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Local stability of an endoreversible heat pump with linear phenomenological heat transfer law working in an ecological regime

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Abstract Based on the optimal ecological performance parameters of a heat pump with linear phenomenological heat transfer law between working fluid and heat reservoirs, the local stability analysis of the endoreversible heat pump working in an ecological regime is studied. The steady state of the heat pump working at the maximum ecological function is steady. After a small perturbation, the system state exponentially decays to steady state with either of the two relaxation times. The effects of temperatures of heat reservoirs and heat transfer coefficients on the local stability of the system are discussed. Distribution information of phase portraits of the system is obtained. It is concluded that both the energetic properties and local stability of the system should be considered for designing the real heat pumps.

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1. Introduction

Since Curzon and Ahlborn [1] proposed an endoreversible Carnot heat engine with Newton's heat transfer law and calculated its efficiency at maximum power output, i.e. $\eta_{CA} = 1 - \sqrt{T_L/T_H}$ in 1975, Finite Time Thermodynamics (FTT) has been made tremendous progress [2–10]. Blanchard [11] was the first to extend the Curzon–Ahlborn analysis method [1] to the analysis of heat pump cycles, and derived the coefficient of performance bounds for the fixed heating load for an endoreversible Carnot heat pump. Angulo-Brown [12] proposed an ecological criterion $E' = P - T_L\sigma$ for finite time Carnot heat engines, where T_L is the temperature of cold heat reservoir, P is the power output and σ is the entropy generation rate. Arias-Hernandez et al. [13], Barranco-Jimenez et al. [14] and Barranco-Jimenez [15] investigated the thermodynamic optimization of heat engines; the ecological function has been applied with different heat transfer laws. Chen et al. [16]

provided a unified ecological optimization objective function for all of the thermodynamic cycles, that is $E = A/t - T_0\Delta S/t$, where A is the exergy output of the cycle, T_0 is the environmental temperature, ΔS is the entropy generation of the cycle, and t is the cycle period. For heat pump cycles, the exergy output rate of the cycle is $A/\tau = Q_H(1 - T_0/T_H) - Q_L(1 - T_0/T_L)$, where Q_L is the rate of heat transfer supplied by the heat source, Q_H is the rate of heat transfer released to the heat sink, and T_H and T_L are the temperatures of the heat sink and heat source, respectively. The coefficient of performance and the heating load of the heat pump are $\varphi = Q_H/(Q_H - Q_L)$ and $\pi = Q_H$. Therefore, one has:

$$E = \pi \left[\left(1 - \frac{T_0}{T_H} \right) - \left(1 - \frac{1}{\varphi} \right) \left(1 - \frac{T_0}{T_L} \right) \right] - T_0\sigma. \quad (1)$$

Eq. (1) represents the best compromise between the exergy output rate and the exergy loss rate (entropy production rate) of the thermodynamic cycles. Sun et al. [17] investigated the ecological optimal performance of endoreversible Carnot heat pumps with Newton's heat transfer law based on the energy analysis. The finite time thermodynamic performance of heat pump is affected by heat transfer law. Zhu et al. [18] and Chen et al. [19,20] have assessed the effect of the heat transfer law on the ecological performance of endoreversible and irreversible heat pumps.

Most of the previous works of FTT have concentrated on the steady-state energetic properties of the systems.

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Nomenclature

C	heat capacity (WK^{-1})
E	ecological function (W)
F	total surface area of heat exchangers (m^2)
F_1	surface area of the hot-side heat exchanger (m^2)
F_2	surface area of the cold-side heat exchanger (m^2)
J	heat flow (W)
J_1	no-steady-state heat flows from heat pump to warm working fluid (W)
J_2	no-steady-state heat flows from cold working fluid to heat pump (W)
Q	heat exchange (W)
T	temperature of reservoirs (K)
t	relaxation time (s)
\vec{u}	eigenvector
x	temperatures of the warm working fluid at no-steady-state (K)
y	temperatures of the cold working fluid at no-steady-state (K).

Greek symbols

α, β	thermal conductance ($WK (m^2)^{-1}$)
λ	eigenvalue
τ	temperature ratio of heat reservoirs
π	heating load (W)
φ	coefficient of performance.

Subscripts

H	heat sink
L	heat source
max	maximum
0	environment.

Superscripts

-	steady-state values.
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However, it is worthwhile to consider the local stability of the system. Santillan et al. [21] firstly studied the local stability of a Curzon–Ahlborn–Novikov (CAN) engine working in a maximum-power-like regime considering the heat resistance and the equal high- and low-temperature heat transfer coefficients with Newton’s heat transfer law. Chimal-Eguia et al. [22] analyzed the local stability of an endoreversible heat engine working in a maximum-power-like regime with Stefan–Boltzmann heat transfer law. Guzman-Vargas et al. [23] studied the effect of heat transfer laws and heat transfer coefficients on the local stability of an endoreversible heat engine operating in a maximum-power-like regime. Barranco-Jimenez et al. [24] investigated the local stability analysis of a thermo-economic model of a Novikov–Curzon–Ahlborn heat engine. Paez-Hernandez et al. [25] studied the dynamic properties in an endoreversible Curzon–Ahlborn engine using a van der Waals gas as working substance. Chimal-Eguia et al. [26] analyzed the local stability of an endoreversible heat engine working in an ecological regime. Sanchez-Salas et al. [27] studied the dynamic robustness of a non-endoreversible engine working in an ecological regime. Huang et al. [28] studied the local stability analysis of an endoreversible heat pump operating at minimum input power for a given heating load with Newton’s heat transfer law. Huang [29] analyzed the local

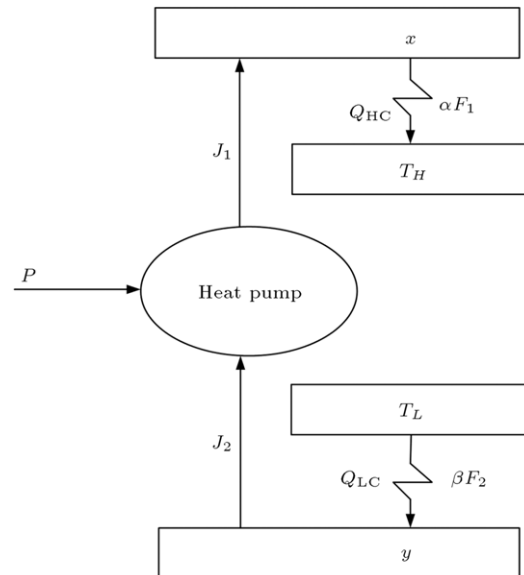


Figure 1: Schematic diagram of an endoreversible heat pump.

asymptotic stability of an irreversible heat pump subject to total thermal conductance constraint. Wu et al. [30] studied the local stability of an endoreversible heat pump with Newton’s heat transfer law working at the maximum ecological function.

This paper will analyze the local stability of an endoreversible heat pump working in an ecological regime based on the optimal ecological performance parameters of the heat pump with linear phenomenological heat transfer law between working fluid and heat reservoirs, and discuss the effects of temperatures of heat reservoirs and heat transfer coefficients on the local stability of the system, and obtain the distribution information of phase portraits of the system.

2. Ecological performance of an endoreversible heat pump

Considering a model of an endoreversible heat pump [11,17,28] as shown in Figure 1, its working conditions are as follows:

(1) The working fluid flows through the system in a steady-state fashion. The cycle consists of two isothermal and two adiabatic processes.

(2) Because of the heat resistance, the working fluid’s temperatures (x and y) are different from the reservoirs’ temperatures (T_H and T_L) and the four temperatures are of the following decreasing order: $x > T_H > T_L > y$. The heat transfer surface areas (F_1 and F_2) of the high- and low-temperature-side heat exchangers are finite. The overall heat transfer surface area (F) of the two heat exchangers is assumed to be a constant: $F = F_1 + F_2$. Assume that the heat transfer surface area ratio is $f = F_1/F_2$, the working fluid’s temperature ratio is $m = y/x$ and the temperature ratio of heat reservoirs is $\tau = T_L/T_H$. Thus, $0 < y/x < T_L/T_H < 1$.

When there is only the heat resistance loss, the second law of thermodynamics requires that

$$\frac{Q_{HC}}{x} = \frac{Q_{LC}}{y}. \quad (2)$$

The first law of thermodynamics gives the heating load, the power input of the cycle and the coefficient of performance of

the heat pump, respectively:

$$\pi = Q_{HC}, \tag{3}$$

$$P = Q_{HC} - Q_{LC}, \tag{4}$$

$$\varphi = \pi/P = Q_{HC}/(Q_{HC} - Q_{LC}). \tag{5}$$

From Eqs. (2) to (4), the rate (Q_{LC}) of heat flow from the heat source to the working fluid and the rate (Q_{HC}) of heat flow from the working fluid to the heat sink can be expressed as:

$$Q_{LC} = \frac{y}{x-y}P, \tag{6}$$

$$Q_{HC} = \frac{x}{x-y}P. \tag{7}$$

Because heat transfer between working fluid and heat reservoirs obeys linear phenomenological heat transfer law, one has:

$$Q_{HC} = \alpha F_1(1/x - 1/T_H), \tag{8}$$

$$Q_{LC} = \beta F_2(1/T_L - 1/y), \tag{9}$$

where α and β are the overall heat transfer coefficients of the high- and low-temperature-side heat exchangers, and are negative, respectively.

When the heat transfer surface area ratio is:

$$f = F_1/F_2 = \sqrt{\beta/\alpha}/m = B/m, \tag{10}$$

the optimal ecological function (E) at a certain temperature ratio (m) is:

$$E = -\frac{\beta F}{(1+B/m)^2} [m(2T_0/T_L - 1) - (2T_0/T_H - 1)] \times \left(\frac{1}{m^2 T_H} - \frac{1}{m T_L} \right). \tag{11}$$

Taking the derivative of E with respect to m and setting it equal to zero ($dE/dm = 0$) yields that the optimal temperature ratio corresponding to the maximum ecological function and the maximum ecological function are [17–20], respectively:

$$m = \frac{(2T_0/T_L - 1)T_L B + (2T_0/T_H - 1)T_H B + 2(2T_0/T_H - 1)T_L}{(2T_0/T_L - 1)T_L + (2T_0/T_H - 1)T_H + 2(2T_0/T_L - 1)T_H B}, \tag{12}$$

$$E_{\max} = -\frac{\beta F [(2T_0/T_L - 1)T_L - (2T_0/T_H - 1)T_H]^2}{4T_H T_L (T_L + T_H B) [T_L B + (2T_0/T_H - 1)]}, \tag{13}$$

where $B = \sqrt{\beta/\alpha}$.

3. The steady state of the heat pump working in an ecological regime

Assume that the working fluid's temperatures of the steady state are \bar{x} and \bar{y} , respectively. In this paper, the variables with over-bars denote the steady-state values and $\bar{x} > \bar{y}$. The rates of heat flows can be given by:

$$\bar{J}_1 = \frac{\bar{x}}{\bar{x} - \bar{y}} \bar{P}, \tag{14}$$

$$\bar{J}_2 = \frac{\bar{y}}{\bar{x} - \bar{y}} \bar{P}, \tag{15}$$

where \bar{J}_1 and \bar{J}_2 are rates of the steady-state heat flows from the heat pump to \bar{x} and from \bar{y} to the heat pump, respectively, and \bar{P} is steady-state power input. When the heat pump operates in a steady state, it means that the rate (Q_{HC}) of heat flow from \bar{x} to

T_H equals to \bar{J}_1 and the rate (Q_{LC}) of heat flow from T_L to \bar{y} equals to \bar{J}_2 , i.e.:

$$\bar{J}_1 = \alpha F_1 (1/\bar{x} - 1/T_H), \tag{16}$$

$$\bar{J}_2 = \beta F_2 (1/T_L - 1/\bar{y}). \tag{17}$$

The coefficient of performance of heat pump is:

$$\bar{\varphi} = \bar{J}_1/\bar{P} = \bar{x}/(\bar{x} - \bar{y}). \tag{18}$$

The working fluid's temperatures (\bar{x} and \bar{y}) can be expressed as:

$$\bar{x} = T_H \frac{1 + B\bar{\varphi}/(\bar{\varphi} - 1)}{1 + B/\tau}, \tag{19}$$

$$\bar{y} = T_L \frac{B + (\bar{\varphi} - 1)/\bar{\varphi}}{B + \tau}. \tag{20}$$

The temperatures of hot reservoir and cold reservoir (T_H and T_L) can be expressed as functions of \bar{x} and \bar{y} , respectively, i.e.:

$$T_H = \frac{\bar{x}\bar{y} + \bar{x}\bar{y}B/\tau}{\bar{y} + \bar{x}B}, \tag{21}$$

$$T_L = \frac{\bar{x}\bar{y}\tau + \bar{x}\bar{y}B}{\bar{y} + \bar{x}B}. \tag{22}$$

The steady-state power input as a function of \bar{x} and \bar{y} can be expressed as:

$$\bar{P} = \frac{\beta F}{\bar{y} + \bar{x}B} \left(\frac{\bar{y} - \bar{x}\tau}{\bar{x}\tau + \bar{x}B} \right) \frac{\bar{x} - \bar{y}}{\bar{y}}. \tag{23}$$

4. Local stability analysis of the heat pump

In order to analyze the local stability of an endoreversible heat pump, assume that the temperatures corresponding to macroscopic objects with heat capacity C are x and y , respectively. The dynamical equations with respect to x and y are [21,28]:

$$dx/dt = [J_1 - \alpha F_1 (1/x - 1/T_H)]/C, \tag{24}$$

$$dy/dt = [\beta F_2 (1/T_L - 1/y) - J_2]/C, \tag{25}$$

respectively, where J_1 and J_2 are rates of heat flows from x to the working fluid and from the heat pump to y , respectively. According to Eqs. (14) and (15), J_1 and J_2 can be written as:

$$J_1 = \frac{x}{x-y}P, \tag{26}$$

$$J_2 = \frac{y}{x-y}P. \tag{27}$$

When the system works in the steady state of the maximum ecological function, the optimal temperature ratio of the working fluid with the case of $T_0 = T_L$ is given by Eq. (12):

$$\bar{m} = \frac{\bar{y}}{\bar{x}} = \frac{\tau B + (2\tau - 1)B + 2\tau(2\tau - 1)}{\tau + (2\tau - 1) + 2B}. \tag{28}$$

The temperature ratio of heat reservoirs can be written as a function of \bar{x} and \bar{y} (see Eq. (29) given in Box 1).

By using Eqs. (18)–(20) and (28), the steady-state values \bar{x} and \bar{y} can be obtained:

$$\bar{x} = 2T_H \frac{(2\tau - 1)B + \tau B + (2\tau - 1)\tau + B^2}{[(2\tau - 1)B + \tau B + 2(2\tau - 1)\tau](1 + B/\tau)}, \tag{30}$$

$$\bar{y} = 2T_L \frac{(2\tau - 1)B + \tau B + (2\tau - 1)\tau + B^2}{[\tau + (2\tau - 1) + 2B](\tau + B)}. \tag{31}$$

$$\tau = \frac{(2 + 3\bar{y}/\bar{x} - 3B) + \sqrt{(3B - 2 - 3\bar{y}/\bar{x})^2 - 16(\bar{y}/\bar{x} - B - 2B\bar{y}/\bar{x})}}{8} \tag{29}$$

Box I

$$dx/dt = \frac{\alpha F}{C} \left[\frac{B^2}{y + xB} \left(\frac{y - x\tau}{\tau + B} \right) \frac{1}{y} - \frac{B}{y + xB} \left(1 - \frac{x}{T_H} \right) \right], \tag{34}$$

$$dy/dt = \frac{\beta F}{C} \left[\frac{1}{y + xB} \left(\frac{y}{T_L} - 1 \right) - \frac{1}{y + xB} \left(\frac{y - x\tau}{x\tau + xB} \right) \right], \tag{35}$$

where

$$\tau = \frac{(2 + 3y/x - 3B) + \sqrt{(3B - 2 - 3y/x)^2 - 16(y/x - B - 2By/x)}}{8}$$

Box II

Substituting Eq. (29) into Eq. (23) yields:

$$\bar{P}(\bar{x} - \bar{y}) = \frac{\beta F}{\bar{y} + \bar{x}B} \left(\frac{\bar{y} - \bar{x}\tau}{\bar{x}\tau + \bar{x}B} \right) \frac{\bar{x} - \bar{y}}{\bar{y}} \tag{32}$$

Consider the Taylor's formula of no-steady-state power input $P(x, y)$ about the point (\bar{x}, \bar{y}) , neglecting the two and more orders, $P(x, y) = \bar{P}(\bar{x}, \bar{y}) + (x - \bar{x})(\partial P/\partial x) + (y - \bar{y})(\partial P/\partial y)$. When the heat pump works out of the steady state but not too far away, the distances $(x - \bar{x})$ and $(y - \bar{y})$ are small enough to be neglected, one can assume that $P(x, y) \approx \bar{P}(\bar{x}, \bar{y})$. It has been applied to the local stability analysis [21–30]. The power input of the heat pump depends on x and y in the same way that it depends on \bar{x} and \bar{y} at the steady state, i.e.

$$P(x, y) = \bar{P}(\bar{x}, \bar{y}) = \frac{\beta F}{y + xB} \left(\frac{y - x\tau}{x\tau + xB} \right) \frac{x - y}{y} \tag{33}$$

Substituting Eqs. (26), (27) and (33) into Eqs. (24) and (25) yields Eqs. (34) and (35) as given in Box II.

To analyze the system stability near the steady state, based on linearization and stability analysis [31], one can define two functions:

$$f(x, y) = \frac{\alpha F}{C} \left[\frac{B^2}{y + xB} \left(\frac{y - x\tau}{\tau + B} \right) \frac{1}{y} - \frac{B}{y + xB} \left(1 - \frac{x}{T_H} \right) \right], \tag{36}$$

$$g(x, y) = \frac{\beta F}{C} \left[\frac{1}{y + xB} \left(\frac{y}{T_L} - 1 \right) - \frac{1}{y + xB} \left(\frac{y - x\tau}{x\tau + xB} \right) \right]. \tag{37}$$

The eigenvalues at maximum ecological function can be obtained [21–31]:

$$\lambda_1 = \left[f_x + g_y - \sqrt{(f_x - g_y)^2 + 4f_y g_x} \right] / 2, \tag{38}$$

$$\lambda_2 = \left[f_x + g_y + \sqrt{(f_x - g_y)^2 + 4f_y g_x} \right] / 2, \tag{39}$$

and the corresponding eigenvectors are:

$$\vec{u}_1 = \left(\left[f_x - g_y - \sqrt{(f_x - g_y)^2 + 4f_y g_x} \right] / 2g_x, 1 \right), \tag{40}$$

$$\vec{u}_2 = \left(\left[f_x - g_y + \sqrt{(f_x - g_y)^2 + 4f_y g_x} \right] / 2g_x, 1 \right), \tag{41}$$

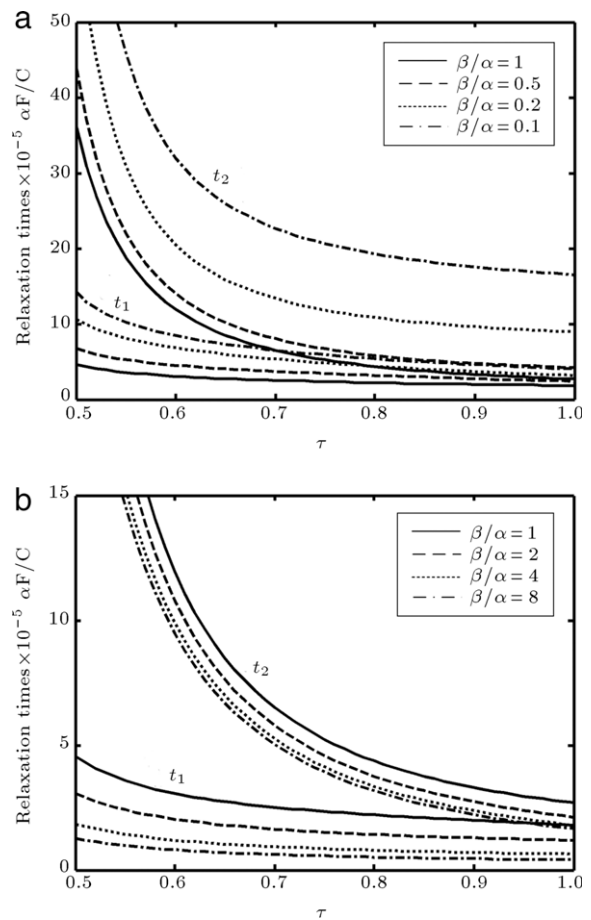


Figure 2: Relaxation times vs. τ with (a) $\beta/\alpha \leq 1$ and (b) $\beta/\alpha \geq 1$.

where $f_x = (\partial f/\partial x)_{\bar{x}, \bar{y}}$, $f_y = (\partial f/\partial y)_{\bar{x}, \bar{y}}$, $g_x = (\partial g/\partial x)_{\bar{x}, \bar{y}}$ and $g_y = (\partial g/\partial y)_{\bar{x}, \bar{y}}$ are given in Appendix.

According to Eqs. (38) and (39) and Appendix, the eigenvalues (λ_1 and λ_2) are function of $C, F, \alpha, \beta, \tau$ and T_L . The final expressions are quite lengthy, moreover, our calculations show that both eigenvalues are real and negative ($\lambda_1 < \lambda_2 < 0$). Thus, the characteristic relaxation times (which are defined as $t_{1,2} = 1/|\lambda_{1,2}|$) can be written as:

$$t_1 = -2/[f_x + g_y - \sqrt{(f_x - g_y)^2 + 4f_y g_x}], \tag{42}$$

$$f_x = \frac{\alpha F}{C} \left(\frac{B^2(1 - \alpha\tau/T_L)}{(Ba + b)^2} + \frac{B}{T_H(Ba + b)} - \frac{B^3(b - ad/8)}{y(Ba + b)^2(B + d/8)} \right. \\ \left. + \left[B^2 \left(\frac{3b}{8a} - \frac{ac}{16(d - 2 + 3B - 3b/a)} - \frac{d}{8} \right) \right] / \left[b(Ba + b) \left(B + \frac{1}{8}d \right) \right] \right. \\ \left. - \left[B^2 \left(-\frac{3b}{a^2} + \frac{c}{2(d - 2 + 3B - 3b/a)} \right) \left(b - \frac{ad}{8} \right) \right] / \left[8b(Ba + b) \left(B + \frac{d}{8} \right)^2 \right] \right), \quad (\text{A.1})$$

$$f_y = \frac{\alpha F}{C} \left(\frac{B(1 - \alpha\tau/T_L)}{(Ba + b)^2} + \frac{B^2(10 - ac/(d - 2 + 3B - 3b/a))}{16b(Ba + b)(B + d/8)} - \frac{B^2(b - ad/8)}{b(Ba + b)^2(B + d/8)} \right. \\ \left. - \frac{B^2(b - ad/8)}{b^2(Ba + b)(B + d/8)} - \left[B^2 \left(\frac{3(b - ad/8)}{a} + \frac{c(b - ad/8)}{2(d - 2 + 3B - 3b/a)} \right) \right] / \left[8b(Ba + b)(B + 1/d)^2 \right] \right), \quad (\text{A.2})$$

$$g_x = \frac{\beta F}{C} \left(-\frac{B(-1 + b/T_L)}{(Ba + b)^2} + \frac{B(b - ad/8)}{(Ba + b)^2(Ba + ad/8)} \right. \\ \left. + \left[\left(B - \frac{3b}{8a} + \frac{ac}{16(d - 2 + 3B - 3b/a)} + \frac{d}{8} \right) \left(b - \frac{ad}{8} \right) \right] / \left[(Ba + b) \left(Ba + \frac{ad}{8} \right)^2 \right] \right. \\ \left. - \left(\frac{3b}{8a} - \frac{ac}{16(d - 2 + 3B - 3b/a)} - \frac{d}{8} \right) / \left[(Ba + b)(Ba + ad/8) \right] \right), \quad (\text{A.3})$$

$$g_y = \frac{\beta F}{C} \left(\frac{1}{T_L(Ba + b)} - \frac{-1 + b/T_L}{(Ba + b)^2} + \frac{b - ad/8}{(Ba + b)^2(Ba + ad/8)} + \left[3 + \frac{ac}{2(d - 2 + 3B - 3b/a)} \right] \right. \\ \left. \times \left(b - \frac{ad}{8} \right) / \left[8(Ba + b) \left(Ba + \frac{ad}{8} \right)^2 \right] - \frac{10 - ac/(d - 2 + 3B - 3b/a)}{(Ba + b)(Ba + ad/8)} \right), \quad (\text{A.4})$$

where:

$$a = 2 \frac{T_L}{\tau} \frac{(2\tau - 1)B + \tau B + (-1 + 2\tau)\tau + B^2}{[(2\tau - 1)B + \tau B + 2(-1 + 2\tau)\tau](1 + B/\tau)},$$

$$b = 2T_L \frac{(2\tau - 1)B + \tau B + (2\tau - 1)\tau + B^2}{[\tau + (2\tau - 1) + 2B](\tau + B)},$$

$$c = \frac{b}{a^2} \left(4 - 14B - \frac{18b}{a} \right),$$

$$d = 2 - 3B + \frac{3b}{a} + \sqrt{\left(2 - 3B + \frac{3b}{a} \right)^2 - 16 \left(-B + \frac{b}{a} - \frac{2Bb}{a} \right)}.$$

Box III

$$t_2 = -2/[f_x + g_y + \sqrt{(f_x - g_y)^2 + 4f_y g_x}]. \quad (43)$$

Because both λ_1 and λ_2 are real and negative, any perturbation decays exponentially to the steady state with time and the steady state of the heat pump working at the maximum ecological function is steady. Eqs. (42) and (43) are general expressions of the characteristic relaxation times. It is shown that the characteristic relaxation times are proportional to heat capacity C , inversely-proportional to the overall heat transfer surface area F , relative to τ , T_L , α and β .

Relaxation times of the system working at the maximum ecological function vs. heat reservoirs' temperature ratio τ for different β/α are shown in Figs. 2. For the given heat transfer

coefficient ratio β/α , it can be seen that t_1 and t_2 decrease as τ increases, that means the stability improves as $\tau \rightarrow 1$. As shown in Figure 2a, in the region of $\beta/\alpha \leq 1$, t_1 and t_2 increase as β/α decreases, if $\beta/\alpha \rightarrow 0$, $t_2 \rightarrow \infty$, the stability of the system is lost. As shown in Figure 2b, in the region of $\beta/\alpha \geq 1$, t_1 and t_2 decrease as β/α increases, the stability of the system is improved.

As mentioned before, eigenvalues are real and negative ($\lambda_1 < \lambda_2 < 0$). Thus, $0 < t_1 < t_2$, i.e., $0 < t_1/t_2 < 1$ and the corresponding eigenvectors \vec{u}_1 and \vec{u}_2 can be described as fast eigendirection and slow eigendirection, respectively. According to the numerical calculations by using the relaxation time ratio and corresponding eigenvectors, the phase portraits can be

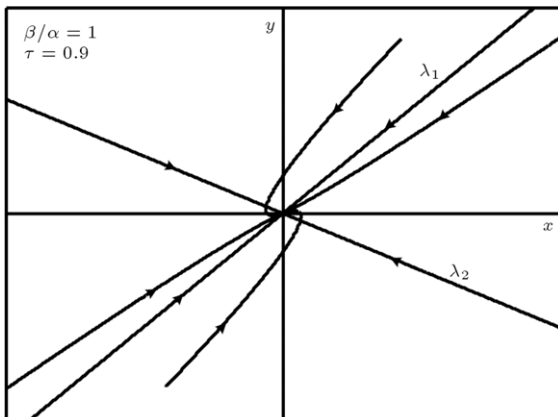


Figure 3: Phase portrait of $x(t)$ vs. $y(t)$ with $\beta/\alpha = 1$, $\tau = 0.9$.

plotted and the distribution information of phase portraits of system may be obtained. The phase portrait of $x(t)$ vs. $y(t)$ for $\beta/\alpha = 1$, $\tau = 0.9$ and $T_L = 300$ K is shown in Figure 3. It is calculated that the relaxation time ratio is $t_1/t_2 = 0.60$ and the eigendirections are $\bar{u}_1 = (1, 1.1)$ and $\bar{u}_2 = (1, -0.56)$. There are two different linear trajectories named fast eigendirection and slow eigendirection, respectively. The phase portraits show that any perturbation on x and y values tend to approach the steady-state point (\bar{x}, \bar{y}) .

5. Conclusion

The local stability of an endoreversible heat pump with linear phenomenological heat transfer law working in an ecological regime is analyzed, the general expressions of the relaxation times with heat capacity C , overall heat transfer surface area F , heat reservoirs' temperature ratio τ , and heat transfer coefficients α and β are obtained. The steady state of the heat pump working at the maximum ecological function is steady, after a small perturbation the system state exponentially decays to steady state with either of two relaxation times. According to the numerical calculations, both relaxation times t_1 and t_2 decrease as τ increases, and decrease as β/α increases, and thus, the local stability of the system is improved. There are two different linear trajectories named fast eigendirection and slow eigendirection, respectively. The phase portraits show that any perturbation on x and y values tend to approach the steady-state point (\bar{x}, \bar{y}) . It is concluded that both the energetic properties and local stability of the system should be considered for designing real heat pumps.

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Appendix

See Box III.

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