Safety Science 53 (2013) 51–60

Contents lists available at SciVerse ScienceDirect

Safety Science

journal homepage: www.elsevier.com/locate/ssci

A network flow model for interdependent infrastructures at the local scale

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ARTICLE INFO

Article history:
Received 29 September 2011
Received in revised form 19 June 2012
Accepted 27 August 2012
Available online 27 October 2012

Keywords:
Infrastructural interdependencies
Computational model
Networks
Costs
Optimisation

ABSTRACT

Infrastructures are becoming increasingly interconnected and it is essential to develop models that account for interdependencies between infrastructure systems at different scales. This paper presents a network model designed to achieve this, aimed at a local scale. Infrastructure systems are considered as a network. Vertices, which represent processes of production, consumption, transhipment and storage of resources (commodities), are connected by edges that capture commodity flows. Optimal performance of the network under normal and extreme conditions may be found by minimising the cost of commodity flow. The model is described, and the performance of interdependent infrastructure systems (energy, water and wastewater) during floods is demonstrated, using Monte Carlo simulation. The advantages and limitations of the model are considered before future developments are outlined.

1. Introduction

The proper performance of infrastructure is essential for supporting security, economic prosperity and social well-being, particularly in industrial societies like the UK. However, achieving this is, and will continue to be, a difficult challenge. The present UK infrastructure is subject to ever increasing loads due to population growth and suffers from deterioration, which makes it more vulnerable (ICE, 2010). In addition, the infrastructure must adapt to climate change; weather-related hazards, such as floods and heatwaves, are projected to increase in frequency and intensity (UKCP09). These events can damage energy and water supply, transport, communication routes and other infrastructure systems.

Therefore, it is essential to develop efficient strategies for improving infrastructure resilience, and ensure continuous and reliable performance in the future. Numerical models that simulate infrastructure performance may significantly assist in the development of such strategies. However, in order to provide a realistic prediction of the performance of various infrastructure systems, especially when they are subject to weather-related hazards, it is important to account for *interdependencies*, i.e., when failure in one system may cause severe disruptions and/or failures in other systems.

In this paper we present an extended, network-flow approach designed to simulate the performance of systems with infrastructural interdependencies. Nodes represent physical infrastructure assets, and associated processes of production, consumption, transhipment and storage of commodities (e.g., water, wastewater, electricity); electricity substations, water treatment works, water pumping stations and road intersections are examples of such assets. The edges of the network model assets associated with the flow/movement of commodities between nodes; e.g. power transmission, water and wastewater pipelines, or roads. The model allows optimisation of infrastructure performance by minimising the total operational cost associated with production, storage and commodity flow. This can be done under normal and extreme/hazardous conditions. A high node resolution, compared to typical models of national infrastructures, is intended to capture assets at local scales where, e.g., storage facilities and emergency generators become very important. To account for the damage of assets, parameters describing the assets can be treated as random variables.

The paper is structured as follows. The next section provides an overview of existing infrastructure models, placing particular emphasis on the modelling of infrastructural interdependencies. Section 3 presents the mathematical formulation as a standard form linear program. Probabilistic analysis of interdependent infrastructure systems, using the model in the context of Monte Carlo simulation, and a simple example problem, is then presented in section 4. We discuss the model (Section 5) and plans for its future development (Section 6), before concluding the paper.

2. Infrastructure modelling

There are many different ways to classify infrastructure models. In the context of this paper, it may be useful to distinguish between two kinds: (1) those concerned with a particular, *individual* infrastructure system (e.g., power, water or transport); and (2) those that consider *interdependencies* between different infrastructure systems.

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http://dx.doi.org/10.1016/j.ssci.2012.08.013

Nomenclature

General, graph-theoretic notation

k current commodity: $k \in \{1,2,\ldots,K\}$
K the number of modelled infrastructures: $k \in \{1,2,\ldots,K\}$
$G(E, V)$; the set of vertices and edges
V the set of vertices
E the set of edges
$e(i, j)$ edge connecting vertex i to vertex j
i used for current and origin vertex
j used for other and terminal vertex
$\gamma_k^{i\leftarrow}$ the set of links that carry commodity k that are incoming to node i
$\gamma_k^{i\rightarrow}$ the set of links that carry commodity k that are outgoing from node i

Set of constants in the model, i.e., those parameters whose values are defined by input data/values

$f_k^{e\,\max}$ maximum flow rate of commodity k on edge e
$\pi_k^{i\,\max}$ maximum production rate of commodity k at node i
$\kappa_k^{i\leftarrow}$ requirement of commodity k at node i
$\alpha_{n,k}^i$ coefficient relating the production of commodity k at node i to corresponding consumption rate of commodity n at node i
$\omega_k^{i\,\max}$ maximum capacities for storage of commodity k at node i
$\omega_k^i(0)$ stored commodity k at node i at initial time ($t = 0$). As the solution progress in time this constant is replaced by $\omega_k^i(t-\Delta t) = \omega_k^i$ – the amounts of stored commodities at the previous time step
ω_k^i the amount of storage of commodity k at node i in the previous time-step, i.e., compressed notation for $\omega_k^i(t-\Delta t)$
c_k^{fe} costs of the flow of commodity k on edge e
$c_k^{\omega i}$ costs of the storage of commodity k at node i
$c_k^{\pi i}$ costs of the production of commodity k at node i
$c_k^{\phi i}$ costs of the discharge of commodity k at node i
$c_k^{\lambda i}$ costs of the shortage of commodity k at node i
C total cost (function to be minimised)

Set of variables in the model, i.e., those parameters whose values are defined in the objective function

f_k^e flow rate of commodity k on edge e
κ_{kn}^i consumption (*uptake*) rates of commodity k, at node i, for the production of commodity n at node i, such that where $n \neq k$
π_{kk}^i production (creation) rate of commodity k at node i. This is commodity that does not require any uptake of other commodity kinds to produce it
λ_k^i unsatisfied demand (shortage) rate of commodity k at node i
$f_k^{i\leftarrow}$ in-flow rate of commodity k at node i
$f_k^{i\rightarrow}$ out-flow rate of commodity k at node i
π_k^i production rate of commodity k at node i
κ_k^i consumption rate of commodity k at node i
s_k^i storage rate of change of commodity k at node i
ϕ_k^i discharge rate of commodity k at node i

2.1. Modelling individual infrastructure systems

Individual infrastructure systems are usually described mathematically as networks that contain vertices (or *nodes*) and edges (or *links*), the elementary objects of graph theory (Wilson, 1996). Network representations underpin many modelling approaches to infrastructure systems. In the current context, vertices typically represent localised assets (e.g. water towers or pumping stations, electricity nodes) that are connected by edges (e.g. water pipes or transmission lines, etc.).

