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Letter to the Editor

Exponential stability for one-dimensional problem of swelling porous elastic soils with fluid saturation

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Abstract

This paper concerns the one-dimensional linear theory of swelling porous elastic soils in the case of fluid saturation. The formulation belongs to the theory of mixtures for porous elastic solids filled with fluid. It proposes some new mathematical difficulties. We prove the exponential stability for the initial-boundary value problem determined by the homogeneous Dirichlet boundary conditions. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

It is accepted that the swelling of soils, drying of fibers, wood, paper, plants, etc. are problems concerning porous media theory. There have been, in fact, several recent articles introducing continuum theories for fluids infiltrating elastic porous media, see e.g. Payne et al. [5] and other references therein. It is only by analysing such theories mathematically that we shall be in a position to assess their suitability for use in a given physical problem. In this paper we concentrate on Eringen's theory presented in [3]. We recall that in the introduction Eringen pointed out that "I believe the present theory can provide basis for the treatment of various practical problems in the field of swelling, oil explanations, slurred and consolidation problems by further simplifications and/or extensions of the theory". It is worth noting that the formulation belongs to the theory of mixtures for porous elastic solids filled with fluid. Heat conduction was also included. In the present paper we are concerned with the linear equations proposed in this theory. When swelling, consolidation and many other problems, such as motion of gas, liquid and solid are small then linear equations can be adequate for the treatment of these problems. Some results concerning this problem have been obtained recently [6,7].

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In this paper, we restrict our attention to the homogeneous displacement-boundary conditions although other conditions could be also considered.

The purpose of this article is to obtain exponential stability for a one-dimensional linear problem in Eringen's theory [3]. Thus, it is worth recalling several results concerning exponential stability in other theories [1,4]. We restrict our attention to the case of fluid saturation. Such studies are important in view to assess whether a given theory is mathematically acceptable. It is worth noting that in dimensions two or three the equations are more complicated. Thus, our approach cannot be used in dimension > 1.

In Section 2, we recall the problem we study in this paper. In Section 3, we use the energy method to prove the exponential stability of solutions in the isothermal case. The extension of this results to the nonisothermal case is sketched in Section 4. In the last section, we use the Hurtwitz theorem to prove exponential stability of solutions when $a_2 \neq 0$ and $\xi = 0$ (see (2.1), (2.2)).

2. Preliminaries

The field equations of the linear theory of swelling porous elastic soils in the case of fluid saturation are (see [3], p. 1345)

$$\rho_z \ddot{z} = a_1 z_{xx} + a_2 u_{xx} + \beta_1 T_x - \xi(\dot{z} - \dot{u}) + \mu_z \dot{z}_{xx}, \tag{2.1}$$

$$\rho_u \ddot{u} = a_2 z_{xx} + \mu u_{xx} + \beta_2 T_x + \xi(\dot{z} - \dot{u}), \tag{2.2}$$

$$c\dot{T} = \beta_1 \dot{z}_x + \beta_2 \dot{u}_x + kT_{xx}. \tag{2.3}$$

Eqs. (2.1)–(2.3) constitute a system of three partial differential equations with three unknown functions (z, u, T) that represent the displacements of fluid and solid elastic material, respectively, and the temperature. The constants ρ_z, ρ_u are the densities of each constituent and c is the heat capacity. The parameters a_1, a_2, a_3, β_1, β_2, ξ, μ, μ_z and k are the constitutive constants in this theory.

To define a problem we need boundary and initial conditions. When the system (2.1)–(2.3) is considered the initial conditions are

$$z(x,0) = z^0, \quad u(x,0) = u^0, \quad T(x,0) = T^0, \quad x \in [0,L], \tag{2.4}$$

$$\dot{z}(x,0) = y^0, \quad \dot{u}(x,0) = v^0, \quad x \in [0,L], \tag{2.5}$$

and the homogeneous boundary conditions are

$$z(x,t) = u(x,t) = T(x,t) = 0, \quad x = 0, L. \tag{2.6}$$

It is worth noting that the existence of solutions of the problem determined by (2.1)–(2.6) can be obtained directly by means of the semigroup theory.

From a mathematical point of view system (2.1)–(2.3), propose new stimulating questions. For instance, does the dissipation of the fluid imply the exponential stability even in the isothermal case?

3. Isothermal case

The aim of this section is to obtain an exponential stability result in the isothermal case. To this end we use the energy methods (see [2,4]). In this section, we assume that

$$\rho_z > 0, \quad \rho_u > 0, \quad \mu_z > 0, \quad \xi > 0 \tag{3.1}$$

and that the matrix

$$\begin{pmatrix} a_1 & a_2 \\ a_2 & \mu \end{pmatrix} \tag{3.2}$$

is definite positive.

The system of equations we consider here introduces new mathematical difficulties in order to use the usual energy method.

