Life Prediction based on Degradation Amount Distribution using Composite Time Series Analysis

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Abstract

This paper presents a life prediction method to predict Degradation Amount Distribution (DAD) of products using a composite time series modeling procedure based on degradation data. Product DAD data are treated as composite time series and described using composite time series model and utilized to predict long-term trend of degradation. A degradation test is processed for a certain electronic product and degradation data are collected for life prediction. A comparison between the predicted DAD using composite time series analysis and the predicted DAD using regression analysis of the electronic product is processed and the results show that the DAD prediction of the product using composite time series analysis is more effective than regression analysis.

Keywords: Degradation Test, Life Prediction, Degradation Amount Distribution, Composite Time Series

1. Introduction

Degradation Testing (DT) is presented to deal with the cases that no fault data could be obtained but degradation data of the primary parameter of the product are available. Environmental variables have an important effect on the performance degradation process of many products. At present, there are mainly two ways to predict product life by DT: one is based on degradation path, that is, product life prediction is

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obtained by prediction of each sample degradation path; the other is based on DAD, that is, product life prediction is obtained by prediction of all samples DAD parameters. Most previous works use deterministic model to represent the degradation path or parameters of DAD. However, long-term life prediction must take into account the stochastic nature of environmental variables. A few literatures study DT life prediction using time series method for its excellent capable of stochastic and periodic information mining. However, life prediction using time series method in present literatures is all based on degradation path. Due to several special advantages of life prediction based on DAD, it is important to study DT life prediction based on DAD using time series method.

In this paper, two assumptions of DT are put forward:
1. The samplings instant of all products are same.
2. The distribution pattern of degradation amount at each sampling instant is unchanged, only parameters of degradation amount distribution change.

2. DAD Parameters Estimation

In DT, degradation amounts of all product samples at same time usually obey a certain location-scale distribution, and location and scale parameter at time \( t \) is denoted as \( \mu_t \) and \( \sigma_t \), which describes DAD at time \( t \). DAD type is determined by Goodness of Fit Test, such as Pearson chi-square test.

In this paper, when degradation amount distribution is location-scale distribution, location and scale parameter at time \( t \) is denoted as \( \mu_t \) and \( \sigma_t \), which describes degradation amount distributing situation at time \( t \).

Estimation of location and scale parameter of distribution of degradation amount \( y_i \) at time \( t \) is obtained by MLE. This paper denotes degradation amount of \( i^{th} \) product at time \( t \) as \( y_{ti} \), when total number of products is \( n \), then maximum likelihood function of distribution of \( y_t \) is

\[
L(\beta_t) = \prod_{i=1}^{n} f(y_{ti}, \beta_t)
\]

Estimation of location and scale parameter at time \( t \) is \( \hat{\beta}_t = (\hat{\mu}_t; \hat{\sigma}_t)^T \). By solving MLE equation of location and scale parameter at each time, location and scale parameter series \( \{\hat{\mu}_t\} \) and \( \{\hat{\sigma}_t\} \), are obtained.

3. DAD Stability Analyses

Location and scale parameter series is arranged in order of time. Thus, \( \{\hat{\mu}_t\} \) and \( \{\hat{\sigma}_t\} \) is time series.

Location and scale parameter describes different characteristic of distribution of \( y_t \). Hence, time series model structure of \( \{\hat{\mu}_t\} \) and \( \{\hat{\sigma}_t\} \) is different. This paper models them respectively.

3.1. Location parameter time series modeling

In DT, for monotonous degraded nature of product performance, periodic nature of test equipment control and stochastic nature of environmental variables, degradation amount \( y_t \) varies with time monotonously, periodically and randomly.

Location parameter describes central tendency location of distribution of \( y_t \). Hence, location parameter varies with time monotonously, periodically and randomly, too.

Hence, this paper decomposes location parameter time series \( \{\hat{\mu}_t\} \) into three components, which are trend, seasonal and random component, to take into account the monotonous degraded nature, periodic nature and stochastic nature, and then describes them using a composite time series model.

The location parameter time series model structure is
Here, $T_{\mu}$, $S_{\mu}$ and $R_{\mu}$ denote trend, seasonal and random component at time $t$.

