Economics Letters 120 (2013) 542–545

Contents lists available at SciVerse ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Level-k reasoning and time pressure in the 11–20 money request game

Florian Lindner[a,*], Matthias Sutter[a,b]

[a] University of Innsbruck, Department of Public Finance, Universitaetsstrasse 15, A-6020 Innsbruck, Austria
[b] University of Gothenburg, Department of Economics, Box 640, 405 30 Gothenburg, Sweden

HIGHLIGHTS

- We study level-k reasoning and learning in a repeated 11–20 game.
- We also examine level-k reasoning with and without time pressure.
- Behavior is robust to repetition, which means that no learning is observed.
- Under time pressure, choices are indistinguishable from the Nash equilibrium.
- Without time pressure, this is not the case.

ARTICLE INFO

Article history:
Received 30 April 2013
Received in revised form
30 May 2013
Accepted 7 June 2013
Available online 19 June 2013

JEL classification:
C91
C72

Keywords:
Level-k reasoning
Time pressure
Repetition
Experiment

ABSTRACT

Arad and Rubinstein (2012a) have designed a novel game to study level-k reasoning experimentally. Just like them, we find that the depth of reasoning is very limited and clearly different from that in equilibrium play. We show that such behavior is even robust to repetitions; hence there is, at best, little learning. However, under time pressure, behavior is, perhaps coincidentally, closer to that in equilibrium play. We argue that time pressure evokes intuitive reasoning and reduces the focal attraction of choosing higher (and per se more profitable) numbers in the game.

1. Introduction

In a recent paper, Arad and Rubinstein (2012a) — henceforth, AR — introduced a new two-player game, called the 11–20 money request game. This game can be regarded as a useful tool to explaining behavior and is related to the prominent concept of level-k reasoning, which was introduced by Stahl and Wilson (1994, 1995) and Nagel (1995).[1] In general, the letter k denotes the steps (or depth) of reasoning of a particular subject when facing a strategic problem. Therefore, a level-0 type is a non-strategic decision maker who randomizes over the available strategies. In contrast, the behavior of a level-k type (Lk), for any $k = 1$, is a best response to the belief that other players are a level $k - 1$ type. This implies that level-k models feature (i) an $L0$ behavior, which is the starting point for iterative reasoning, and (ii) a distribution of types.

The 11–20 money request game of AR is a simultaneous move game in which two players request a number of points between 11 and 20, which they receive for sure. One player may receive 20 extra points if he/she requests one point less than the other player does.[2] This very simple and straightforward game has a number of notable features that make it suitable for studying level-k reasoning (see AR for an elaborate discussion). Compared to many

* Corresponding author. Tel.: +43 512 5077163.
E-mail address: florian.lindner@uibk.ac.at (F. Lindner).

[1] For an up-to-date account of models of strategic thinking, see Crawford et al. (2013).

[2] AR also describe two variations of this game — the Cycle Version and the Costless Iterations Version — and in Arad and Rubinstein (2012b) they discuss another version (the 91–100 game). These variations show very similar results to the basic game that we are going to investigate in our paper.

0165-1765
http://dx.doi.org/10.1016/j.econlet.2013.06.005

Table 1
Relative frequencies of actions in different treatments.

Action	11 (%)	12 (%)	13 (%)	14 (%)	15 (%)	16 (%)	17 (%)	18 (%)	19 (%)	20 (%)	*N*
Equilibrium					25	25	20	15	10	5	
AR (2012)	4	0	3	6	1	6	32	30	12	6	108
BASE	1	3	4	6	3	6	20	38	14	6	80
BASE-T	4	1	1	8	6	5	15	34	19	8	80
TIME	3	3	11	5	20	5	18	17	11	8	65

games, like the beauty-contest game or normal-form games, actions in the 11–20 game create no payoff externalities, for which reason social preferences become unimportant and negligible. The level-0 type specification is appealing, as choosing 20 is (the most profitable) anchor for an iterative reasoning process. Furthermore, best-responding is straightforward, as it requires undercutting the other player by one unit.

