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# Magnetic solutions in $AdS_5$ and trace anomalies

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**Abstract**

We discuss black hole and black string solutions in $d = 5$ Einstein–Yang–Mills theory with negative cosmological constant, proposing a method to compute their mass and action. The magnetic gauge field of these configurations does not vanish at infinity. We argue that this implies a nonvanishing trace for the stress tensor of the dual $d = 4$ theory.



## 1. Introduction

As originally found in $d = 4$ spacetime dimensions [1,2], a variety of well-known features of asymptotically flat self-gravitating non-Abelian solutions are not shared by their anti-de Sitter (AdS) counterparts. In the presence of a negative cosmological constant $\Lambda < 0$, the Einstein–Yang–Mills (EYM) theory possesses a continuum spectrum of regular and black hole non-Abelian solutions in terms of the adjustable parameters that specifies the initial conditions at the origin or at the event horizon, rather then discrete points. The gauge field of generic solutions does not vanish asymptotically, resulting in a nonzero magnetic flux at infinity.

For positive values of the cosmological constant, the solutions found in [3] are linearly unstable as shown in [4,5]. In contrast with the $\Lambda \geqslant 0$ case, some of the AdS configurations are stable against linear perturbations [6]. As found in [7,8] these features are shared by higher-dimensional spherically symmetric AdS non-Abelian solutions.

Since gauged supergravity theories generically contain non-Abelian matter fields in the bulk, these configurations are relevant in an AdS/CFT context, offering the possibility of studying some aspects of the nonperturbative structure of a CFT in a background gauge field [9]. On the CFT side, the boundary non-Abelian fields correspond to external source currents coupled to various operators.

However, in contrast with the four-dimensional case, a generic property of $d > 4$ non-Abelian solutions is that their mass and action, as defined in the usual way, diverge [7,8], which may raise questions about their physical relevance. For example, in the best understood $d = 5$ case [7], although the spacetime still approaches asymptotically the maximally symmetric background, the total action presents a logarithmically divergent part. The coefficient of the divergent term is proportional to the square of the induced non-Abelian field on the boundary at infinity.[1]

Here we argue that the logarithmic divergence of the non-Abelian $AdS_5$ configurations does not signal a problem with these solutions, but rather provides a consistency check of the AdS/CFT conjecture. The coefficient of the divergent term in the action is related in this case to the trace anomaly of the dual CFT defined in a background non-Abelian magnetic field. In this context,

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[1] The existence of a logarithmic divergence in the action is a known property of some classes of $AdS_5$ solutions with a special boundary geometry [10]. The coefficients of the divergent terms there are related to the conformal Weyl anomaly in the dual theory [11,12]. However, this is not the case of the non-Abelian $AdS_5$ configurations in [7], which have the same boundary metric as the Schwarzschild–AdS (SAdS) solution and thus no Weyl anomaly in the dual CFT.

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we propose to compute the mass and action of these solutions by using a counterterm prescription. This enables us to discuss the thermodynamical properties of two classes of $AdS_5$ non-Abelian black objects.

## 2. Non-Abelian black hole solutions

The action of the $d=5$ gauged supergravities usually contain the YM term $L_{\rm YM}=-1/(2e^2)\,{\rm Tr}\{F_{\mu\nu}F^{\mu\nu}\}$ as a basic building block (with $F_{\mu\nu}$ the field strength and $e$ the gauge coupling constant). In what follows we consider a truncation of such models corresponding to a pure EYM theory with a Lagrangian density[2] $L=1/(16\pi G)(R-2\Lambda)+L_{\rm YM}$, with $\Lambda=-6/\ell^2$. The first class of solutions we consider corresponds to spherically symmetric or topological black holes with a metric ansatz

$$ds^2=\frac{dr^2}{N(r)}+r^2\,d\Omega_{3,k}^2-N(r)\sigma^2(r)\,dt^2, \tag{1}$$

where $d\Omega_{3,k}^2=d\psi^2+f_k^2(\psi)(d\theta^2+\sin^2\theta\,d\varphi^2)$ denotes the line element of a three-dimensional space $\Sigma$ with constant curvature. The discrete parameter $k$ takes the values $1, 0$ and $-1$ and implies the form of the function $f_k(\psi)$: when $k=1$, $f_1(\psi)=\sin\psi$ and the hypersurface $\Sigma$ represents a 3-sphere; for $k=-1$, it is a 3-dimensional negative constant curvature space and $f_{-1}(\psi)=\sinh\psi$. The case $k=0$ is with $f_0(\psi)=\psi$ and $\Sigma$ a flat surface.