When a model is intended to be applied to the design and/or management of an infrastructure, it is usually formulated as an optimisation problem. The objective function can be associated with the allocation of resources, for example: of various energy sources to various demand patterns (Tan, 2011); of flows in order to minimise overflow and maintain balance within wastewater systems subject to inhomogeneous rainfall (Burkhard, 2000); or of water supply within the constraints of a given drainage basin (Hsu, 2002); to name a few.

Accurate simulation typically requires solution of a nonlinear problem. For example, alternating currents (AC) in an electrical network are described by a system of nonlinear equations (Wang et al., 2008), as are the flows and pressures in water supply networks, which can also be solved as a nonlinear optimisation problem (Collins et al., 1978). Solutions of nonlinear problems often require time-consuming, iterative techniques that do not always converge. These physically accurate, non-linear formulations, are therefore often simplified by linear approximations, especially when large-scale networks are being analysed; examples include the direct current (DC) load flow model for AC electrical networks (Wang et al., 2008), and a linear programming algorithm for analysis of water pipe networks (Berghout and Kuczera, 1997). Although these linear models provide quite accurate predictions of the performance of individual infrastructures, they require details of network-specific information (e.g., admittances of power lines, diameters of water pipes, etc.). Therefore, the same model cannot be used to capture different infrastructure networks (e.g., power and water).

Analysis of infrastructure networks can be further simplified by the use of network flow models (Ahuja et al., 1993). Such models only ensure flow continuity at nodes, while physical laws governing the flow of commodities within infrastructure systems are not fully satisfied. However, the accuracy of these models is acceptable (e.g., for the optimisation and planning of power and water systems) and they continue to be used (e.g., Padiyar and Shanbhaq, 1988; Sun et al., 1995; Manca et al., 2010). Furthermore, a major advantage of network flow models is that a single mathematical formulation can describe flows of commodities in *different* infrastructure systems, e.g., power, water, gas, transport networks. One the other hand, because network flow models do not fully satisfy physical laws they may not be appropriate for simulating the effect of local failures and disruptions, and associated overloads, on the performance of neighbouring components. Consequently, they are not useful for examining failure propagation (i.e., cascading failure) in individual systems (e.g., in power transmission networks). Moreover, using network flow models for the analysis of large-scale infrastructure networks may be time-consuming and this further complicates their use in the assessment of vulnerability of such networks.

Alternatively, *complex network* models, which have less emphasis on optimisation, have targeted these kinds of behaviours by examining the role of connectivity in large-scale networks, such as power grids (Wang and Rong, 2009; Duenas-Osorio and Vemuru, 2009), transport topologies (Doménech, 2009; Sun, 2009), and various ICT systems large enough, e.g., to contain heterogeneous network topologies amenable to multilayer network descriptions (Tsirakakis and Clarkson, 2009). Empirical work supports the idea that it is often the *topology* in large-scale systems that matters, not necessarily processes at the node. For example, the high degrees of complexity in systems such as the world-wide web deeply affect network functionality (Broder et al., 2000). Therefore, another important concept that can be explored by such models is *resilience*, where this means the ability of an infrastructure to recover from failure while delivering a service; topology is important in complex network approaches because it has the potential capacity to hold a network together when it is 'under attack' (i.e., subject to a disruptive event).

The resilience of individual infrastructures can also be investigated by system dynamic methods. Here, system behaviour is described by linear, dynamic models, i.e., sets of first-order linear ordinary differential equations (ODEs) that relate system states over time. The problem is formulated in the context of optimal feedback control, such that control (input) variables can be used to represent recovery efforts. For example, the approach has been employed to assess resilience of infrastructure systems by taking into account the recovery costs (Vurgin and Camphouse, 2011). The practical implementation of such methods requires a large amount of data about system states and their relationships, and analyses may be computationally inefficient, especially for large-scale networks.

2.2. Modelling infrastructure interdependencies

Modern infrastructure systems are highly *interdependent* in the sense that failures in one infrastructure can propagate, causing failures at nodes or edges considered to belong to a 'separate' system. Therefore, studying infrastructure systems in isolation may not provide adequate information about performance. Rinaldi et al. (2001) define four categories of infrastructure interdependencies: (1) *geographical*; (2) *physical*; (3) *cyber*; and (4) *logical*. Geographic dependencies refer to infrastructure assets 'linked' by proximity. Physical relationships refer to physically-connected assets. For example, a water pumping station (a water network asset) depends on electricity supplied by an electricity substation (an electricity network asset) to function. Cyber relationships refer to dependencies between infrastructure sectors that are connected via the internet. Logical relationships are more abstract; they might refer to the 'connection' between various financial state variables, or human behaviour. A review by Pederson et al. (2006), and reports on the UK floods of 2007 (Pitt, 2008), contain related conceptual discussions of infrastructures with the aim of organising knowledge schematically to help make sense of the complexity seen in such inter-related systems.

A methodology for the assessment of infrastructure interdependencies, and associated with them cascading failures, has been proposed by Franchina et al. (2011). This is a high-level, qualitative approach to assess the criticality of different infrastructures, based on the eventual impact of failures on quality of life. Another high-level, but quantitative, approach to modelling infrastructure interdependencies is based on the Leontief input–output model of equilibrium in regional and national-scale economies (Leontief, 1951). The use of the Leontief model to analyse interconnections between various infrastructure sectors was initially proposed by Haimes and Jiang (2001). The model is a system of linear equations that connect the inoperability of infrastructure sectors (i.e., inability to produce as-planned) with demand perturbations through an interdependency matrix. The model has been further developed (e.g., Haimes et al., 2005a, 2005b) and extended to include a probabilistic formulation of uncertainties associated with estimated demand perturbations (Barker and Haimes, 2009; Xu et al., 2012). This high-level approach considers *sector* interdependencies and component-level detail is not represented explicitly.

Agent-based modelling is often used to simulate infrastructure interdependencies (Pederson et al., 2006; Ghorbani and Bagheri, 2008). In agent-based modelling, the actions (and interactions) of agents are defined by simple sets of rules. By definition these are often coded at an individual level, but aggregated via simulations in which rules and patterns of interaction can be seen to develop over time. In principle, this approach allows any level of refinement (scalability) depending on the agent definition. In this way, infrastructures are viewed as communities of interacting agents (physical assets and decision-making entities) and behave, together, as a complex adaptive system (Rinaldi et al., 2001; Brown et al., 2004; van der Lei et al., 2010). It has been also suggested that combining ABM at the level of a single infrastructure with the High Level Architecture (HLA) simulation standard can facilitate the interoperability of multiple-type models and simulations (Eusgeld et al., 2011). The level of detail included is often very fine, i.e., thousands of agents are often required. This makes such models computationally expensive. Another disadvantage is that input parameters map to model behaviour, which is typically said to *emerge*, in complex ways; the danger is that the usefulness of abstraction is lost.

