It is well known that if $\hat{X}$ is a Hausdorff compactification of a Tychonoff space $X$, then there is a precompact uniformity compatible with the topology of $X$ such that the Samuel compactification obtained from it is $\hat{X}$. We say that a compactification is Wallman if it can be constructed from a normal base of closed sets as described by Frink [1].

The purpose of this note is to characterize those precompact uniformities whose Samuel compactification is Wallman. The criterion is surprisingly simple.

Let $\mu$ be the family of uniform covers of some precompact uniformity. If there is some base $B$ for open sets in the uniform topology such that the family of all finite covers from $B$ is a base for $\mu$, we will say that $\mu$ is generated by $B$.

**THEOREM.** The Samuel compactification is Wallman if and only if the associated precompact uniformity possesses a generating base of open sets.

**Proof.** Let $\mu$ be a precompact uniformity compatible with the topology of $X$, $B$ a generating base of open sets, and $\beta\mu X$ the Samuel compactification. To show $\beta\mu X$ is a Wallman compactification, it suffices to find a base $\mathcal{F}$ for closed sets in $\beta\mu X$ with the trace property with respect to $X$ (i.e. if $F_i \in \mathcal{F}$, $i = 1, \ldots, n$, then $\bigcap \{F_i\} \neq \emptyset$ implies $\bigcap \{F_i\} \cap X \neq \emptyset$, cf. [4]).

Let $\mathcal{F} = \{(X - O)^{-} | O \in B\}$, where $A = \text{cl}_{\beta\mu X} A$. Suppose $F_i = (X - O_i)^{-}$, $O_i \in B$, $i = 1, \ldots, n$ and $\bigcap \{F_i\} \cap X = \emptyset$. Then the family $\{O_i\}$ is a cover in $\mu$ and there exists a finite open cover $\mathcal{U}$ of $\beta\mu X$ such that the trace of $\mathcal{U}$ on $X$ refines $\{O_i\}$. Since the complement of each member of $\mathcal{U}$ contains some $F_i$, it follows that $\bigcap \{F_i\} = \emptyset$.

It remains to show that $\mathcal{F}$ is a base for closed sets. Let $U$ be an open set containing $x$ and let $V$ and $W$ be open sets satisfying $x \in V \subset \overline{V} \subset C W \subset \overline{W} \subset U$. The family $\{W, \beta\mu X - \overline{V}\}$ covers $\beta\mu X$ so its trace on $X$ is refined by a finite open cover $D$ of sets in $B$.

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Letting \( D' = \beta \mu X - (X - D)^{-} \), we see that \( D' \cap X = D \) and \( \{ D' | D \in \mathcal{D} \} \) is a cover of \( \beta \mu X \). Thus \( x \in D'_k \) for some \( D_k \in \mathcal{D} \). Since \( X \) is dense, \( D'_k = D_k \). Now, \( D_k \) is either in \( W \) or \( \beta \mu X - \bar{V} \), but \( x \in D'_k \subset D_k \) implies \( D_k \subset W \). Hence, \( x \in D'_k \subset U \) and the family of complements of elements in \( \mathcal{I} \) forms a base for the open sets. Thus \( \mathcal{I} \) is a base for the closed sets and \( \beta \mu X \) is a Wallman compactification.

Now suppose \( \hat{X} \) is a Wallman compactification of \( X \). It is also \( \beta \mu X \) for some precompact uniformity \( \mu \). There is a family \( \mathcal{J} \) which is a base for closed sets in \( \hat{X} \) and has the trace property with respect to \( X \). We will show that \( \mu \) is generated by \( \mathcal{B} = \{ X - (F \cap X) | F \in \mathcal{I} \} \).

Let \( \{ O_i | i = 1, \ldots, n \} \) be an open cover from \( \mathcal{B} \), where \( O_i = X - (F_i \cap X) \), \( F_i \in \mathcal{I} \). It follows from the trace property that \( \{ \hat{X} - F_i \} \) is an open cover of \( \hat{X} \). Since every open cover is in the uniformity for a compact space, \( \{ O_i \} \in \mu \).

Let \( \mathcal{U} \in \mu \). Then it is induced by some member \( \mathcal{U}' \) in the uniformity of \( \beta \mu X \). Since \( \hat{X} \) is compact and \( \mathcal{I} \) is a base for closed sets, \( \mathcal{U}' \) can be refined by a finite open cover \( \mathcal{U}'' \) consisting of complements of members of \( \mathcal{I} \). The trace of \( \mathcal{U}'' \) on \( X \) is a finite cover from \( \mathcal{B} \) which refines \( \mathcal{U} \). Thus \( \mu \) is generated by \( \mathcal{B} \).

Recent work (cf. [2], [3], [4], [5]) has shown that a vast number of compactifications are Wallman. Possibly they all are. Thus, at least all of the common precompact uniformities have generating bases.

**REFERENCES**