Restricting to an SU(2) gauge field, the YM ansatz compatible with the symmetries of the line-element (1) reads [13,14] (with $\tau_a$ the Pauli spin matrices)

$$A=\frac{1}{2}\left\{\tau_3\big(\omega(r)\,d\psi+\cos\theta\,d\varphi\big)-\frac{df_k(\psi)}{d\psi}(\tau_2\,d\theta+\tau_1\sin\theta\,d\varphi)+\omega(r)f_k(\psi)(\tau_1\,d\theta-\tau_2\sin\theta\,d\varphi)\right\}, \tag{2}$$

the radial function $\omega(r)$ is to be determined, together with the metric functions $N(r)$, $\sigma(r)$, by solving the field equations. The resulting set of three ordinary differential equations is solved with suitable boundary conditions. Supposing the existence of an event horizon for some $r_h>0$, one imposes $N(r_h)=0$, $\sigma(r_h)=\sigma_h>0$, $w(r_h)=w_h$. By going to the Euclidean section (or by computing the surface gravity) one finds the black holes Hawking temperature $T_H=1/\beta=\sigma_hN'(r_h)/4\pi$. (One should note that these non-Abelian magnetic solutions extremize also the Euclidean action, the Wick rotation $t\to it$ having no effect at the level of the equations of motion.) For $k=\pm1$, the EYM equations have a nontrivial exact solution [7]

$$N(r)=k+\frac{r^2}{\ell^2}-\frac{M+8\pi G(k^2/e^2)\log r}{r^2},\qquad \sigma(r)=1,\qquad \omega(r)=0, \tag{3}$$

which retains the basic features of the general configurations. Solutions with a nonvanishing $w(r)$ are constructed numerically, the $k=1$ case being considered in [7] (in the numerics we set $4\pi G/e^2=1$). As $r\to\infty$, the spacetime is locally isometric to AdS spacetime, and we find the following asymptotic expression of the solutions (with $M$, $w_0$, $w_2$ arbitrary parameters[3])

$$N(r)=k+\frac{r^2}{\ell^2}-\frac{M}{r^2}-\frac{8\pi G}{e^2}\frac{(w_0^2-k)^2}{r^2}\log\left(\frac{r}{\ell}\right)+\cdots,\qquad \sigma(r)=1-\frac{16\pi G}{3e^2}\ell^4w_0^2\frac{(w_0^2-k)^2}{r^6}\log^2\left(\frac{r}{\ell}\right)+\cdots,$$
$$w(r)=w_0+\frac{w_2}{r^2}-\frac{\ell^2}{r^2}w_0\big(w_0^2-k\big)\log\left(\frac{r}{\ell}\right)+\cdots. \tag{4}$$

For all considered values of $(\Lambda, r_h)$, we find black hole solutions with regular horizon for only one interval $0\leqslant w_h<w_h^c$. The spherically symmetric black holes with $w\neq0$ have a nontrivial globally regular limit $r_h\to0$. In contrast, the topological black holes possess minimal event horizon radius, for any $w_0$. An extremal black hole is found for the $w(r)=0$ solution (3) with $r_h^2=\ell^2(-k+|k|\sqrt{32\pi G/(e^2\ell^2)+1})/4$, the parameter $M$ being also fixed by the value of the cosmological constant.

The action and mass of the $AdS_5$ non-Abelian configurations is computed by using a boundary counterterm prescription. As found in [15], the following counterterms are sufficient to cancel divergences in five dimensions, for SAdS black hole solution:

$$I_{\rm ct}=-\frac{1}{8\pi G}\int\limits_{\partial\mathcal{M}_r}d^4x\,\sqrt{-h}\left[\frac{3}{\ell}+\frac{\ell}{4}{\rm R}\right], \tag{5}$$

with R the Ricci scalar for the boundary metric $h$. However, in the presence of matter fields, additional counterterms may be needed to regulate the action [16]. This is the case for the non-Abelian solutions discussed in this Letter, whose total action (where we have included also the Gibbons–Hawking boundary term [17]) diverges logarithmically, $I=V_k(\frac{3\beta}{16\pi G}(M+\frac{k^2\ell^2}{4})-\frac{1}{4G}r_h^3)+\frac{3\beta V_k}{2e^2}(w_0^2-k)^2\log(\frac{r}{\ell})$ (with $V_k$ the area of the surface $\Sigma$). This divergence is cancelled by a supplementary counterterm of the form (with $a$, $b$

[2] Usually, one has also to consider a non-Abelian Chern–Simon term. However, for purely magnetic solutions discussed here, this term vanishes identically.