Some of the network-based models described in the previous section have also been applied to the analysis of infrastructure interdependencies. For large-scale examples, the emphasis has been placed on vulnerability, which has been examined through topological analysis (e.g., Ouyang and Duenas-Osorio, 2011; Wang et al., 2012). A number of extensions to the traditional network flow model have also been proposed. For example, in order to model infrastructure systems subject to disruptions, when the total supply of a commodity can be less than its total demand, Lee et al. (2007) introduce slack variables to a model that uses node types (supply, demand and transhipment) based on the traditional network flow model. Slack variables represent the shortfall in meeting demand at nodes and corresponding weighting factors are set for the cost of unmet demand. Individual infrastructures are modelled by separate networks and additional variables connecting interdependent nodes are therefore introduced. Time-invariance requires that any change in network state is given by a new set of input data, which needs to be prepared before a new iteration. This model has been applied to the analysis of an infrastructure system supplying a hospital during a disaster event (Arboleda et al., 2009). Svendsen and Wolthusen (2007) propose a conceptual network flow model with multifunctional nodes, where a *general* node acts as a consumer and a producer, and also represents changing storage. Time thus requires explicit consideration, which is achieved by considering network states at discrete time points. These additions are appropriate for systems that have become damaged and exploit storage capacities during recovery time, although it should be noted that to the best of our knowledge the model is not fully detailed and example analysis deals only with topological analysis.

An interesting model, including the simulator I2Sim, was presented by Rahman et al. (2011). The authors call it a cell-channel model, where interdependencies between infrastructures are described using an extended Leontief input–output model (see above). We mention the model here because it has similarities with the model proposed by Svendsen and Wolthusen (2007); a cell is very similar to a multi-functional node; a 'channel' is an edge; commodities are referred to as 'tokens'; and simulations are carried out at discrete time.

Finally, for the purposes of this paper we chose to distinguish between *individual interdependent* infrastructure models, although various reviews that contain alternative classifications, and more inclusive discussion, can be found elsewhere (Pederson et al., 2006; Ghorbani and Bagheri, 2008; Xiao et al., 2008; Griot, 2010; Satumtira and Dueñas-Osorio, 2010).

3. Model description

3.1. General network description

The model developed in this study is intended to simulate the operation of interdependent infrastructure systems at the local (community) scale, under normal and extreme (i.e., when the infrastructure assets are damaged) conditions. The model takes into account uncertainties associated with the performance of damaged assets by treating relevant parameters as random variables. In order to be suitable for probabilistic analysis the model needs to be computationally efficient.

The model description contains graph-theoretic notation. Again, edges represent interactions between vertices. For example, edges can be power lines, water pipes, roads, etc., whereas vertices can be power plants, water treatment plants, railway stations, etc. It is assumed that any directed edge $e(i,j)$ defines an interaction between vertex i and j. For example, an electricity power station might supply a substation that in turn supplies a water pumping station. In this way, when we consider a number of connections between different vertices, edges often connect together various assets such that a path is formed from a *source* vertex to a *consumption* vertex. For example, a source vertex might be a power plant that delivers electricity to a consumption vertex (a house or hospital, etc.) via a number of intermediate edges.

Although the scope of the model is restricted to the community scale, it may be necessary to ignore administrative/geographical boundaries, if important supply vertices are located outside the modelled area. In other words, in order to allow important supply vertices to be included, the boundaries of the modelled infrastructure need not be entirely contained within the geographical boundaries of a chosen community.

In order to model a set of different infrastructures we define a digraph $G(E,V)$, where E is the set of directed edges of the form $e(i,j)$ and V is the set of vertices. Vertex i has a neighbourhood $\Gamma(i) \subseteq G$ containing vertices adjacent to i. It is useful also to distinguish between edges $e(i,j)$ coming into i and the edges $e(j,i)$ leaving i – we label these as $\gamma^{\leftarrow} \subseteq \Gamma$ and $\gamma^{\rightarrow} \subseteq \Gamma$, respectively. Edges are similar to those defined in standard network flows approaches (Ahuja et al., 1993; Lee et al., 2007). However, in contrast to these standard approaches, vertices are defined in a *general* way to accommodate function-specific asset types, similarly to the multi-functional node model proposed by Svendsen and Wolthusen (2007). Instead of supply and demand vertices, which are typical in the standard approaches, we introduce additional processes of production, consumption and storage to the node, whereas transhipment remains the same as standard approaches. Moreover, a single vertex represents all these functions, in a *general* way. This means that a commodity can be supplied from one vertex to the next by being taken from storage. Similarly, a commodity can be consumed in order to produce another commodity or simply to satisfy a demand. To simulate the operation of damaged infrastructure it is necessary to deal with unsatisfied demand. Following Lee et al. (2007), a variable representing shortfall (or shortage) is introduced, along with the corresponding cost (or penalty).

It is expected that there will be uncertainties associated with damage caused by hazards. To take this into account, various parameters related to the infrastructure operation (e.g., flow capacities of edges, production capacities of vertices) can be treated as random variables.

Finally, in order to describe different kinds of commodities (e.g., water, electricity, gas) associated with different infrastructure systems, we say that there are K commodities such that k: $k \in \{1,2,\ldots,K\}$. Some of the definitions introduced above are illustrated in Fig. 1.

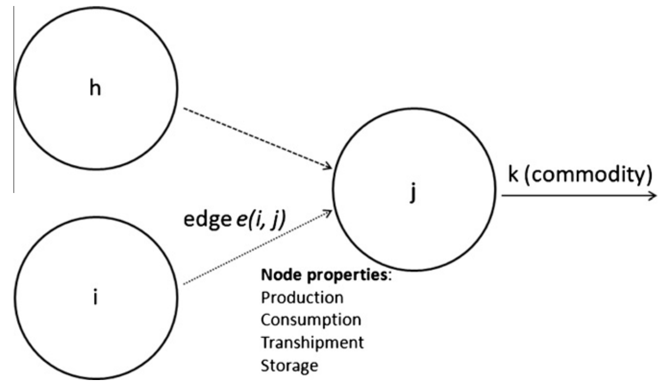

Fig. 1. Elements of the model: each node, connected to other nodes by edges, has a number of associated properties.

