Simulation and stability study of the flow around a cylinder in infinite domain

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Abstract

The flows around a cylinder in infinite domain are simulated using CFD method. The simulation results are found to be in agreement with the data in literatures. The flow stability is investigated with the energy gradient theory. It is found that the two zero velocity gradient points at lateral sides of the cylinder lead to flow instability which causes vortex shedding. The flow transition to turbulence are dominated by two aspects, i.e. the zero velocity gradient at the lateral sides of the cylinder, and the zero velocity gradient inner the region of the vortex in the cylinder wake.

1. Introduction

The flow around a cylinder and the unsteady wake behind the cylinder are classical issues of fluid mechanics [1]. The phenomenon exists in many engineering problems and in nature [2], such as the water past the bridge pier, the wind past a building and the air past the airfoil etc.

It has been a long time for investigating the flow around a bluff body. Von Karmam firstly studied and analyzed the phenomenon, and then he found the relationship between the structure of vortex and the drag acting on the cylinders in 1912. Then a large number of researchers devoted to revealing the physical feature of the flow around the cylinders after that, for example, Roshko[3], Taneda [4,5], Mathis et al [6], Sreenivasan et al [7], Provansal et al [8], Williamson[9], Zdravkovich[10], Gerrard [11], Coutanceau[12] and Perry et al [13], etc. Among these studies, the mechanism of Karman vortex street and the flow instability of flow field were investigated with various methods,

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However, many problems behind this flow phenomenon have not been solved so far due to its complexity. In this paper, the flows at $Re=26$ and 100 are simulated within the frame of laminar flow and the flow instabilities are investigated with the energy gradient theory.

2. Briefly introduction of the energy gradient theory

Dou et al. [14-16] proposed a new theory, i.e., the energy gradient theory, which is based on Newtonian mechanics and compatible with the Navier-Stokes equations. It uses the energy gradient function $K$ to characterize the feature of flow stability. A larger value of $K$ means the local flow tends to be unstable than that with lower value of $K$. The function $K$ is defined as the ratio of the transversal gradient of the total mechanical energy and the rate of loss of the total mechanical energy in streamwise direction. Now, the theory has been successfully applied to plane Couette flow, plane Poiseuille flow, pipe Poiseuille flow, and Taylor-Couette flow, etc. The theory has been obtained consistent agreement with experimental data.

3. Physical models and the numerical method

3.1. Governing equations and numerical method

In this study, the flow keeps as laminar, so the governing equations are as follows:

$$\nabla \cdot \mathbf{v} = 0$$

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{v}$$

Here, $\rho$ is the density, $\mathbf{v}$ is the velocity director. The governing equation are discretized with finite volume method (FVM). The coupling of pressure and velocity is done using PISO algorithm. The Reynolds number based on the cylinder is defined as $Re=U_\infty \frac{D}{\nu}$, and $U_\infty$ is the average velocity from the inlet. $D$ is the diameter of the cylinder, and $\nu$ is the kinematic viscosity.

3.2. Geometric model

The computational domain is shown in Fig.1. The flow field is modeled in two dimensions with the axes of the cylinder perpendicular to the direction of flow. The diameter of the cylinder is $D=0.02m$. The computational domain is $25D \times 31.5D$. The upstream and downstream are $11.5D$ and $20D$ from the center of the cylinder respectively.

3.3. Boundary conditions

As shown in Fig.1, the average velocity boundary condition is applied at the upstream of the cylinder. At the downstream boundary, a Neumann-type boundary condition is defined. The cylinder is defined as wall, i.e., $u=0$, $v=0$. Two lateral sides of the cylinder are treated as symmetry boundary condition.
4. Results and Analysis

4.1. Stability analysis of laminar flow with no Karman vortex street

In this part, the condition of $Re=26$ is calculated, since the experimental data of $Re=26$ can be found from the literature. The calculating results are shown as follows:

As is shown in Fig.2, there is no vortex street formed in the cylinder wake at $Re=26$. The flow is in steady state and remains as laminar according to the Reynolds number. Comparing the contour of the energy gradient function $K$ and the streamline contour (Fig.2), it can be found that the value of $K$ at the position with two eddies just behind the cylinder is low. Thus, it can be deduced that the two eddies have no effect on the flow stability according to the energy gradient theory.