[3] By using similar techniques to those employed in the globally regular case [7], one can prove the absence of non-Abelian black hole solutions with $w_0^2=k$.

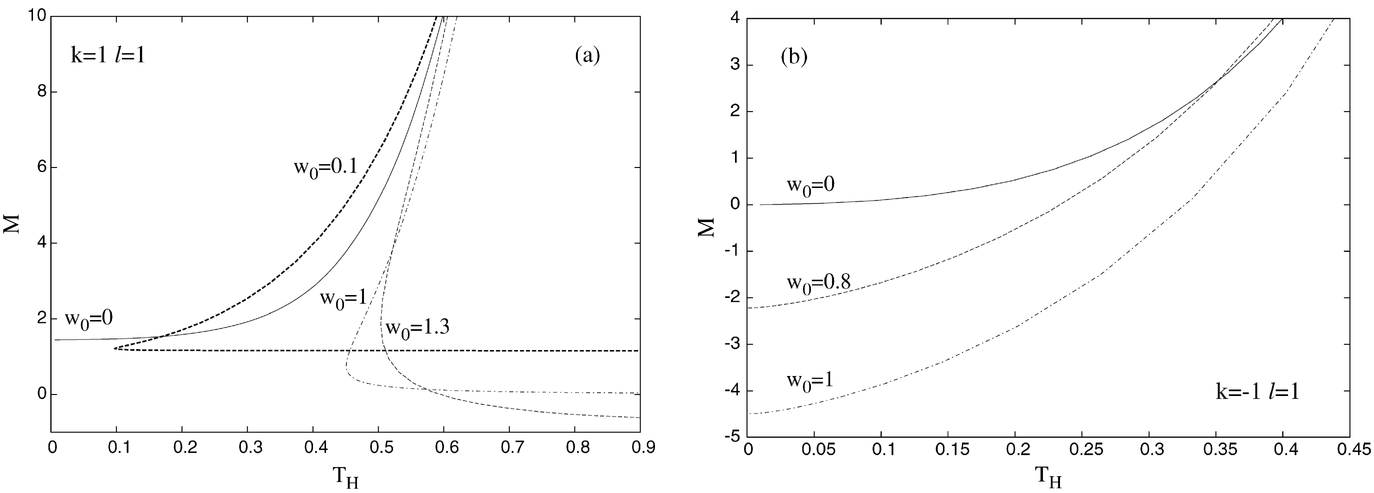


Fig. 1. The mass-parameter $M$ is plotted as a function of temperature for $k=1,-1$ black hole solutions and several values of the magnetic potential at infinity.

boundary indices):

$$I_{\mathrm{ct}}^{\mathrm{YM}} = -\log\left(\frac{r}{\ell}\right)\int_{\partial\mathcal{M}_r} d^4x\,\sqrt{-h}\,\frac{\ell}{2e^2}\,\mathrm{tr}\{F_{ab}F^{ab}\}. \tag{6}$$

Using these counterterms, one can construct a divergence-free boundary stress tensor $T_{ab}$,

$$T_{ab} = \frac{1}{8\pi G}\left(K_{ab} - Kh_{ab} - \frac{3}{\ell}h_{ab} + \frac{\ell}{2}E_{ab}\right) - \frac{2\ell}{e^2}\log\left(\frac{r}{\ell}\right)\mathrm{tr}\left\{F_{ac}F_{bd}h^{cd} - \frac{1}{4}h_{ab}F_{cd}F^{cd}\right\}, \tag{7}$$

where $E_{ab}$ and $K$ are the Einstein tensor and the trace of the extrinsic curvature $K_{ab}$ for the induced metric of the boundary, respectively. In this approach, the mass $\mathcal{M}$ of the solutions is the conserved charge associated with the Killing vector $\partial/\partial t$ [15]:

$$\mathcal{M} = \frac{3V_kM}{16\pi G} + M_c^{(k)}, \quad \text{with } M_c^{(k)} = \frac{3k^2V_k\ell^2}{64\pi G}. \tag{8}$$

We have found that $\mathcal{M}$ coincides with the mass computed from the first law of thermodynamics, up to the constant term $M_c^{(k)}$ which is usually interpreted as the mass of the pure global $AdS_5$.