3.2. Consideration of time factor

The formulation presented below is given for a single point in time t. The solution starts at $t = t_0$, a network under normal conditions. Time progresses in increments Δt that do not need to be constant and can thereby simulate the performance of the network as it is being damaged then restored. However, all variables and constants remain unchanged within a time step. This approach to time dependency is similar to those in (Svendsen and Wolthusen, 2007; Rahman et al., 2011). Note: mathematically it is necessary that variables change during the solution (or search) process, although the final solution gives the optimal values of variables for that time step.

It is also important to note that minimising the total costs separately at each time step does not give an optimal solution for the network performance over the *entire* period. For example, when a network is under normal conditions it is obvious that the optimal amount of storage is zero; if everything works properly there is no need for stored commodities because storage incurs additional cost. However, when the network is damaged by a hazard, the availability of commodities from storage may reduce the total cost (since there are additional high costs penalising unsatisfied demand). In order to take this into account we can set artificially negative costs for storage (to prevent the use of stored commodities under normal conditions) or minimise the total cost over all the considered time steps, i.e., the objective function should be the sum of the costs associated with each time step. The second approach seems better. In order to use this approach we need to know the time required for restoring the network to its original undamaged conditions and very often this cannot be predicted with certainty.

3.3. Description of processes at a node

A number of variables ($f_k^e \pi_{kk}^i \kappa_{kn}^i$, λ_k^i) relate processes concerning commodity k at node i. These variables are the flow of commodity k on edge e, denoted f_k^e, the production of a commodity k at node i, denoted π_{kk}^i, the consumption of commodity k for the production of commodity n, denoted κ_{kn}^i, and the shortage of commodity k at node i, denoted λ_k^i. The variable λ_k^i representing shortage is thus similar to the slack variable introduced by Lee et al. (2007).

We now explain relations between these variables and their associated constants. We consider, in turn, the three aspects at the node: (a) *in-flow*, and *out-flow*, (b) *consumption*, and (c) *production*.

3.3.1. Flow

In-flow rates to vertex i are given by $f_k^{i\leftarrow} = \sum_{e\in\gamma_i^{\leftarrow}} f_k^e$ and outflow rates by $f_k^{i\rightarrow} = \sum_{e\in\gamma_i^{\rightarrow}} f_k^e$ where $\gamma^{\leftarrow}$ and $\gamma^{\rightarrow}$ are the incoming and outgoing edges of i, respectively.

3.3.2. Consumption

The consumption rate is summarised as:

$$\kappa_k^i = \kappa_{k,k}^i + \sum_{n \neq k} \kappa_{k,n}^i \tag{1}$$

where two kinds of consumption are defined by the first and second terms in the right-hand-side. In the first right-hand-side term, commodity k is simply *digested* $\kappa_{k,k}^i = \kappa_k^{i\leftarrow} - \lambda_k^i$ on the basis of need $\kappa_k^{i\leftarrow}$, which is unmet if there is a shortage λ_k^i. In the second term, there is straightforward *uptake* $\kappa_{k,n}^i$ of commodity k as raw material in production of commodity n. Thus, consumption rate is defined as:

$$\kappa_k^i = \kappa_k^{i\leftarrow} - \lambda_k^i + \sum_{n \neq k} \kappa_{k,n}^i \tag{2}$$

3.3.3. Production

The production rate is summarised as $\pi_k^i = \pi_{kk}^i + \sum_{n \neq k} \pi_{nk}^i$ such that two kinds of production are defined by the first and second terms in the right-hand-side. The first right-hand-side term is the *creation* $\pi_{k,k}^i$ of a commodity where required consumables are undefined in $\gamma^{i\leftarrow}$ – i.e., for supply nodes. In the second *manufacture* $\pi_{n,k}^i = \alpha_{n,k}^i \kappa_{n,k}^i$ is the rate of production of commodity k that consumes commodity n. Note, we assume that production rate is linearly proportional to consumption rate, ensuring compatibility with the standard form of linear optimisation problem (see below). Thus, production rate is:

$$\pi_k^i = \pi_{k,k}^i + \sum_{n \neq k} \alpha_{n,k}^i \kappa_{n,k}^i \tag{3}$$

The relationships of the processes at the node are illustrated at two levels of depth, one more schematic and the other related to the in-text equations (see Fig. 2).

3.4. Node balance and other general constraints

The balance equation for a node is formulated as:

$$(f_k^{i\leftarrow} - f_k^{i\rightarrow}) + (\pi_k^i - \kappa_k^i) - (s_k^i + \phi_k^i) = 0 \tag{4}$$

where the third term is the rate of unbalanced flow occurring in storage (s_k^i) or that discharged (ϕ_k^i). Substituting Eqs. (2) and (3) into Eq. (4) gives:

$$(f_k^{i\leftarrow} - f_k^{i\rightarrow}) + \left(\pi_{kk}^i + \sum_{n \neq k} \alpha_{nk}^i \kappa_{nk}^i - \kappa_k^{i\leftarrow} + \lambda_k^i - \sum_{n \neq k} \kappa_{kn}^i\right) - (s_k^i + \phi_k^i) = 0 \tag{5}$$

It is assumed that a commodity must be discharged only when its storage capacity at the node is exceeded. This leads to another general constraint:

$$\omega_k^i + s_k^i \Delta t + \phi_k^i \Delta t \geqslant \omega_k^{i,\max} \tag{6}$$

where ω_k^i is the amount of stored commodity at the previous time-step, i.e., $\omega_k^i = \omega_k^i(t - \Delta t)$, as mentioned above, and $\omega_k^{i,\max}$ the maximum storage capacity.

The standard form for a linear optimisation problem, Eq. (10), determines that only variables multiplied by constant coefficients should be in the left hand side of a general constraint. Thus, Eq. (5) is re-written as:

$$(f_k^{i\leftarrow} - f_k^{i\rightarrow}) + \left(\pi_{kk}^i + \sum_{n \neq k} \alpha_{nk}^i \kappa_{nk}^i + \lambda_k^i - \sum_{n \neq k} \kappa_{kn}^i\right) - (s_k^i + \phi_k^i) = \kappa_k^{i\leftarrow} \tag{7}$$

and Eq. (6) as:

$$s_k^i + \phi_k^i \geqslant \frac{\omega_k^{i,\max} - \omega_k^i}{\Delta t} \tag{8}$$

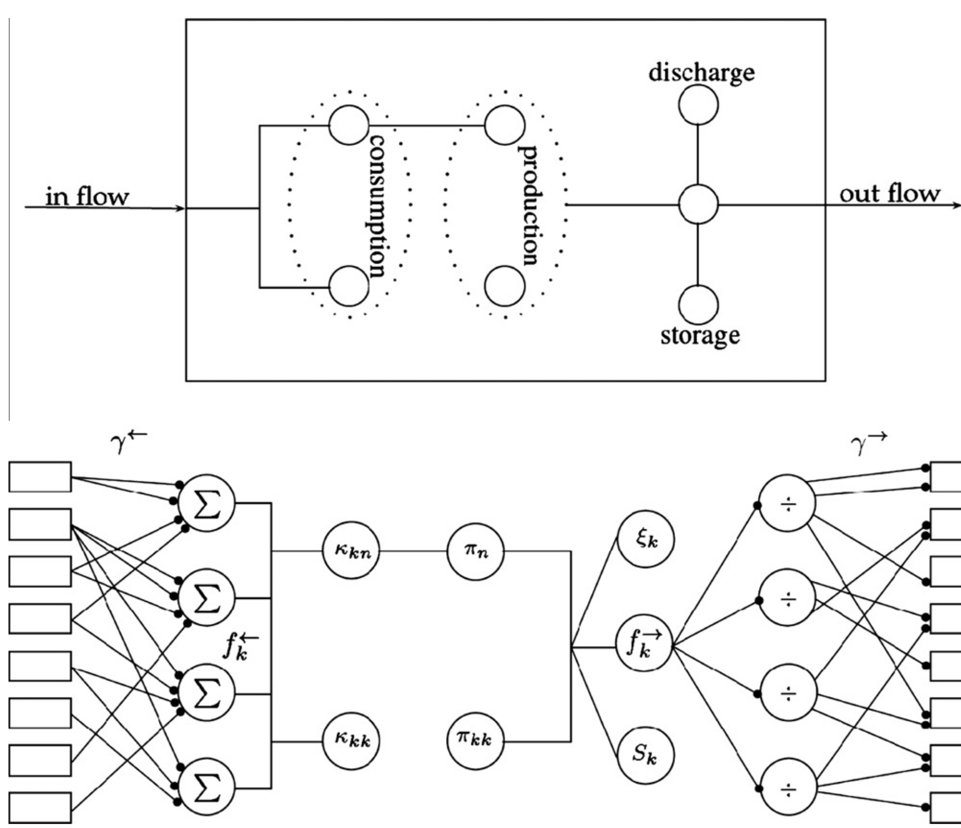

Fig. 2. Top: schematic showing the overall processes associated with the node. Bottom: schematic relating the processes to the equations.

It is worth noting that the last constraint does not prevent discharge even when the storage in not full (i.e., it may be discharged more than required in order not to overfill the storage capacity). It will depend on the relation between the cost of storage and the cost of discharge. If the latter is larger than the cost of storage, then unnecessary discharge should not occur.

Finally, the production of the commodity k at node i cannot exceed its limit, ($\pi_k^{i,\max}$). This determines another general constraint:

$$\pi_{k,k}^i + \sum_{m \neq k} \alpha_{m,k}^i \kappa_{m,k}^i \leqslant \pi_k^{i,\max} \tag{9}$$

3.5. Formulation of optimisation problem

The model presented below will be formulated according to the structure of the *standard form* of a linear optimisation problem:

$$\begin{aligned} &\min_{\mathbf{x} \in \Re^n} \quad \mathbf{c}'\mathbf{x} \\ &\text{s.t.}: \quad \mathbf{b}_l \leqslant \mathbf{A}\mathbf{x} \leqslant \mathbf{b}_u \quad \text{(general constraints)} \\ &\qquad\quad \mathbf{x}_l \leqslant \mathbf{x} \leqslant \mathbf{x}_u \quad \text{(bounds on variables)} \end{aligned} \tag{10}$$

where x is the vector of variables, $\mathbf{c}$ is the objective coefficient vector (in our case, costs), $\mathbf{A}$ is the coefficient matrix, and the vectors $\mathbf{b_l}$, $\mathbf{b_u}$, $\mathbf{x_l}$ and $\mathbf{x_u}$ are lower and upper bounds on the constraints and the variables, respectively.

As should be clear from this formulation, the constraints should be expressed only in terms of the variables appearing in the objective function. This means that, if the total cost (i.e., objective function) depends on discharge and unsatisfied demand (see below), then variables representing these quantities should be in the constraints. Moreover, all constants except for the coefficients of the matrix $\mathbf{A}$ should be moved into the bounds.

3.5.1. Objective function and constraints: general case

The total cost C at a given point in time t can be expressed as

$$\begin{aligned} C = &\sum_{e \in E} c_k^{fe} f_k^e + \sum_{i \in V} c_k^{\omega i} \frac{1}{2}((\omega_k^i(t) + \omega_k^i(t - \Delta t)) + \sum_{i \in V} c_k^{\pi i} \pi_k^i \\ &+ \sum_{i \in V} c_k^{\phi i} \phi_k^i + \sum_{i \in V} c_k^{\lambda i} \lambda_k^i, \forall k \in K \end{aligned} \tag{11}$$

where the terms from one to five represent costs associated with *flow*, *storage*, *production*, *discharge* and *shortage*, respectively. Note that the cost of storage is based on the average amount of stored commodity within the time interval ($t - \Delta t$, t), where the storage $\omega_k^i(t)$ for the kth commodity at the ith node is:

$$\omega_k^i(t) = \omega_k^i(t - \Delta t) + s_k^i \Delta t \leqslant \omega_k^{i,\max} \tag{12}$$

Substituting Eq. (12) for the second term of Eq. (11) gives $\sum c_k^{\omega i} \omega_i(t - \Delta t) + \sum c_k^{\omega i} \frac{1}{2}(s_k^i \Delta t)$ where s_k^i is the rate of change of the storage of commodity k at node i. The constant term $\sum c_k^{\omega i} \omega_i(t - \Delta t)$ is excluded from the objective function; in order to obtain the total cost, including the full cost of storage, this term can be added after optimisation. Substituting Eq. (3) for π_k^i into Eq. (11) gives the total cost:

$$\begin{aligned} C = &\sum_{e \in E} c_k^{fe} f_k^e + \sum_{i \in V} c_k^{\omega i} \frac{1}{2}(s_k^i \Delta t) + \sum_{i \in V} c_k^{\pi i} \left(\pi_{kk}^i + \sum_{i \in V} \alpha_{n,k}^i \kappa_{kk}^i \right) \\ &+ \sum_{i \in V} c_k^{\phi i} \phi_k^i + \sum_{i \in V} c_k^{\lambda i} \lambda_k^i, \end{aligned} \tag{13}$$