1. Trend component modeling

Monotonic degradation trend of $\hat{\mu}_t$ is represented by linear or monotonic nonlinear regression model, such as logarithmic function, exponential function, power function and so on. The model of trend component $T_{\mu}$ is

$$T_{\mu} = b_{\mu} g_{\mu}(t) + T_{\mu0}$$

Here, $b_{\mu}$ is degradation rate of $\hat{\mu}_t$, $g_{\mu}(t)$ is linear or monotonic nonlinear function, $T_{\mu0}$ is initial value of $T_{\mu}$.

2. Season component modeling

Controlled by test equipment, the test stress level usually fluctuates periodically around set level. Product performance reflects periodic fluctuation of stress level. Removing trend component from $\hat{\mu}_t$, this paper regards this fluctuation as seasonal component $S_{\mu}$ and represents it using Hidden Periodicity (HP) regression model

$$S_{\mu} = \sum_{j=1}^{q} a_j \cos(\omega_j t + \varphi_j)$$

Here, $q$ is number of angular frequency, $a_j$ is $j^{th}$ amplitude, $\omega_j$ is $j^{th}$ angular frequency, $\varphi_j$ is $j^{th}$ phase.

3. Random component modeling

Removing $T_{\mu}$ and $S_{\mu}$ from $\hat{\mu}_t$, the random component $R_{\mu}$ is represented by Auto Regressive (AR) model

$$R_{\mu} = \sum_{j=1}^{p_{\mu}} \eta_{\mu j} R_{\mu(-j)} + \varepsilon_{\mu}$$

Here, $p_{\mu}$ is order number of AR model, $\eta_{\mu j}$ is coefficient of $j^{th}$ order, $\varepsilon_{\mu}$ is independent white noise.

Adding Eq.3 and Eq.4 to Eq.5, a complex time series model of $\hat{\mu}_t$ is put forward.

$$\hat{\mu}_t = T_{\mu} + S_{\mu} + R_{\mu} = b_{\mu} g_{\mu}(t) + T_{\mu0} + \sum_{j=1}^{q} a_j \cos(\omega_j t + \varphi_j) + \sum_{j=1}^{p_{\mu}} \eta_{\mu j} R_{\mu(-j)} + \varepsilon_{\mu}$$

3.2. Scale parameter time series modeling

Scale parameter describes dispersion scale of distribution of $y_t$. In practice, performance degraded nature of different product sample is different. Hence, there is different degradation rate of different product sample. When initial value of $y_t$ of different product samples are same, dispersion scale of distribution of $y_t$ must be increase with time. Hence, scale parameter not only varies with time monotonically, but also monotonically increase.

In DT, periodic nature of test equipment control has the same effect on life of different product sample. That means, periodical variation trend of $y_t$ of different product sample are same. Hence, dispersion scale of $y_t$ doesn’t vary with time periodically.

Implement of stochastic nature of environmental variables on different product sample is stochastic too. Hence, random variation of $y_t$ of different product sample is different, which is reflected at scale parameter variation.

Hence, this paper decomposes scale parameter time series $\{\hat{\sigma}_t\}$ into two components, which are trend and random component, to take into account the different degraded nature of product performance and different stochastic nature of environmental variables, and then describes them using a composite time series model.

The scale parameter time series model structure is
\[
\hat{\sigma}_t = T_{\sigma t} + R_{\sigma t}, t = 1, 2, \ldots
\]  

(7)

Here, \( T_{\sigma t} \) and \( R_{\sigma t} \) denote trend and random component of \( \hat{\sigma}_t \) at time \( t \).

1. Trend component modeling
   Monotone increasing trend of \( \hat{\sigma}_t \) is represented by linear or monotonic nonlinear regression model. The model of trend component \( T_{\sigma t} \) is
   \[
   T_{\sigma t} = b_{\sigma t} g_{\sigma t}(t) + T_{\sigma 0}
   \]
   Here, \( b_{\sigma t} \) is degradation rate of \( \hat{\sigma}_t \), \( g_{\sigma t}(t) \) is linear or monotonic nonlinear increasing function, \( T_{\sigma 0} \) is initial value of \( T_{\sigma t} \).