Based on these results, one can discuss the thermodynamics of the non-Abelian black hole solutions in a canonical ensemble, holding the temperature $T_H$ and the magnetic potential at the boundary at infinity (i.e. the “magnetic charge”) fixed. Upon application of the Gibbs–Duhem relation $S = \beta\mathcal{M} - I$, one finds that the entropy $S$ of these solutions is one quarter of the event horizon area. The response function whose sign determines the thermodynamic stability is the heat capacity $C = (\partial\mathcal{M}/\partial T_H)_{w_0}$. In Fig. 1 we plot the $M(T_H)$ curves for several values of $w_0$ for spherically symmetric and hyperbolic black holes with $\ell = 1$ (the results for $k = 0$ are rather similar to the $k = -1$ case). For spherically symmetric black holes with $w_0 \neq 0$, the usual SAdS behaviour (corresponding to the $w_0 = 1$ curve in Fig. 1(a)) is reproduced: the curves first decrease toward a minimum, corresponding to the branch of small unstable black holes, then increase along the branch of large stable black holes. The $w(r) = 0$ solutions are rather special, since $C > 0$ in this case for any $r_h$. As seen in Fig. 1(b), the heat capacity is always positive for $AdS_5$ non-Abelian topological black holes. As a result, the $k = 0, -1$ black hole solutions are always thermodynamically locally stable.

From the AdS/CFT correspondence, we expect the non-Abelian hairy black holes to be described by some thermal states in a dual theory formulated in a metric background given by $\gamma_{ab}\,dx^a\,dx^b = -dt^2 + \ell^2(d\psi^2 + f_k^2(\psi)(d\theta^2 + \sin^2\theta\,d\varphi^2))$. One should also consider the interaction of the matter fields in the dual CFT with a background non-Abelian field, whose expression, as read from (2), (4) is

$$A_{(0)} = \frac{1}{2}\left\{\tau_3(\omega_0\,d\psi + \cos\theta\,d\varphi) - \frac{df_k(\psi)}{d\psi}(\tau_2\,d\theta + \tau_1\sin\theta\,d\varphi) + \omega_0 f_k(\psi)(\tau_1\,d\theta - \tau_2\sin\theta\,d\varphi)\right\}. \tag{9}$$

The expectation value $\langle\tau_b^a\rangle$ of the dual CFT stress tensor can be calculated using the relation [18] $\sqrt{-\gamma}\gamma^{ab}\langle\tau_{bc}\rangle = \lim_{r\to\infty}\sqrt{-h}h^{ab}T_{bc}$. Employing also (7), we find the finite and covariantly conserved stress tensor (with $x^1 = \psi$, $x^2 = \theta$, $x^3 = \varphi$, $x^4 = t$)

$$8\pi G\langle\tau_b^a\rangle = \frac{1}{2\ell}\left(\frac{M}{\ell^2} + \frac{k^2}{4}\right)\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-3\end{pmatrix} - \frac{4\pi G(w_0^2-k)^2}{e^2\ell^3}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\end{pmatrix}. \tag{10}$$

Different e.g. from the case of Reissner–Nordström–AdS Abelian solutions, this stress tensor has a nonvanishing trace, $\langle\tau_a^a\rangle = \mathcal{A}_{\rm YM} = -3(w_0^2-k)^2/(2\ell^2e^2)$. This agrees with the general results [16,19,20] on the trace anomaly in the presence of an external gauge field, $\mathcal{A}_{\rm YM} = \mathcal{R}F_{(0)}^2$, the coefficient $\mathcal{R}$ being related to the charges of the fundamental constituent fields in the dual CFT.

## 3. Non-Abelian black strings solutions

For the situation discussed above, the gravitational Weyl anomaly $\mathcal{A}_g$ vanishes, since $\mathcal{A}_g = -\frac{\ell^3}{8\pi G}(-\frac{1}{8}\mathsf{R}_{ab}\mathsf{R}^{ab}+\frac{1}{24}\mathsf{R}^2)$ is zero for the induced metric of the boundary. Here we present an example of configurations where both types of anomalies are present. This occurs for the non-Abelian version of a class of solutions recently considered in [21,22] and describing $\mathrm{AdS}_5$ black strings and vortices. The metric ansatz in this case reads

$$ds^2 = \frac{dr^2}{p(r)} + r^2\,d\Omega_{2,k}^2 + a(r)\,dz^2 - b(r)\,dt^2, \tag{11}$$