To minimise the total cost the optimisation problem can thus be formulated as:

$$\begin{aligned} \min z = &\sum_{e \in E} c_k^{fe} f_k^e + \sum_{i \in V} c_k^{\omega i} \frac{1}{2}(s_k^i \Delta t) + \sum_{i \in V} c_k^{\pi i} \left(\pi_{kk}^i + \sum_{i \in V, n \neq k} \alpha_{n,k}^i \kappa_{nk}^i \right) \\ &+ \sum_{i \in V} c_k^{\phi i} \phi_k^i + \sum_{i \in V} c_k^{\lambda i} \lambda_k^i, \quad \forall k \in K, \quad \text{and} \quad \forall e \in E, \quad \text{and} \quad \forall i \in V \end{aligned} \tag{14}$$

subject to general constraints:

$$\begin{aligned} &(f_k^{i\leftarrow} - f_k^{i\rightarrow}) + \left(\pi_{kk}^i + \sum_{n \neq k} \alpha_{nk}^i \kappa_{nk}^i + \lambda_k^i - \sum_{n \neq k} \kappa_{kn}^i \right) - (s_k^i + \phi_k^i) \\ &\quad = \kappa_k^{i\leftarrow} \quad \forall k \in K \quad \text{and} \quad \forall i \in V \end{aligned} \tag{15}$$

$$s_k^i + \phi_k^i \geqslant \frac{\omega_k^{i,\max} - \omega_k^i}{\Delta t}, \quad \forall i \in V \quad \text{and} \quad \forall k \in K \tag{16}$$

$$\pi_{k,k}^i + \sum_{m \neq k} \alpha_{m,k}^i \kappa_{m,k}^i \leqslant \pi_k^{i,\max}, \quad \forall i \in V \quad \text{and} \quad \forall k \in K \tag{17}$$

and variable bounds:

$$0 \leqslant f_e \leqslant f_e^{\max}, \quad \forall e \in E \tag{18}$$

$$0 \leqslant \pi_{kk}^i, \quad \forall i \in V \quad \text{and} \quad \forall k \in K \tag{19}$$

$$0 \leqslant \kappa_{nk}^i, \forall i \in V \text{ and } \forall k \in K \text{ and } \forall n \in K \quad \text{and} \quad \forall n \neq k \tag{20}$$

$$-\frac{\omega_k^i}{\Delta t} \leqslant s_k^i \leqslant \frac{\omega_k^{i,\max} - \omega_k^i}{\Delta t}, \quad \forall i \in V \quad \text{and} \quad \forall k \in K \tag{21}$$

$$0 \leqslant \phi_k^i, \quad \forall i \in V \quad \text{and} \quad \forall k \in K \tag{22}$$

$$0 \leqslant \lambda_k^i \leqslant \kappa_k^{i\leftarrow}, \quad \forall i \in V \quad \text{and} \quad \forall k \in K \tag{23}$$

where $f_e^{\max}$ is the maximum flow capacity of an edge.

3.5.2. Objective function and constraints: normal conditions

Under normal conditions, when storage is not used and discharge and unsatisfied demand do not occur, the above general formulation can be significantly simplified. Variables representing storage, discharge and shortage can be excluded from the analysis. Thus, the objective function and the constraints become:

$$\min \ z = \sum_{e \in E} c_k^{fe} f_k^e + \sum_{i \in V} c_k^{\pi i} \left(\pi_{kk}^i + \sum_{i \in V} \alpha_{n,k}^i \kappa_{kk}^i \right), \tag{24}$$

subject to general constraints:

$$(f_k^{i\leftarrow} - f_k^{i\rightarrow}) + \left(\pi_{kk}^i + \sum_{n \neq k} \alpha_{nk}^i \kappa_{nk}^i - \sum_{n \neq k} \kappa_{kn}^i \right) = \kappa_k^{i\leftarrow} \tag{25}$$

$$\pi_{k,k}^i + \sum_{m \neq k} \alpha_{m,k}^i \kappa_{m,k}^i \leqslant \pi_k^{i\,\max} \tag{26}$$

and variable bounds:

$$0 \leqslant f_e \leqslant f_e^{\max}, \quad \forall e \in E \tag{27}$$

4. Example

The application of the proposed model is illustrated by a hypothetical example that simulates the interactions between several energy system and water system components, shown in Fig. 3. The energy system is represented by an electricity substation (node 1), which is connected by power lines to a water tower (node 3) and several other electricity 'consumers', including a residential area (node 4), two nursing homes (node 5) and a community

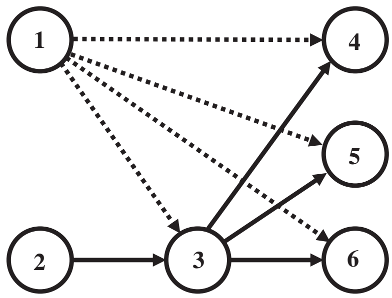

Fig. 3. Network model of considered infrastructure. Nodes: node 1 – electricity substation; node 2 – water treatment plant; node 3 – water tower; node 4 – residential area; node 5 – nursing homes; node 6 – community hospital. Edges: water pipes (solid lines); power lines (dashed lines).

hospital (node 6). In addition to the water tower, the water system includes a water treatment plant (node 2). Thus, flows of two commodities – electricity supplied by the electricity substation (whose maximum capacity is assumed to be 2000 kW h/day) and water supplied by the water treatment plant (with maximum capacity of 100 m^3/day) – are considered. The three main consumer nodes – the residential area, the two nursing homes and the community hospital – have daily demands that are given in Table 1.

The water tower, which requires electricity for pumping water into the tower, is a form of transhipment node in the water system; we assume that 1 kW h is consumed to pump 7.5 m^3 of water. The tower capacity is 160 m^3, i.e., it contains double the daily demand of water. It is also assumed that the water tower, each of the nursing homes, and the hospital, have the same kind of emergency diesel generators. Each generator has a power of 12 kW, a storage tank for 100 l of fuel, and can produce 3 kW h per litre of consumed diesel. Thus, diesel fuel is a third commodity included in the example, although its flow is not simulated. However, this can be done with the proposed model, for example, if the transport system (i.e., roads) is also considered.

The costs of flow, storage, production and shortage do not represent actual costs and are assigned to ensure that the commodities are distributed between the consumers as intended. For example, in order to prevent the use of stored commodities (i.e., water and fuel), until there is no other way to meet demands, the costs of their storage are set negative. It is also assumed that the community hospital has the highest priority in receiving required commodities, followed by the nursing homes and then the residential area. The costs of shortages of water and electricity for these consumers are set accordingly, i.e., the highest costs for the hospital, a lower costs for the nursing home and the lowest ones for the residential area.

The performance of this infrastructure network, subject to a natural hazard (e.g., flood), is simulated. It is assumed that the natural hazard partially damages the production capacities of the electricity substation and the water treatment plant. This is taken into account by treating these capacities as random variables which are described by beta distributions defined on the interval (0, π_k^{imax}), where $\pi_1^{1max} = 2000$ kW h/day is the maximum production capacity of the electricity substation and $\pi_2^{2max} = 120$ m^3/day of the water treatment plant, respectively. It is also assumed that the means of the random variables are equal to $\pi_k^{imax}/2$, and coefficients of variations equal to 0.40. It is worth noting, again, that the example is purely hypothetical; the above assumptions are made for illustrative purposes only.

Table 1
Electricity and water demands.

Consumer	Demands	
	Electricity (kW h/day)	Water (m^3/day)
Residential area	1200	45
Nursing homes	400	25
Community hospital	200	10

Generally, natural hazards may damage infrastructure components that are represented by nodes *and* edges, resulting in partial or full failure, which may be taken into account in the model in a number of ways. The infrastructure network will function in a partially damaged condition until the production capacities of the electricity substation and the water treatment plant will be restored, which may take several days. The example examines what happens with the services provided by the infrastructure (i.e., supply of electricity and water to the consumers) when it takes up to 10 days to restore these capacities. The analysis is carried out using a daily time step ($\Delta t = 1$ day). There are a number of parameters characterising the performance of the infrastructure that have been estimated. However, results for only one parameter (relative satisfied demand) are presented herein. For the commodity k at node i the latter is defined as $(1 - \lambda_k^i/\kappa_k^{i\leftarrow})$ – i.e., when it equals unity the demand is fully satisfied and when it is zero the node receives none of the commodity.

The results (shown in Figs. 4 and 5) are obtained by Monte Carlo simulation and are presented in terms of the expected value and standard deviation of the relative satisfied demand for the three consuming nodes: residential area ($i = 4$); nursing homes ($i = 5$); and community hospital ($i = 6$); and two commodities: electricity ($k = 1$); and water ($k = 2$). As can be seen, the community hospital is well protected in the case when the production capacities of the electricity substation and the water treatment plant are partially damaged. It will continue to receive the required electricity and water supply with very little disruption, even if 10 days are required to restore the damaged capacities. The nursing homes are reasonably protected. During the first 2 days after the damage their demand for electricity and water will be fully satisfied, after that at around 90%, but with relatively high uncertainty (the corresponding standard deviations are greater than 0.2). As expected, the residential area is the least protected. Its water demand will be satisfied in full only for a day after the damage while the supply of electricity will drop immediately. In 3 days its supply of electricity will be reduced on average to about 40% of demand, and water to 50%, with very high uncertainty – i.e., if it will take more than 3 days to restore the electricity substation and the water treatment plant, then it is highly probable that the residential area will receive almost no electricity and water.

5. Discussion

The purpose of our model is to simulate commodity flows within local communities, especially during times when infrastructual damage has resulted from a natural hazard. Since highly accurate prediction of the extent of damage is not possible, uncertainties regarding damaged infrastructures require consideration. To achieve this, various infrastructure parameters (e.g., maximum production capacities at nodes, maximum flow capacities of edges) are treated as random variables. These properties, in principle, make the model suitable for probabilistic analysis.

In Section 2, a number of different approaches to modelling the performance of a single infrastructure system and interconnected infrastructure systems have been discussed. Based on this discussion, it is clear that network models are the most suitable for our purposes. However, taking into account the scale of infrastructure networks at the community level, and the need to obtain flows of commodities at each node, the models applied to the topological analysis of 'vulnerability' in large-scale networks are of little rele-

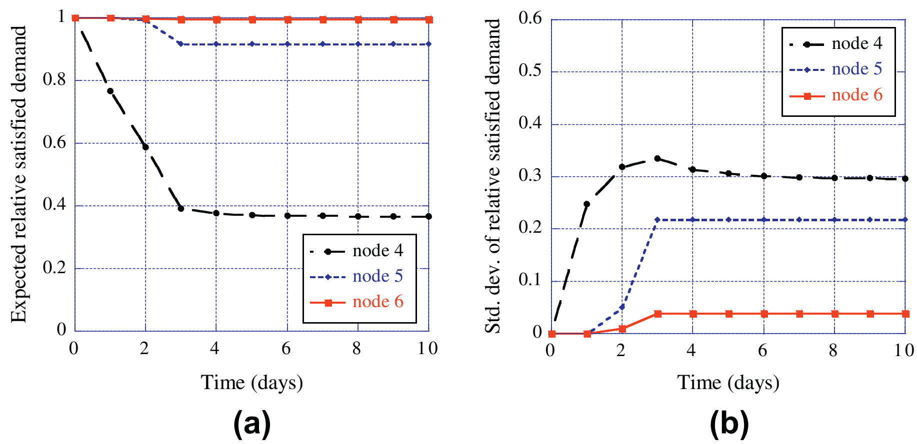

Fig. 4. Relative satisfied demand of electricity: (a) expected value; (b) standard deviation.

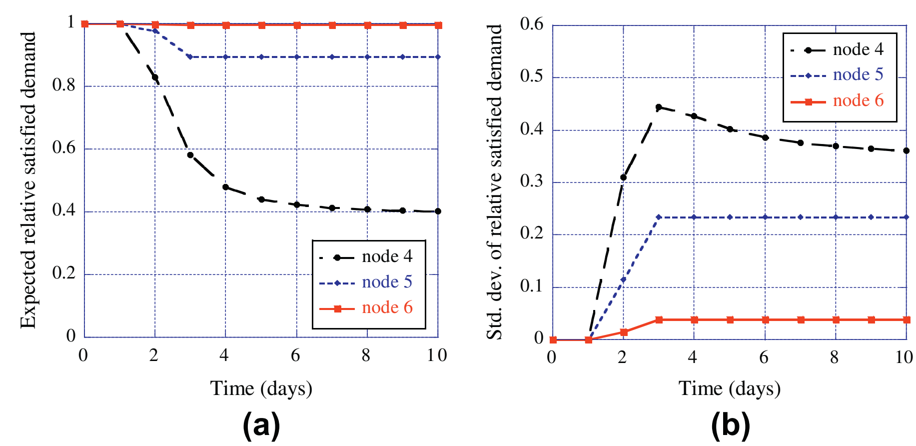

Fig. 5. Relative satisfied demand of water: (a) expected value; (b) standard deviation.

vance to the current work. Furthermore, computational efficiency matters. Restrictions on the computational time required for an analysis therefore prevent (or, more correctly, make very difficult) the use of infrastructure-specific models where the flow of commodities involve non-linear equations. Given the number (this depends on the probabilistic technique used) of iterations required, along with the fact that non-linear models would add complexity to the model description (individual infrastructure systems would require specific equations) a linear formulation was adopted.