It is observed that the maximum of $K$, $K_{\text{max}}$, occurs in the lateral side of the cylinder by further investigating the distribution of $K$, and the detailed distributions of $K$ and velocity are shown in Fig.3. It can be determined that the flow at two lateral sides of the cylinder, i.e., the two free shear layers will firstly lose its stability according to the energy gradient theory. Then, it can be deduced that the origin of the formation of Karman vortex street is from the interaction of two free shear layers. It is the interaction of two free shear layers that squashes two eddies leading to the vortex shedding, and then the Karman vortex street is formed. Gerrard[11] found that, the determining factor leading to vortex shedding is the interaction of two separating free shear layers. So the finding that the large $K$ at lateral sides of the cylinder leading to the flow losing its stability, in fact, is agreement with Gerrard[11].

In a summary, it can be concluded that the formation of Karman vortex originates from the area with large $K$ at the lateral sides of the cylinder. The two eddies are squashed by the free layers leading to vortex shedding.

![Fig.2 Distribution of physical quantities at Re=26.](image)

![Fig.3 Distribution of velocity and K at Section 1 of Fig.2](image)

4.2. Stability analysis of laminar flow with Karman vortex street

In this part, the flow at the condition of $Re=100$ is calculated. As is shown in Fig.4, it can be found that there exists vortex street in the cylinder wake and the flow keeps as laminar according to Reynolds number. After studying the distribution of $K$, it is found that $K_{\text{max}}$ of the entire flow fields occurs at lateral sides of the cylinder, i.e., the position of free shear layers. It is also found that a large $K$ occurs in the region inner a vortex in the cylinder wake as shown in Fig.4. According to the energy gradient theory [14-16], the position with $K_{\text{max}}$ will firstly lose its stability in entire flow field, which means that the two free shear layers will firstly lose its stability in entire flow...
field, and the region inner a vortex will firstly lose its stability in the cylinder wake and two free shear layers will be earlier losing its stability than the region inner a vortex.

To investigate and clarify the mechanism of the problem, three sections, i.e., 1, 2 and 3 in Fig.4 (a), of the flow field are intercepted and their detailed distribution of $K$ are shown in Fig.5, Fig.6 and Fig.7 respectively. It is found that $K_{\text{max}}$ at each section always appears in the location where the gradient of velocity is zero after comparing the distribution of the velocity and $K$ respectively. So it can be concluded that it is the zero velocity gradient that leads to a large $K$, and that is in agreement with the instability criteria proposed by Dou [14-16], i.e., for shear driven flow, the necessary and sufficient condition for turbulent transition is the existence of zero velocity gradient on the velocity profile in the averaged flow profile. According to the criteria, there has a large energy gradient at the position with zero velocity gradients. And the flow will firstly lose its stability if some suitable disturbances are introduced.

According to the distribution of $K$ and the energy gradient theory, it can be predicted that the free shear layers at lateral sides of the cylinder will firstly lose its stability in the entire flow field and the locations with a vortex in the cylinder wake will firstly lose its stability because of their zero velocity gradients as the Re increase.

![Fig.4 Contours of velocity, vorticity and $K$ at Re=100](image)

![Fig.5 Profile at Section 1](image)

![Fig.6 Profile at Section 2](image)
5. Conclusions

In this paper, the flows around a cylinder in infinite domain are simulated using CFD method. The simulation results are found to be in agreement with experimental ones. Based on the simulation results, the flow stabilities are investigated with the energy gradient theory. The conclusions obtained are as follows:

1. For the flow without Karman vortex street, it is the zero velocity gradients at the lateral sides of the cylinder that leads to the instability. And the two eddies at the rear of the cylinder have no effect on the stability.

2. For the flow with Karman vortex, the flow losing its stability can attributes two aspects, the zero velocity gradient at the lateral sides of the cylinder and the zero velocity gradient inner a vortex.

3. The energy gradient theory can be used to predict the position firstly losing its stability in the flow field. The present study confirms the criterion proposed in previous work: for shear driven flows, the necessary and sufficient condition for turbulent transition is the existence of zero velocity gradient on the velocity profile in the averaged flow.

References: