Calibration of a $\gamma$-$Re_\theta$ transition model and its validation in low-speed flows with high-order numerical method

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Abstract Based on the Reynolds-averaged Navier–Stokes (RANS) equations and structured grid technology, the calibration and validation of $\gamma$-$Re_\theta$ transition model is performed with fifth-order weighted compact nonlinear scheme (WCNS), and the purpose of the present work is to improve the numerical accuracy for aerodynamic characteristics simulation of low-speed flow with transition model on the basis of high-order numerical method study. Firstly, the empirical correlation functions involved in the $\gamma$-$Re_\theta$ transition model are modified and calibrated with experimental data of turbulent flat plates. Then, the grid convergence is studied on NLR-7301 two-element airfoil with the modified empirical correlation. At last, the modified empirical correlation is validated with NLR-7301 two-element airfoil and high-lift trapezoidal wing from transition location, velocity profile in boundary layer, surface pressure coefficient and aerodynamic characteristics. The numerical results illustrate that the numerical accuracy of transition length and skin friction behind transition location are improved with modified empirical correlation function, and obviously increases the numerical accuracy of aerodynamic characteristics prediction for typical transport configurations in low-speed range.

1. Introduction
High-lift flow-fields are characterized by confluent wakes, wake/boundary layer merging, regions of separated flow and regions of laminar-to-turbulent transition. The prediction of boundary-layer transition plays a key role in the simulation of high-lift flow-field and is an important component in commercial transport aircraft design. The correct prediction of transition locations significantly influence lift and drag forces and, thus, the overall performance. The study of laminar-to-turbulent transition is a hotspot in aerodynamics in recent decades.1–4 In today’s applied aerodynamics, numerical methods based on Reynolds averaged Navier–Stokes (RANS) equations are still the most popular tools, and, widely used transition-prediction approaches include empirical correlative method,
\( e^N \) method, and intermittency model. As the non-local operation in calculating momentum-thickness, empirical correlation method and intermittency model are incompatible with modern RANS CFD technology, especially for unstructured mesh and parallel program. It is very difficult for \( e^N \) method to deal with bypass transition and extend to three-dimensional problems. Local correlation-based \( \gamma-Re_\theta \) transition model is proposed by Menter et al.\(^5,6\) for modern CFD computation. This model combines empirical correlation methods with intermittency methods and avoids the computation of momentum-thickness via the relation between the momentum-thickness Reynolds number and the local shear strain rate, therefore, the whole procedure of the transition modeling is localized. This model and its modification have made great successes.\(^3,4,7-10\) Recently, the \( \gamma-Re_\theta \) transition model is implemented into CFD code CFX and OVERFLOW and successfully applied to the simulation of three-dimensional high-lift trapezoidal wing (trap wing).\(^11,12\) These results are all obtained with second-order finite-volume schemes, and the coupling of \( \gamma-Re_\theta \) transition model to high-order finite-difference methods is not reported up to now. In comparison with high-order spectrum method, finite volume method and discontinuous Galerkin finite element method, high-order finite difference scheme is easier to construct and needs less computer resource, but mainly applied to simulating complicated flow around simple configuration, such as boundary layer receptivity, cavity flow, acoustic mechanism and separated flow around delta wing, because of problems associated with high-order physical properties exchange among multi-block interfaces and geometric conservation law (GCL). A series of high-order weighed compact nonlinear scheme (WCNS)\(^13,14\) is proposed by Deng et al. in 2000. With continuous research work in GCL\(^15,16\), remarkable progress has been obtained in the simulation of complex flow around complex configuration, and WCNS method illustrates the potential priority in the resolution of space vortex and prediction of the maximum lift coefficients of transport configuration, but all these results do not contain the effect of laminar-to-turbulent transition.

This paper is organized as follows. First, high-order numerical method, Menter's original \( \gamma-Re_\theta \) transition model and the modified empirical correlation are described briefly. Next, the modified empirical correlation is validated with the convective term is discretized by fifth-order WCNS difference scheme, the viscous terms is discretized by six-order central difference scheme, and boundary condition is discretized by fourth-order upwind difference scheme. Menter's shear stress transport (SST) turbulent modes is employed and the discretization methods of every terms involved are the same as those of the mean control equations. The fifth-order WCNS difference scheme and the fourth-order boundary and nearby boundary scheme are briefly introduced.

The space interval in \( \xi \) direction is denoted by \( h \), and the fifth-order explicit WCNS scheme and the fourth-order boundary and near boundary scheme are

\[
\frac{\partial E_1}{\partial \xi} = \frac{75}{64h} \left( \bar{E}_{i+1/2} - \bar{E}_{i-1/2} \right) - \frac{25}{384h} \left( \bar{E}_{i+3/2} - \bar{E}_{i-3/2} \right) + \frac{3}{640h} \left( \bar{E}_{i+5/2} - \bar{E}_{i-5/2} \right)
\]

(1)

\[
\frac{\partial E_1}{\partial \xi} = \frac{1}{24h} \left( -22 \bar{E}_{i+1/2} + 17 \bar{E}_{i+1/2} + 9 \bar{E}_{i+3/2} - 5 \bar{E}_{i+7/2} + \bar{E}_{i+9/2} \right)
\]

(2)

\[
\frac{\partial E_2}{\partial \xi} = \frac{1}{24h} \left( \bar{E}_{i+1/2} - 27 \bar{E}_{i+3/2} + 27 \bar{E}_{i+5/2} - \bar{E}_{i+7/2} \right)
\]

(3)

\[
\frac{\partial E_{N-1}}{\partial \xi} = -\frac{1}{24h} \left( \bar{E}_{N-1/2} - 27 \bar{E}_{N-3/2} + 27 \bar{E}_{N-5/2} - \bar{E}_{N-7/2} \right)
\]

(4)

\[
\frac{\partial E_N}{\partial \xi} = -\frac{1}{24h} \left( -22 \bar{E}_{N-1/2} + 17 \bar{E}_{N-3/2} + 9 \bar{E}_{N-5/2} - 5 \bar{E}_{N-7/2} + \bar{E}_{N-9/2} \right)
\]

(5)

where \( \frac{\partial E_i}{\partial \xi} \) is the space derivative at cell node, and \( \bar{E}_{i+\nu/2} (m=\pm 1, \pm 3, \pm 5, \pm 7, \pm 9) \) the numerical flux at cell edge; details of WCNS scheme are given in Ref.\(^13,14\).

2.2. Menter's \( \gamma-Re_\theta \) transition model

Menter's \( \gamma\)-hyphen\( Re_\theta \) transition model includes the transport equations for the intermittency \( \gamma \) and transition momentum-thickness Reynolds number \( Re_\theta \). The final object of the two-functions transition model is the computation of intermittency \( \gamma \), which is used to turn on the production term of the turbulent kinetic energy within Menter's SST turbulence model. The dimensionless intermittency equation is formulated as

\[
\frac{\partial (\rho \gamma)}{\partial t} + \frac{\partial (\rho \gamma U_i)}{\partial x_j} = \frac{1}{Re} \frac{\partial}{\partial x_j} \left( \mu + \sigma \mu_i \frac{\partial \gamma}{\partial x_j} \right) + P_t - E_t
\]

(6)

\[
\begin{cases}
    P_t = c_{\text{at}} F_{\text{length}} \rho S \left( \gamma F_{\text{onset}} \right)^{\gamma} \left( 1 - c_{\text{at}} \gamma \right) \\
    E_t = c_{\text{at}} F_{\text{turb}} \rho \alpha_t \left( 1 - c_{\text{at}} \gamma \right) \\
    F_{\text{turb}} = \exp \left( - \left( R_T / 4 \right)^4 \right) \\
    R_T = \frac{p \kappa}{\mu} \\
    F_{\text{onset}} = \max \left( 0, F_{\text{onset}2} - F_{\text{onset}3} \right) \\
    F_{\text{onset}2} = \max \left( 0, \frac{Re_v}{2.193 Re_{\varepsilon}} \right) \\
    F_{\text{onset}3} = \min \left( 2.0, \max \left( F_{\text{onset}1}, \frac{F_{\text{onset}}}{2.5^3} \right) \right)
\end{cases}
\]

details of intermittency equation are given in Ref.\(^5,6\). Both \( F_{\text{length}} \) and \( Re_{\varepsilon} \) in the above equation are empirical.
correlations and they are functions of transition momentum-thickness Reynolds number \( \textit{Re}_{h} \), which are also functions of the turbulence intensity \( Tu \) and the streamwise pressure gradient \( \partial h / \partial x \) in the free flow. The present calibration work is based on the empirical correlations provided by Menter and Langtry.\(^7\)

2.3. Modification of Menter’s \( \gamma \)-\( \textit{Re}_h \) transition model

As mentioned in Section 2.2, \( F_{\text{length}} \) and \( \textit{Re}_{h} \) in Eq. (6) are empirical correlations. With the empirical correlations provided by Menter and Langtry\(^7\) and the present high-order numerical methods, the calibration work with the four turbulent flat-plate test cases provided in the next section is not satisfactory. In the present work, the empirical correlation of transition length \( F_{\text{length}} \) from Ref.\(^7\) is fixed and the empirical correlation of critical Reynolds number \( \textit{Re}_{h} \) from Ref.\(^7\) is modified as

\[
\begin{align*}
\textit{Re}_{h} & = \textit{Re}_{h}^{\text{Menter}} + a \left( \textit{Re}_{h} - b \right) \\
\textit{Re}_{h} & = \min \left( \textit{Re}_{h}, 0.99 \textit{Re}_{h}^{\text{Langtry}} \right)
\end{align*}
\]

where \( a \) and \( b \) are constants; \( \textit{Re}_{h}^{\text{Langtry}} \) is the original function from Refs.\(^6,7\). According to the transition locations of experiments, \( a \) is 0.0445 and \( b \) is 150.0 by iterative calculations.

3. Calibration of empirical correlation with high-order numerical method

The turbulent flat-plate test cases in Ref.\(^5\), including T3A, T3B, T3A- and S&K, are used to calibrate empirical correlations.
correlations with high-order numerical method. The inlet conditions for these test cases are summarized in Table 1.

The flat-plate is 2.0 in length, inflow boundary is 0.22 ahead of the flat-plate, and far-field boundary in normal direction is 0.22 away from the wall. The mesh used for calibration is shown in Fig. 1. H-type mesh is adopted, and the distance in normal direction is \(1.0 \times 10^{-3}\) for the first grid beyond the wall, while the distance in streamwise direction is \(1.5 \times 10^{-3}\) for the first grid ahead of the flat-plate. The mesh consists of 425 streamwise nodes, with 392 nodes on the wall, and 129 nodes in normal direction.

When the empirical correlations provided by Ref. 7 are directly employed, the distribution of skin friction coefficient \(C_f\) along the surface as a function of Reynolds number \(Re_x\), based on the distance from the leading edge, is presented in Fig. 2. For T3A and T3A- cases, the numerical transition locations do not correctly reproduce those of experiments. For S&K case, the numerical transition length is shorter than that of experiment.

With the modified empirical correlation \(Re_{h,c}\) of Eq. (7), Fig. 3 shows the distribution of skin friction along the surface. For the T3A, T3A- and S&K cases, the numerical results are improved obviously and agree well with experimental data.

4. Grid refinement of NLR-7301 two-element airfoil

NLR-7301 two-element airfoil is one of the most commonly used configurations for CFD validation. This configuration is designed with moderate flap angle \((20^\circ)\), which is a typical take-off flap setting. The overlap is 5.3\% and the gap width is 2.6\%\(c\), of which \(c\) is the chord of the basic airfoil.

Numerical tests have indicated that the aerodynamic character is most sensitive to the resolution of the grid in the wall normal direction in the boundary layer. To gain some insight into how much grid clustering is needed near the solid surface to obtain convergent aerodynamic characteristics, three multi-block structured grids with different first cell distances \(\Delta y\) in boundary layer are created with a given total number of grid points, which is 191376. Fig. 4 gives the topology and local grid.

Numerical simulation without transition model is performed on each of the three grids to obtain lift coefficient \(C_L\), drag coefficient \(C_D\), skin friction drag coefficient \(C_{Df}\) and pressure drag coefficient \(C_{Dp}\). The freestream condition is \(Ma = 0.185, \alpha = 13.1^\circ, Re = 2.51 \times 10^6\). The results are listed in Table 1. Both of \(C_L\) and \(C_D\) decrease monotonically and converge as the grids further cluster in the boundary layer, and the decrease of \(C_D\) is a synthetical effect from \(C_{Df}\) and \(C_{Dp}\). The second grid of \(y^+ \approx 1.0\) in Table 2 is chosen for the rest of work in the next section.
5. High-order simulation of NLR-7301 two-element airfoil

The experiment was conducted in the NLR 3 m × 2 m low-speed wind tunnel in 1970s. The experimental data include the aerodynamic forces, pressure distribution on the surface and velocity profiles in the boundary layer and wakes. No obvious flow separation occurs on the flap and a small laminar separation bubble appears on the wing nose.

(1) Pressure coefficient

Fig. 5 is the surface pressure coefficient $C_p$ distribution from the fully turbulent and transition solution, compared with experimental data. In general, the numerical results with/without transition model agree very well with experimental data. The result with transition model captures the small separation bubble on the wing nose, which is not presented in the fully turbulent solution.

(2) Skin friction coefficient and transition location

Fig. 6 shows the surface skin friction coefficient $C_f$ distribution on each of the elements from the fully turbulent and transition solutions, compared with experimental data. The shadow bars in this figure mark the experimental transition location. The separation-induced transition location computed with transition model is in good agreement with experimental data on the upper surface of the main wing, while on the lower surface of main wing and on the upper surface of the flap, the transition locations are slightly further downstream compared with experimental data.

(3) Velocity profile

Fig. 7 shows the dimensionless velocity $u$ in $x$-direction at different streamwise stations from the fully turbulent and transition solution, compared with experimental data. The two stations, $x = 0.60$ and $x = 0.88$, on the upper surface of the main wing are located in fully turbulent region and the thickness of boundary layer is improved obviously with the transition model. The $x = 0.60$ station on the lower surface of the main wing is located in laminar region, whereas, $x = 0.88$ in fully turbulent region. The thickness of boundary layer with the transition model is in good agreement with experiment. The $x_f = 0.119$ station on the upper surface of the flap is located in laminar region, whereas, $x_f = 0.219$ in fully turbulent region. The thickness of the main wake is improved evidently.
with the transition model, although there is still difference between computation and experiment.

(4) Aerodynamic characteristics

The aerodynamic characteristics obtained with different methods are summarized in Table 3, where $C_m$ is the pitch moment coefficient. With transition model, the lift coefficient increases slightly; both of friction drag and pressure drag decrease, whereas, pitch moment coefficient does not change. The increase of $C_L$ and decrease of $C_{D_p}$ are results of the increase of suction peak at the leading edge of the main wing (see Fig. 5); the reduction of turbulent area results in the decrease of $C_{D_f}$. The drag coefficient from transition solution agrees better with experiment.

6. High-order simulation of 3-D high-lift trap wing

Trap wing is a three-element semi-span wing attached to a body pod, with 30° slat deflection and 25° flap deflection, which is one of the configurations selected for 1st American Institute of Aeronautics and Astronautics (AIAA) CFD High Lift Prediction Workshop (HiLiftPW-1). The experimental data from 14 × 22 foot subsonic wind tunnel, Langley, National Aeronautics and Space Administration (NASA) are adopted for comparison.19 The freestream condition is $Ma = 0.2$, $Re = 4.5 \times 10^6$ and $x = 28°$. A structured patch grid (point to point) with 14.65 million cells for trap wing is created, initial spacing normal to all viscous walls is 0.004 mm, $y^+ = 1.0$, growth rate of cell sizes in the viscous boundary layer is equal to 1.20. Grid convergence study is presented in Ref.20. Fig. 8 shows the surface grid for the configuration.