Thus, the model we present should be seen as a reasonable compromise, between computational efficiency and accuracy, given that traditional network flow models are still used for analysis of individual infrastructures. The model has a number of desirable features, as follows: it is capable of providing very high resolution; it can account for storage of commodities that are drawn on in times of crisis; it is time dependent, which is important for simulating the performance of infrastructures during processes of damage and repair; it can also represent different infrastructure systems as a single network, which simplifies the modelling of interdependencies; computationally efficiency allows probabilistic analysis of local networks, as is demonstrated with Monte Carlo simulation. On the other hand, the model has the following limitations: it is not useful when the performance of large-scale infrastructure systems is of interest – e.g., cascading failures in large-scale electrical (or water) networks not only because it would involve very long computational times, but because the flow of commodities is simulated in a simplified manner. Thus vulnerability assessment of large-scale networks is not possible; it is a linear model and, therefore, predictions of commodity flows, which in reality involve nonlinearities, may not be highly accurate (further discussion on the accuracy of linear models may be found in a number of publications citied in Section 2.1).

Finally, the formulation of the model as a linear optimisation problem (where the objective function represents the total cost of flow, storage, production and shortage of commodities) serves two purposes. When the cost values in the analysis represent actual costs associated with these activities (i.e., production costs, fines for unmet demand as shortage costs, etc.) the model can be used to optimise the infrastructure performance in purely economic terms. However, when the distribution of commodities during disruptions caused by natural hazards is of interest, notional costs representing the preferences of a decision-maker may be employed. This is a mechanism that allows choices regarding the allocation of resources between various consumers to be made, as the example simulation in Section 4 demonstrates.

6. Further model development

As this suggests, one of the main intentions is to explore the network flow model in the context of real, weather-related hazards. This will be achieved, initially, by coupling it to a flood model. The coupling could be achieved by using a geographical information system (GIS) to link to an existing flood models. Once augmented with spatial data, the infrastructure nodes and

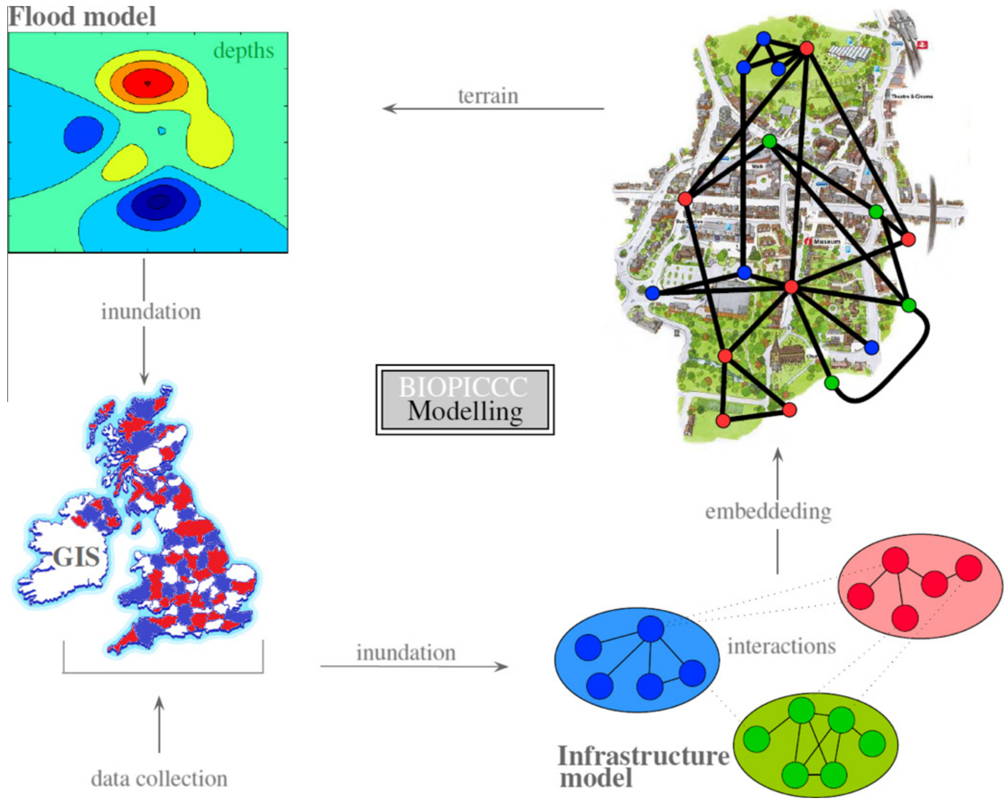

Fig. 6. Modelling components. The street map (top right), not including network overlay, is included with permission (©Cityvisions by Paul Holland, www.cityvisions.co.uk).

connections should be overlaid with a flood plain. Then it will be possible to compare the flood depths at nodes to model thresholds or known failure thresholds; information, where available, will be obtained – e.g., substations should be resilient to a flood depth of 300 mm, (NGE, 2010). Existing models of node failure within infrastructure systems tend to rely on the random selection of nodes. In real hazard situations, however, it is unlikely that such methods reflect real processes. This is certainly true for flood situations where vulnerable areas – e.g., with small heights above the water table – are often unevenly distributed, and better represented by models that capture topographical features.

Similarly, over the longer-term, we would like to allow the infrastructure model to be exploited in the context of other weather-related hazards. We illustrate the components of the intended approach in Fig. 6. In the short-term we will be developing our approach at two test sites within the UK, but the intention is that the tool can be deployed in any given region.

7. Conclusion

In this paper we have introduced a network flow model that has at its heart a general node model. Any number of different infrastructures can thus be conceived of as a single graph, which incorporates representations of infrastructural interdependencies. The novel aspects of the model relate to the integration, at the nodes, of processes – consumption, production and storage – important at the community scale in times of crisis. We have demonstrated the potential of the model by providing an example application involving two infrastructures and three commodity types, in terms of the controlled supply of two of those types to consumer nodes of varying vulnerability. The example demonstrates how the model might be used with respect to the prioritised control of commodity flows. In the future, we envision that such a model can be applied by stakeholders – e.g. managers of community infrastructure; engineers concerned with the protection of public and private assets in the local community; emergency response units – in local communities throughout the UK. We have outlined some future directions of our research that will support this development.

Acknowledgements

This research has been carried out as part of the EPSRC-funded project Built Infrastructure for Older People in Conditions of Climate Change (BIOPICCC). The financial support provided by the EPSRC is gratefully acknowledged. The authors also acknowledge the insightful and helpful comments provided by the anonymous reviewers.

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