where $d\Omega_{2,k}^2 = d\theta^2 + f_k^2(\theta)\,d\varphi^2$ denotes the line element of a two-dimensional space with constant curvature, and the direction $z$ is periodic with period $L$. Considering again an SU(2) YM field, the gauge field ansatz has two magnetic potentials and reads

$$A = \frac{1}{2}\left\{\omega(r)\tau_1\,d\theta + \left(\frac{d\ln f_k(\theta)}{d\theta}\tau_3 + \omega(r)\tau_2\right)f_k(\theta)\,d\varphi + H(r)\tau_3\,dz\right\}. \tag{12}$$

Similar to the black hole case, we have found a continuum of black string solutions presenting an event horizon at $r=r_h$, where $p(r_h)=b(r_h)=0$, while $a(r_h)=a_h>0$, $w(r_h)=w_h$, $H(r_h)=H_h$. The Hawking temperature of the black strings is $T_H = \sqrt{b'(r_h)p'(r_h)}/4\pi$. The solutions have the following asymptotic expression in terms of four arbitrary constants $c_t$, $c_z$, $H_0$ and $w_2$:

$$\begin{aligned}
a(r) &= \frac{k}{2} + \frac{r^2}{\ell^2} + c_z\left(\frac{\ell}{r}\right)^2 + \frac{k^2}{2}\left(\frac{1}{6} - \frac{8\pi G}{e^2\ell^2}\right)\log\frac{r}{\ell}\left(\frac{\ell}{r}\right)^2 + \cdots,\\
b(r) &= \frac{k}{2} + \frac{r^2}{\ell^2} + c_t\left(\frac{\ell}{r}\right)^2 + \frac{k^2}{2}\left(\frac{1}{6} - \frac{8\pi G}{e^2\ell^2}\right)\log\frac{r}{\ell}\left(\frac{\ell}{r}\right)^2 + \cdots,\\
p(r) &= \frac{2k}{3} + \frac{r^2}{\ell^2} + \left(c_t + c_z + \frac{8\pi G}{e^2\ell^2}\right)\left(\frac{\ell}{r}\right)^2 + k^2\left(\frac{1}{6} - \frac{8\pi G}{e^2\ell^2}\right)\log\frac{r}{\ell}\left(\frac{\ell}{r}\right)^2 + \cdots,\\
w(r) &= \frac{w_2}{r^2} + \cdots, \qquad H(r) = H_0\left(1 + \frac{w_2^2\ell^2}{12r^6}\right) + \cdots.
\end{aligned} \tag{13}$$

The basic features of the black strings are similar to the black hole case. Again, the $k=1$ solutions possess nontrivial globally regular limits, representing the AdS counterparts of the $\Lambda=0$ non-Abelian vortices in Ref. [23]. The $k=0,-1$ topological black strings present a minimal event horizon radius. For given $(r_h,\Lambda)$ the solutions' global charges depend on the value of the magnetic gauge potential $H$ at infinity, which is a free parameter. The solutions with $w(r)=0$, $H(r)=\text{const}$ represent Abelian black strings, generalizing the exact BPS solutions in [24]. These configurations exist for values of the event horizon radius greater than a minimal value $r_h^c$, an extremal solution being approached in that limit. The non-Abelian solutions depend on the value $H_0$ and exist on a finite interval of $r_h$. In the limit $r_h \to r_h^c$ the gauge function $w(r)$ vanishes identically and the branch of non-Abelian solutions bifurcates into the Abelian branch.

The action and global charges of these configurations are computed by employing again the counterterm formalism. As found in [22] the action of the vacuum solutions presents a logarithmic divergence which is regularized by adding the following term to the boundary action [11]:

$$I_{\rm ct}^s = \frac{1}{8\pi G}\log\left(\frac{r}{\ell}\right)\int_{\partial\mathcal{M}_r} d^4x\,\sqrt{-h}\frac{\ell^3}{8}\left(\frac{1}{3}\mathsf{R}^2 - \mathsf{R}_{ab}\mathsf{R}^{ab}\right), \tag{14}$$

which implies a supplementary contribution to the boundary stress tensor (7). The bulk YM fields give another logarithmic divergence, which is regularized by the matter counterterm (6). As usual with black strings [25], apart from mass $\mathcal{M}$, there is also a second global charge associated with the Killing vector $\partial/\partial z$ and corresponding to the solutions' tension $\mathcal{T}$:

$$\begin{aligned}
&\mathcal{M} = M_0 + M_c^{(k)}, \quad M_0 = \frac{\ell L V_k}{16\pi G}[c_z - 3c_t],\\
&\mathcal{T} = \mathcal{T}_0 + \mathcal{T}_c^{(k)}, \quad \mathcal{T}_0 = \frac{\ell V_k}{16\pi G}[3c_z - c_t], \quad \text{with } M_c^{(k)} = L\mathcal{T}_c^{(k)} = \frac{\ell}{16\pi G}V_k L,
\end{aligned} \tag{15}$$

where $V_k$ is the total area of the angular sector, $M_c^{(k)}$ and $\mathcal{T}_c^{(k)}$ being Casimir-like terms. In Fig. 2 we plot the mass-parameter $M_0$ as a function of temperature for $k=1$ black strings with several values of $H_0$ (in a $d=4$ picture, this corresponds to different vacuum

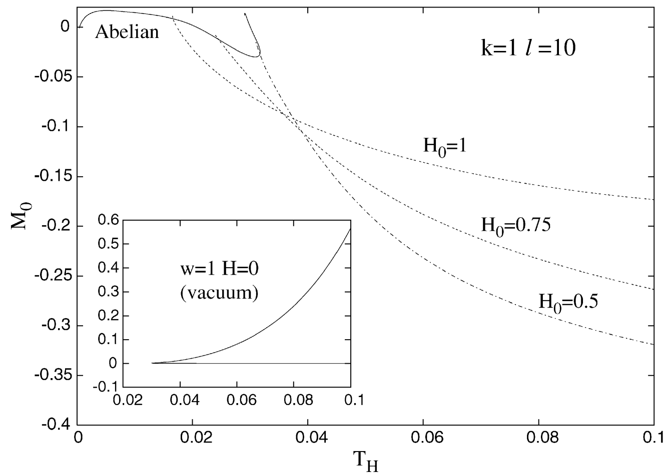


Fig. 2. The mass-parameter $M_0$ is plotted for $k=1$ black string solutions.

expectation values of the Higgs field [23]). One can see that, in contrast with the vacuum case, the non-Abelian black strings are thermally unstable. The situation is more complicated in the Abelian case, the solutions near extremality possessing a positive heat capacity.

For these black strings solutions, the background metric upon which the dual field theory resides is $\gamma_{ab}\,dx^a\,dx^b=-dt^2+dz^2+\ell^2(d\theta^2+f_k^2(\theta)\,d\varphi^2)$. The boundary CFT is formulated in this case in a background Abelian gauge field, with

$$A_{(0)}=\frac{\tau_3}{2}\left\{\frac{df_k(\theta)}{d\theta}d\varphi+H_0\,dz\right\}. \tag{16}$$

The expectation value of the stress tensor of the dual CFT contains four different parts (with $x^1=\theta$, $x^2=\varphi$, $x^3=z$, $x^4=t$)

$$8\pi G\langle\tau_b^a\rangle=-\frac{c_z}{2\ell}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-3&0\\0&0&0&1\end{pmatrix}-\frac{c_t}{2\ell}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-3\end{pmatrix}$$
$$+\frac{k^2}{24\ell}\begin{pmatrix}2&0&0&0\\0&2&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}-\frac{2\pi G}{e^2\ell^3}k^2\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}. \tag{17}$$

The trace of this tensor is equal to the sum of the gravitational and external gauge field contributions $\mathcal{A}=\mathcal{A}_g+\mathcal{A}_{\mathrm{YM}}=k^2(\frac{1}{96\pi G\ell}-\frac{1}{2e^2\ell^3})$, vanishing for the Abelian BPS solutions in [24].

## 4. Further remarks

On general grounds, one expects that extending the known classes of solutions of the $d=5$ supergravity to a non-Abelian gauge group would lead to a variety of new physical effects. The black objects discussed in this Letter are perhaps the simplest solutions relevant in this context. We expect a much richer structure to be found when relaxing the spacetime symmetries, or when taking a more general gauge group. However, the generic non-Abelian solutions will always present a nonvanishing magnetic gauge field on the boundary which appears as a background for the dual theory. Also, similar to the $d=4$ case [9], the existence of both spherically symmetric globally regular and hairy black hole solutions with the same set of data at infinity raises the question as to how the dual CFT is able to distinguish between these different bulk configurations.

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