2. Random component modeling
   Removing \( T_{\sigma t} \) from \( \hat{\sigma}_t \), the random component \( R_{\sigma t} \) is represented by AR model
   \[
   R_{\sigma t} = \sum_{j=1}^{p_s} \eta_j R_{\sigma t-j} + \epsilon_{\sigma t}.
   \]
   (9)

Here, \( p_s \) is order number of AR model, \( \eta_j \) is coefficient of \( j^{th} \) order, \( \epsilon_{\sigma t} \) is independent white noise.

Adding Eq.2 to Eq.6, a complex time series model of \( \hat{\sigma}_t \) is put forward.

\[
\hat{\sigma}_t = T_{\sigma t} + R_{\sigma t} = b_{\sigma t} g_{\sigma t}(t) + T_{\sigma 0} + \sum_{j=1}^{p_s} \eta_j R_{\sigma t-j} + \epsilon_{\sigma t}.
\]

(10)

4. Life Prediction

In DT, fault occurs as product performance level achieves a specified failure threshold. Product reliability is probability of achieving. In this paper, life prediction is obtained by prediction of parameters of DAD.

4.1. Parameter Prediction

The \( l \)-step prediction of \( \hat{\mu}_t \) is obtained using best linear unbiased prediction of Eq.6. The prediction formula is

\[
\hat{\mu}_{t+l} = T_{\mu t+l} + S_{\mu t+l} + R_{\mu t+l} = b_{\mu t+l} g_{\mu t+l}(\tau+l) + T_{\mu 0} + \sum_{j=1}^{q} a_j \cos(\omega_j(\tau+l)+\varphi_j) + \sum_{j=1}^{p_\mu} \eta_{\mu j} R_{\mu t+j}.
\]

(11)

Here, \( \tau \) is time scale before prediction.

The \( l \)-step prediction of \( \hat{\sigma}_t \) is obtained using best linear unbiased prediction of Eq.10. The prediction formula is

\[
\hat{\sigma}_{t+l} = T_{\sigma t+l} + R_{\sigma t+l} = b_{\sigma t+l} g_{\sigma t+l}(\tau+l) + T_{\sigma 0} + \sum_{j=1}^{p_\sigma} \eta_{\sigma j} R_{\sigma t+j}.
\]

(12)

4.2. Reliability Evaluation

Mostly, product failure threshold is supposed a constant, which is denoted as \( D \), reliability evaluation based on constant failure threshold is obtained by traditional reliability formulas.

5. Example Verification

A DT of 13 certain products is conducted as an example to verify the suggest method. Sampling interval is 1 min. The construction of DT system is shown in Fig.1.
In this paper, DT has been conducted for 17000 min. DT data of each product is preprocessed for eliminating influence of its initial value difference. Fig.2 shows the preprocessed DT data until all products failure.

According to Tab.1, Lognormal distribution is the best fitted distribution. Thus, logarithm of degradation amount obeys normal distribution. It is regarded as degradation data in this example.

Fig.3 shows trend component of degradation data.
Random component of degradation data is obtained by removing trend component, Fig.4 shows them.

Stationary test for random component of degradation data is conducted by round test. The test result shows that random component of degradation data is stationary.

Secondly, location and scale parameter of DAD is modeled by composite time series model. Location parameter trend component is power function. Scale parameter trend component is modeled by power function too.

Thirdly, predictions of DAD parameters are obtained by time series model prediction. Fig.5 shows location parameter prediction. Fig.6 shows scale parameter prediction.
According to practical experience, reliability evaluation is obtained by a constant failure threshold 93%. Fig. 7 shows reliability evaluation based on the suggested time series model and traditional regression model respectively to compare.
According to Fig.7, it is obviously that compared with regression model, reliability curve based on the suggested time series model can reflect implement of stochastic nature of test equipment and environmental variables on product reliability.

6. Conclusions

This paper proposes a DT life prediction method based on DAD using a composite time series modeling procedure. Based on degradation amount distribution, a composite time series modeling procedure is put forward to modeling parameters MSE of degradation amount distribution. According to example verification, compared with regression model, DT life prediction based on DAD using composite time series method is more realistic.

References