The game has no pure-strategy Nash equilibrium, but a mixed-strategy equilibrium in which the strategies from 15 to 20 are chosen with positive probabilities (see Table 1). In their paper, AR find that experimental play is significantly different from equilibrium play, and that the level of reasoning is fairly limited, with subjects typically not choosing more than three steps of reasoning.[3]

In this paper, we study behavior in the 11–20 game by considering (i) whether time pressure has an effect on experimental (equilibrium) play and (ii) whether subjects converge to equilibrium with repetition. The second question touches upon learning. Previous work on level-*k* reasoning in beauty-contest games (Nagel, 1995; Ho et al., 1998), for instance, has shown that repeated choices converge towards the equilibrium solution (even if the level of reasoning from one period to the next does not increase). So far, the 11–20 game has not been played in a repeated-game setting, and hence we test the robustness of the one-shot findings of AR. The first question is related to a growing body of literature which is interested in how time pressure affects economic decision making (see Kocher et al., forthcoming for a recent contribution and a brief review of economic literature on the topic). Since many important decisions — e.g., last-minute bidding in auctions (Roth and Ockenfels, 2002), bargaining decisions (Sutter et al., 2003), or decisions in financial markets — have to be made under severe time pressure, it is important to understand whether economic behavior changes in such a stressful environment. An earlier contribution of Kocher and Sutter (2006) on the effects of time pressure in the beauty-contest game has shown that time pressure dampens the speed with which choices converge to the equilibrium (of choosing zero). As argued above (and more extensively in AR), the 11–20 game provides a much simpler setting to study level-*k* reasoning than the beauty-contest game, and hence we are interested in the effects of time pressure in such an environment.

2. Experimental design

Our experiment consists of two parts. Part one is a one-shot version of the 11–20 game (the rules of which have been introduced above; see the online supplement (Appendix) for experimental instructions). Part one has three different treatments, explained below. Part two is a five-fold repetition of the 11–20 game with the same partner (fixed partner matching, where partners differ from part one, though). It is explained only after the conclusion of part one, and no feedback about part one is given until the whole experiment is finished. Part two is identical in all experimental sessions and has no treatment variations.

The first treatment in part one, **BASE**, is a replication of the basic version of AR. The second treatment, **BASE-T**, is a modification of BASE in order to make it comparable to the third treatment, **TIME**. In the treatment with time pressure, TIME, we explain only the basic features of the game in the instructions, i.e., that it is a two-person game, that subjects will be able to choose a number from an interval that will be disclosed when the experiment starts, that subjects earn the amount of points equivalent to the chosen number, and that subjects may earn 20 bonus points under specific conditions. The details — that the interval was [11, 20] and that 20 bonus points could be earned if one player's number was one unit below the other player's number — are only disclosed when the experiment starts and subjects have only 15 s time to read this information and make a choice. If these details had been disclosed before the start of the experiment, it would not have been possible to implement time pressure, because subjects would have had the chance to make up their mind while the experiment was still explained.

In order to have a comparison treatment that mimics TIME as far as possible, we implement our second treatment, BASE-T, which is identical to TIME, except that subjects have 3 min time — instead of 15 s — to read the information on the first screen and enter their decision.[4]

After part one, subjects in all three treatments are told that this game (with the same rules as in treatment BASE) is played for five more periods in partner matching, with 3 min time for making decisions in each period, and with feedback after each period. We denote the data in the repetition as **R-BASE**, **R-BASE-T**, and **R-TIME** to identify which treatment subjects have had in part one.

We have four sessions with 20 subjects each for each of the three treatments (using zTree Fischbacher, 2007, and ORSEE, Greiner, 2004). This yields 80 subjects per treatment, and a total of 240 participants. Experiments were conducted at the University of Innsbruck using a standard subject pool across all disciplines. In our sample, 52% of participants were female. On average, subjects earned 7.90 Euros.

3. Results

3.1. Part one — one-shot game

In treatment TIME, 65 out of 80 subjects were able to make a decision within the time constraint (the 15 remaining subjects did not receive any payment for part one; we also exclude their data in the analysis of part two below). Table 1 presents in the first row the unique symmetric Nash equilibrium distribution (assuming that players maximize the expected monetary payoff). In the second row, we show the data of AR for comparison reasons; and below we present the data of our three treatments.

Recall that BASE is a replication of AR. While the data look slightly different, there is no significant difference between AR and

[3] This feature of limited levels of reasoning is strongly reminiscent of findings for the beauty-contest game (see Nagel, 1995; Costa-Gomes and Crawford, 2006 or Grosskopf and Nagel, 2008) or normal-form games (Costa-Gomes and Weizsäcker, 2008; Danz et al., 2012).

[4] Treatment BASE-T was run immediately before treatment TIME. In Treatment BASE-T, 15% of subjects entered a decision within 15 s. For this reason, we restricted the available decision time in treatment TIME to 15 s, which should guarantee a highly demanding time pressure.

Table 2
Actions in the repeated game.