Fig. 9 shows the chordwise pressure coefficients distribution at different spanwise stations ($\eta = y/b$, $b$ is the half span) from

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Aerodynamic characteristics of NLR-7301 two-element airfoil.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>$C_L$</td>
</tr>
<tr>
<td>Fully turbulent</td>
<td>3.061</td>
</tr>
<tr>
<td>Transition</td>
<td>3.190</td>
</tr>
<tr>
<td>Experiment</td>
<td>3.141</td>
</tr>
</tbody>
</table>

Fig. 7 Velocity profiles at typical station of NLR-7301 two-element airfoil.

Fig. 8 Surface grid for trap wing configuration.
the fully turbulent and transition solution, compared with experimental data. From the station near the root \((\eta = 0.17, 0.28)\) and in the middle part of the wing \((\eta = 0.41)\) to the station near the wing tip \((\eta = 0.98)\), the transition effect enhances gradually on the upper surface around the trailing edge of the main wing and the upper surface near the leading edge of the flap, and the pressure from transition solution moves upward in these regions and is closer to the experiment. The reason of this improvement is attributed to the more accurate resolution of wing tip vortex with transition model. The aerodynamic character obtained with different methods is listed in Table 4. In comparison with fully turbulent solution, the application of the transition model increases lift coefficient, drag coefficient and the magnitude of pitch moment, and are slightly higher than experiment. According to the study of Ref. \(^{11}\), the increments between transition simulation and experiment are attributed to the lack of slat and flap supports in numerical simulation.

### Table 4 Effect on aerodynamic characteristics of trap wing configuration with transition model.

<table>
<thead>
<tr>
<th>Method</th>
<th>(\alpha (\degree))</th>
<th>(C_L)</th>
<th>(C_D)</th>
<th>(C_m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fully Turbulent</td>
<td>28.00</td>
<td>2.8443</td>
<td>0.6579</td>
<td>-0.4295</td>
</tr>
<tr>
<td>Transition</td>
<td>28.00</td>
<td>2.9516</td>
<td>0.6881</td>
<td>-0.4718</td>
</tr>
<tr>
<td>Experiment 1</td>
<td>27.38</td>
<td>2.8732</td>
<td>0.6648</td>
<td>-0.4608</td>
</tr>
<tr>
<td>Experiment 2</td>
<td>28.41</td>
<td>2.9096</td>
<td>0.6860</td>
<td>-0.4526</td>
</tr>
</tbody>
</table>

7. Conclusions

Based on Menter’s \(\gamma-\text{Re}_h\) transition model, we have proposed a modified empirical correlation of critical Reynolds number \(\text{Re}_{h_c}\), with this modified empirical correlation and high-order numerical method, we are able to obtain more accurate numerical results in simulating typical low-speed flows, such as capturing the location of transition more reasonably, and with a improved drag accuracy in two-dimensional case, and the more accurate aerodynamic characteristics in three-dimensional case.

(1) For the turbulent flat plates, the modification of empirical correlation \(\text{Re}_{h_c}\) effectively improves the prediction accuracy of transition locations.

(2) For NLR-7301 two-element airfoil, the modification of empirical correlation \(\text{Re}_{h_c}\) can reasonably predict separation induced transition, resolve the laminar separation bubble on the wing nose and improve the velocity distribution in the boundary layer.

(3) For trap wing high-lift configuration, the modification of empirical correlation \(\text{Re}_{h_c}\) improves the pressure distribution at different spanwise stations and more reasonable aerodynamic characteristics are obtained.

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References


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