as follows. Let α_i be an isomorphism from graph G to its ith isomorphic copy G_i, for $i = 1, 2, 3$, which are vertex disjoint. Let $V_j \subset V(G)$ be the set of vertices of degree j. We define $V(G') = \bigcup_{i=1}^{3} V(G_i) \cup \bigcup_{v \in V_1} \{x_v^1, x_v^2\} \cup \bigcup_{u \in V_2} \{y_u\}$ and $E(G') = \bigcup_{i=1}^{3} E(G_i) \cup \bigcup_{v \in V_1} \bigcup_{i=1}^{3} \{\{x_v^1, \alpha_i(v)\}, \{x_v^2, \alpha_i(v)\}\} \cup \bigcup_{u \in V_2} \bigcup_{i=1}^{3} \{\{y_u, \alpha_i(u)\}\}$ (see Fig. 2). Assuming that x_v^j and y_u are different vertices for $j = 1, 2$ and $v, u \in V(G)$, it is obvious that G' is a cubic graph.

Now, suppose $g : V(G) \to V(C_{2k+1})$ is a homomorphism. Let $g' : V(G') \to V(C_{2k+1})$ be defined $g'(w) = g(v)$ for $w \in \{\alpha_1(v), \alpha_2(v), \alpha_3(v)\}$ and $v \in V(G)$, $g'(x_v^i) = g(z)$ for $\{z, v\} \in E(G)$, $g'(y_v) = g(z)$ for any z adjacent to v. Thus g' is a well-defined homomorphism. Conversely, if g' is a homomorphism from G' to C_{2k+1} then $g = g' \circ \alpha_1$ is a homomorphism from G to C_{2k+1}. □

Theorem 13. *The problem* Hom(C_{2k+1}) *is NP-complete on r-regular graphs for every fixed integer* $k \geqslant 1$ *and* $r \geqslant 4$.

Proof. By induction on $r \geqslant 4$, consider $r + 1$ isomorphic copies of any r regular graph. Using the analogous method as that in Theorem 12 we can show that the problem Hom(C_{2k+1}) is NP-complete for any $k \geqslant 2$ and for all $r \geqslant 4$. In [10] the author proved NP-completeness of edge 3-chromaticity of 3-regular graphs. Since line graphs of 3-regular graphs are 4-regular, the problem of 3-chromaticity of 4-regular graphs is NP-complete. The construction from Theorem 12 is carried over to the case Hom(C_3) on r-regular graphs with $r \geqslant 4$. □

4. Polynomial algorithm for cycles

We show a polynomial-time algorithm for graphs with $\Delta \leqslant 2$.

Theorem 14. *The T-Coloring Problem on graphs with degree not exceeding* 2 *can be solved in time* $O(n|T|^2 \log|T|)$.

Proof. Bipartite graphs can be optimally colored with 1 and $\min \mathbb{N} \setminus T + 1$, thus all we need is considering odd cycles. Let T be any set and a be an arbitrary integer. We ask if $\mathrm{sp}_T(C_{2k+1}) \leqslant a-1$. By Theorem 8 we have $\mathrm{sp}_T(C_{2k+1}) \leqslant 2|T|$. Thus using the standard bisection method we need only check $1+\log_2|T|$ inequalities to find $\mathrm{sp}_T(C_{2k+1})$.

In the following, we sketch the idea of the algorithm. Let $\mathrm{TAB}(v_i)[1\dots a]$ be a table of logical values associated with vertex v_i and defined as follows: $\mathrm{TAB}(v_i)[j] = \mathrm{TRUE}$ if and only if there exists a T-coloring of path $v_1,\dots,v_i$ using colors not greater than a such that v_1 is colored with 1 and v_i is colored with j. So, $\mathrm{TAB}(v_1)$ has value TRUE only on its first position and $\mathrm{TAB}(v_{i+1})[y] = \mathrm{TRUE}$ if and only if there exists $z \in \{1,\dots,a\}$ such that $|z-y| \notin T$ and $\mathrm{TAB}(v_i)[z] = \mathrm{TRUE}$. We see that there exists a T-coloring iff $\mathrm{TAB}(v_{2k+1})[j] = \mathrm{TRUE}$ for some $j-1 \notin T$, so constructing the T-coloring is straightforward. It is obvious that the complexity of the above algorithm is $O(k|T|^2 \log|T|)$. □

5. Main results

Based on Theorem 11 we can prove the main result of this paper. Before doing this, we introduce the following notion.

Definition 15. For a given set T, by d_T we mean the number such that $G_T^{d_T}$ is bipartite and $G_T^{d_T+1}$ is not bipartite.

Lemma 16. *For any set T the following inequality holds*:

$$d_T \leqslant \mathrm{sp}_T(K_3)$$

and, *moreover*, d_T *can be determined in polynomial time.*

Proof. Let us notice that $\chi(G_T^{d_T+1}) = \chi(G_T^{d_T}) + 1 = 3$. Thus from Corollary 5 it follows $G_T^{d_T+1} \to K_3$, hence by Proposition 7 $\mathrm{sp}_T(G_T^{d_T+1}) \leqslant \mathrm{sp}_T(K_3)$. By Proposition 4 $\mathrm{sp}_T(G_T^{d_T+1}) \leqslant d_T$. Assuming $\mathrm{sp}_T(G_T^{d_T+1}) \leqslant d_T - 1$ we get at once $G_T^{d_T+1} \to G_T^{d_T}$ but this contradicts the definition of d_T. So, we get $d_T = \mathrm{sp}_T(G_T^{d_T+1}) \leqslant \mathrm{sp}_T(K_3)$. By

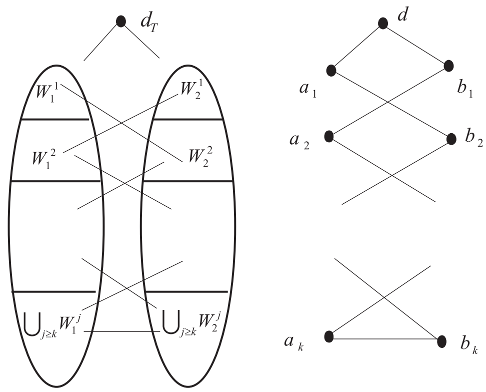

Fig. 3. A graph $G_T^{d_T+1}$ (left) and a cycle C_{2k+1} (right).

Theorem 8 $\mathrm{sp}_T(K_3) \leqslant 2|T|$, hence using the bisection method we can determine the greatest d_T such that $G_T^{d_T}$ is bipartite. This can be done in time $O(|T|^2 \log|T|)$. □

Lemma 17. *Given any set T, we have $d_T = \mathrm{sp}_T(K_3)$ if and only if $K_3 \tilde{\subset} G_T^{d_T+1}$.*

Proof. By Corollary 6 $K_3 \tilde{\subset} G_T^{d_T+1}$ is equivalent to $K_3 \to G_T^{d_T+1}$. Assume $K_3 \tilde{\subset} G_T^{d_T+1}$, then by Proposition 4 $\mathrm{sp}_T(K_3) \leqslant d_T$, hence from Lemma 16 it follows that $d_T = \mathrm{sp}_T(K_3)$. The converse implication is straightforward by Proposition 4. □

Let us denote by C_T the shortest odd-length cycle in graph $G_T^{d_T+1}$.

Lemma 18. *There exists a homomorphism $h : V(G_T^{d_T+1}) \to V(C_T)$.*

Proof. We only have to construct a homomorphism on the vertices of the connected component of $G_T^{d_T+1}$ containing vertex d_T, because the other components are bipartite. So let V_1 and V_2 be a bipartition of a bipartite graph obtained from this component by removing d_T and let W_i^j, $i = 1, 2$ and $j \geqslant 1$, be the vertex subset of V_i of distance j from vertex d_T in the graph $G_T^{d_T+1}$. Finally, let $W_1^0 = W_2^0 = \{d_T\}$. Let $C_{2k+1} = (\{d, a_1, b_1, \ldots, a_k, b_k\}, \{\{d, a_1\}, \{d, b_1\}, \{a_1, b_2\}, \{b_1, a_2\}, \ldots, \{a_{k-1}, b_k\}, \{b_{k-1}, a_k\}, \{a_k, b_k\}\})$ be any cycle isomorphic to C_T. Let us define $h(d_T) = d$, $h(W_1^j) = \{a_j\}$ and $h(W_2^j) = \{b_j\}$ for $j = 1, \ldots, k$ and $h(W_i^j) = h(W_i^k)$ for $j > k$, $i = 1, 2$ (see Fig. 3).

The construction of h is correct because any vertex from W_i^j, $j > 0$, can have neighbours only in the sets $W_{3-i}^{j\pm 1}$ and W_{3-i}^j, and the latter case is impossible for $j < k$. □

Lemma 19. *For any graph G the following equivalence holds: $G \to G_T^{d_T+1}$ if and only if $G \to C_T$.*

Proof. Let $G \to G_T^{d_T+1}$, hence from Lemma 18 it follows $G \to C_T$. Conversely, assume that $G \to C_T$. By definition $C_T \tilde{\subset} G_T^{d_T+1}$, thus we get $G \to G_T^{d_T+1}$. □

Theorem 20. *The T-Span Problem can be solved in polynomial time on subcubic graphs for all sets T satisfying $K_3 \tilde{\subset} G_T^{d_T+1}$. The Fixed T-Span Problem is NP-complete on cubic graphs for all sets T not satisfying $K_3 \tilde{\subset} G_T^{d_T+1}$.*

Proof. Let T be a fixed set and k be any positive integer. By Theorem 8 the case $G = K_4$ is polynomial and can be solved in $O(|T|^3)$ time (by Proposition 4 it reduces to the problem of finding the smallest d such that $K_4 \tilde{\subset} G_T^d$; by Theorem 8 $K_4 \tilde{\subset} G_T^{3|T|+1}$ and the fact that 0 is a vertex of a maximal clique of G_T^d, it reduces to searching all the triples of vertices of $G_T^{3|T|+1}$). For any subcubic graph $G \neq K_4$ we ask if $\mathrm{sp}_T(G) \leqslant k$.

Suppose that $K_3 \tilde{\subset} G_T^{d_T+1}$. Brooks' theorem implies $G \to K_3$, thus by Lemma 17 and Proposition 7 $\mathrm{sp}_T(G) \leqslant d_T$. According to Proposition 4 we have $\mathrm{sp}_T(G) < d_T$ iff G is bipartite, hence to solve T-Span Problem for graph G we only need to check if G is bipartite ($O(n+m)$ time) and if it is so then $\mathrm{sp}_T(G)$ equals the smallest positive integer not belonging to T (which we can find in $O(|T|)$ time). Otherwise, $\mathrm{sp}_T(G) = d_T$, computable in time $O(|T|^2 \log|T|)$.

Now assume that K_3 is not isomorphic to any subgraph of $G_T^{d_T+1}$ and let $k = d_T$. From Proposition 4 we have $\mathrm{sp}_T(G) \leqslant k$ iff $G \to G_T^{d_T+1}$. By Lemma 19 we get $\mathrm{sp}_T(G) \leqslant k$ iff $G \to C_T$ and, moreover, C_T is an odd cycle of length greater than 4. By Theorem 12 the problem $\mathrm{Hom}(C_T)$ on cubic graphs is NP-complete and so is the Fixed T-Span Problem. □

Corollary 21. *The T-Span Problem is NP-complete in the strong sense on 3-regular graphs.*

Proof. By Theorem 20 and Lemma 17 it suffices to verify that for $T = \{0, 2, 3\}$ we have $d_T = 4 < \mathrm{sp}_T(K_3) = 5$. □

It is worth observing that if for some set T we put $k = d_T$, then by Lemma 19 for any graph G the question if $\mathrm{sp}_T(G) \leqslant k$ is equivalent to $G \to C_T$. So, if for every $k \geqslant 1$ the problem $\mathrm{Hom}(C_{2k+1})$ is NP-complete on a class $\mathcal{G}$, then the Fixed T-Span Problem on the class $\mathcal{G}$ is NP-complete as well. Thus from Theorem 13 we have the following:

Theorem 22. *For every set T and integer $r \geqslant 4$ the Fixed T-Span Problem is NP-complete on r-regular graphs.*

Corollary 23. *The T-Span Problem is NP-complete in the strong sense on r-regular graphs for any $r \geqslant 3$.*

Table 1 Now we sum up all the above results in the following table. Recall that the numbers appearing in the third column are polynomially computable functions of T.

Table 1
The complexity of the T-Span Problem and T-Coloring Problem on graphs with bounded degree

Graph	Problem	Property of T	Complexity	Reference
$\Delta \leqslant 2$	T-Coloring Problem	any	$O(n\|T\|^2 \log\|T\|)$	Theorem 14
$\Delta \leqslant 3$	T-Coloring Problem	$\omega(G_T^{d_T+1}) \geqslant 3$	$O(n^2 + \|T\|^3)$	Theorem 20
3-regular	Fixed T-Span Problem	$\omega(G_T^{d_T+1}) \leqslant 2$	NPC	Theorem 20
r-regular, $r \geqslant 4$	Fixed T-Span Problem	any	NPC	Theorem 22

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