Action	11 (%)	12 (%)	13 (%)	14 (%)	15 (%)	16 (%)	17 (%)	18 (%)	19 (%)	20 (%)	N
Equilibrium					25	25	20	15	10	5	
R-BASE	0	1	2	2	7	11	20	24	21	13	400
R-BASE-T	1	3	2	4	8	9	21	22	18	12	400
R-TIME	3	3	3	5	8	11	18	24	12	13	325

BASE ($p = 0.46$; χ^2-test), indicating that we are able to confirm the data of AR. Furthermore, BASE is not significantly different from BASE-T ($p = 0.84$; χ^2-test), showing that the different presentation of the game does not make a difference. Both treatments, BASE and BASE-T, are significantly different from that in equilibrium play, though ($p < 0.01$; χ^2-tests), again confirming AR.

In contrast to these findings, the results in TIME look markedly different. With time pressure, the distribution of chosen actions shifts leftward. It is significantly different from that in BASE-T ($p = 0.025$; χ^2-test). It is also no longer significantly different from that in equilibrium play ($p = 0.18$; χ^2-test). One might think that the latter result would be a consequence of random play in TIME — given the demanding environment — but this is not the case, since the distribution of choices is significantly different from that in random play ($p < 0.01$; χ^2-tests).[5] Rather, we see that less time leads to a lower frequency of sophisticated choices 17–19, and that the choice of 15 (in the middle of the range) becomes fairly prominent. We summarize this subsection as follows.

Result 1. Time pressure leads to a situation where subjects' strategy choices correspond — perhaps coincidentally — more closely to those in equilibrium play compared to a situation without time constraints.

3.2. Part two — five-fold repetition of the game

Table 2 presents the relative frequency of chosen actions across all five periods, and Fig. 1 shows the average chosen number in each of the five periods of part two (and the average in part one). Recall that part two was identical in all sessions and thus did not involve any time pressure. The data in Table 2 show that all three distributions are significantly different from that of equilibrium play ($p < 0.01$; χ^2-tests). Hence, without any time pressure, numbers go up again, as can also be seen in Fig. 1. Interestingly, the exposure to time pressure in part one (in TIME) has an enduring effect on part two insofar as the chosen numbers in part two are significantly smaller in R-TIME than in R-BASE-T (which is the proper comparison), as becomes clear from a regression not reported here ($p = 0.002$). Table 3 presents pairwise comparisons of the three treatments for each period separately. In periods one, two, and five there is no statistical difference in any of the treatment comparisons. We find significant differences in two out of three pairwise comparisons in period three and in all comparisons in period four.[6] Overall, Fig. 1 suggests that repetition does *not* lead to convergence towards the equilibrium distribution of actions, showing that the one-shot results established in AR are robust to potential learning effects when repeating the task.

Result 2. Behavior in the 11–12 game is robust over periods, i.e. the game is rather stable against learning issues and experience of different one-shot treatments.

Table 3
Pairwise comparisons of treatments by period with Fisher's exact tests (p-values).

Period in part two	R-BASE versus R-BASE-T	R-BASE versus R-TIME	R-BASE-T versus R-TIME
Period 1	0.43	0.18	0.33
Period 2	0.42	0.58	0.54
Period 3	0.05	0.16	0.00
Period 4	0.02	0.00	0.02
Period 5	0.41	0.42	0.19

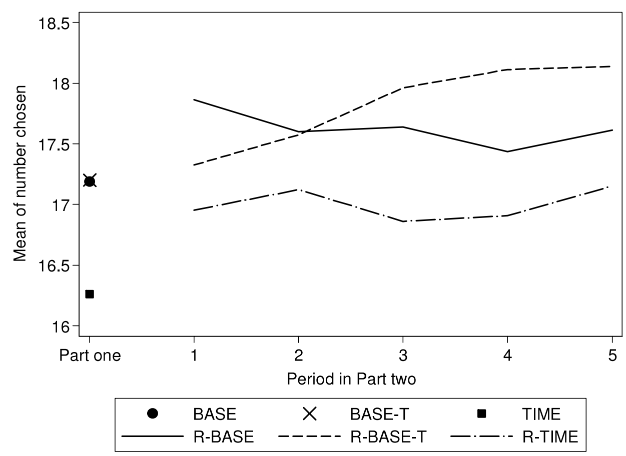

Fig. 1. Average actions per period in part two.

4. Conclusion

We have been able to replicate the results of Arad and Rubinstein (2012a) on behavior in a novel two-person game that allows studying strategic behavior. The level of reasoning has been fairly limited, and the overall behavior diverges considerably from that of equilibrium play. This is also true when the game is repeated, which is the first and main contribution of our paper, indicating that behavior is robust against learning. The finding that no "adjustment" of level-0 types took place in later rounds is quite surprising, and might be further explored in future research. However, and this is the second contribution of our paper, introducing time pressure in the 11–20 game has shifted behavior in the direction of (mixed-strategy) equilibrium, although the shift may be coincidental. Decisions that are made faster are described by Rubinstein (2007) as intuitive. In contrast, cognitive decisions require more time, which might lead subjects in the 11–20 game to be attracted by the higher sure payoffs the longer they think about the game.

Acknowledgments

We thank Martin G. Kocher, Jörg Oechssler, and an anonymous referee for helpful comments and suggestions. Financial support from the Austrian Science Fund (FWF) through project P22772-G11 is gratefully acknowledged.

Appendix. Supplementary data

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.econlet.2013.06.005.

[5] Another way to look at the possibility that random play might partly drive our results is to assume that half of participants choose strategies according to the Nash equilibrium and half play random strategies. This would lead to a 5–5–5–5–17.5–17.5–15–12.5–10–7.5 distribution of choices from 11 to 20. The result of a χ^2-test shows that we can only reject the hypothesis that subjects behave this way on a 15% significance level ($p = 0.15$).

[6] More details are presented in Table A in the online supplement (Appendix).

References

Arad, Ayala, Rubinstein, Ariel, 2012a. The 11–20 money request game: a level-k reasoning study. American Economic Review 102 (7), 3561–3573.

Arad, Ayala, Rubinstein, Ariel, 2012b. Multi-dimensional iterative reasoning in action: the case of the Colonel Blotto game. Journal of Economic Behavior and Organization 84 (2), 571–585.

Costa-Gomes, Miguel A., Crawford, Vincent P., 2006. Cognition and behavior in two-person guessing games: an experimental study. American Economic Review 96 (5), 1737–1768.

Costa-Gomes, Miguel, Weizsäcker, Georg, 2008. Stated beliefs and play in normal-form games. Review of Economic Studies 75 (3), 729–762.

Crawford, Vincent, Costa-Gomes, Miguel, Iriberri, Nagore, 2013. Structural models of nonequilibrium strategic thinking: theory, evidence, and applications. Journal of Economic Literature 51 (1), 5–62.

Danz, David, Fehr, Dietmar, Kübler, Dorothea, 2012. Information and beliefs in a repeated normal-form game. Experimental Economics 15 (4), 622–640.

Fischbacher, Urs., 2007. *z*-tree: Zurich toolbox for ready-made economic experiments. Experimental Economics 10 (2), 171–178.

Greiner, Ben., 2004. An online recruitment system for economic experiments. In: Kremer, Kurt, Macho, Volker (Eds.), Forschung und Wissenschaftliches Rechnen 2003. Gesellschaft für Wissenschaftliche Datenverarbeitung Göttingen, Göttingen, pp. 79–93.

Grosskopf, Brit, Nagel, Rosemarie, 2008. The two-person beauty contest. Games and Economic Behavior 62 (1), 93–99.

Ho, Teck-Hua, Camerer, Colin, Weigelt, Keith, 1998. Iterated dominance and iterated best response in experimental p-beauty contests. American Economic Review 88 (4), 947–969.

Kocher, Martin G., Pahlke, Julius, Trautmann, Stefan T., 2012. Tempus fugit: time pressure in risky decisions. Management Science forthcoming, published online before print: http://dx.doi.org/10.1287/mnsc.2013.1711.

Kocher, Martin G., Sutter, Matthias, 2006. Time is money — time pressure, incentives, and the quality of decision-making. Journal of Economic Behavior and Organization 61 (3), 375–392.

Nagel, Rosemarie, 1995. Unraveling in guessing games: an experimental study. American Economic Review 85 (5), 1313–1326.

Roth, Alvin E., Ockenfels, Axel, 2002. Last-minute bidding and the rules for ending second-price auctions: evidence from eBay and Amazon auctions on the internet. American Economic Review 92 (5), 1138–1151.

Rubinstein, Ariel, 2007. Instinctive and cognitive reasoning: a study of response times. Economic Journal 117 (523), 1243–1259.

Stahl, Dale O., Wilson, Paul W., 1994. Experimental evidence on players models of other players. Journal of Economic Behavior and Organization 25 (3), 309–327.

Stahl, Dale O., Wilson, Paul W., 1995. On players' models of other players: theory and experimental evidence. Games and Economic Behavior 10 (1), 213–254.

Sutter, Matthias, Kocher, Martin G., Strauß, Sabine, 2003. Bargaining under time pressure in an experimental ultimatum game. Economics Letters 81 (3), 341–347.