If we define the energy function

$$E_1(t) = \frac{1}{2}\int_0^L (\rho_z(\dot z)^2 + \rho_u(\dot u)^2 + a_1(z_x)^2 + 2a_2 z_x u_x + \mu(u_x)^2)\,\mathrm{d}l, \tag{3.3}$$

the evolutionary equations and the boundary conditions imply that

$$\frac{\mathrm{d}E_1(t)}{\mathrm{d}t} = -\int_0^L (\xi(\dot z - \dot u)^2 + \mu_z(\dot z_x)^2)\,\mathrm{d}l. \tag{3.4}$$

In view of the arithmetic geometric inequality and the Poincaré inequality, we may conclude the existence of three positive constants m_1, m_2, m_3 (see the appendix) such that

$$\frac{\mathrm{d}E_1(t)}{\mathrm{d}t} \leqslant -\int_0^L (m_1(\dot z)^2 + m_2(\dot u)^2 + m_3(\dot z_x)^2)\,\mathrm{d}l. \tag{3.5}$$

Now, let us consider the functions

$$W_z(t) = \int_0^L \rho_z \dot z z\,\mathrm{d}l, \qquad W_u(t) = \int_0^L \rho_u \dot u u\,\mathrm{d}l. \tag{3.6}$$

We have

$$\frac{\mathrm{d}W_z(t)}{\mathrm{d}t} = -\int_0^L (a_1 z_x^2 + a_2 u_x z_x + \xi(\dot z - \dot u)z + \mu_z \dot z_x z_x)\,\mathrm{d}l + \int_0^L \rho_z(\dot z)^2\,\mathrm{d}l, \tag{3.7}$$

and

$$\frac{\mathrm{d}W_u(t)}{\mathrm{d}t} = -\int_0^L (a_2 z_x u_x + \mu u_x^2 - \xi(\dot z - \dot u)u)\,\mathrm{d}l + \int_0^L \rho_u(\dot u)^2\,\mathrm{d}l. \tag{3.8}$$

Thus, if we denote

$$W(t) = W_z(t) + W_u(t) + \frac{1}{2}\int_0^L (\xi(z-u)^2 + \mu_z z_x^2)\,\mathrm{d}l, \tag{3.9}$$

it follows that:

$$\frac{\mathrm{d}W(t)}{\mathrm{d}t} = -\int_0^L (a_1 z_x^2 + 2a_2 u_x z_x + \mu u_x^2 - \rho_z(\dot z)^2 - \rho_u(\dot u)^2)\,\mathrm{d}l. \tag{3.10}$$

From the definition of the functions E_1 and W and after a use of the Poincaré inequality, we may obtain the existence of a constant N_0 such that for all $N_1 \geqslant N_0$ we may obtain two positive constants $N_2(N_1), N_3(N_1)$ (see the appendix) such that

$$N_2E_1 \leqslant N_1E_1 + W \leqslant N_3E_1. \tag{3.11}$$

From (3.5) and (3.10), we can always select M_0 such that for all $M_1 \geqslant M_0$ we may obtain a positive constant $M_2(M_1)$ (see the appendix) such that

$$M_1 \frac{\mathrm{d}E_1}{\mathrm{d}t} + \frac{\mathrm{d}W}{\mathrm{d}t} \leqslant -M_2E_1. \tag{3.12}$$

The last two inequalities imply that we may always select two positive constants ϕ, φ such that

$$\phi \frac{\mathrm{d}E_1}{\mathrm{d}t} + \frac{\mathrm{d}W}{\mathrm{d}t} \leqslant -\varphi(\phi E_1 + W). \tag{3.13}$$

A quadrature implies that

$$(\phi E_1 + W)(t) \leqslant (\phi E_1 + W)(0)\exp(-\varphi t). \tag{3.14}$$

Using again inequality (3.11) we may obtain a constant R such that

$$E_1(t) \leqslant RE_1(0)\exp(-\varphi t) \tag{3.15}$$

that is the result of exponential stability.

Remark. To obtain the exponential stability we have used the combination of two damping processes. We have assumed that μ_z and ξ are strictly positive. Now, we see that if at least one of these conditions is not satisfied we can always find solutions that are not damped.

For instance, if $\mu_z = 0$ and

$$\frac{a_1 + a_2}{\rho_z} = \frac{a_2 + \mu}{\rho_u},$$

we may select initial conditions such that $z^0 = u^0$ and $y^0 = v^0$. In this case, the solutions are $z = u$ and correspond to the solutions of the wave equation

$$\rho_u \ddot{u} = (a_2 + \mu)u_{xx}.$$

It is worth recalling Ref. [1], where asymptotic stability is obtained for systems of this kind.

If we assume that $\xi = 0$ and restrict our attention to the case $a_2 = 0$, the system is composed of two separated equations. The equation for u is again the undamped wave equation. Then the solutions do not tend to zero. A natural question is to know if there exists exponential stability when $a_2 \neq 0$. This will be the aim of the last section.

4. Nonisothermal case

It is well known that the combination of the thermal effects with the elastic effects determines exponential stability [4]. In the last section we have seen that the same thing holds in the kind of mixture we consider. Thus, it is natural to expect the same behaviour when we consider the two effects at the same time. We only sketch the proof in this case. It is worth noting that in this case, we also assume that $c > 0$ and $k > 0$.

If we define the function

$$E_1(t) = \frac{1}{2}\int_0^L (\rho_z(\dot{z})^2 + \rho_u(\dot{u})^2 + a_1 z_x^2 + 2a_2 z_x u_x + \mu u_x^2 + cT^2)\,\mathrm{d}l, \tag{4.1}$$

we obtain

$$\frac{\mathrm{d}E_1(t)}{\mathrm{d}t} \leqslant -\int_0^L (m_1(\dot{z})^2 + m_2(\dot{u})^2 + m_3(\dot{z}_x)^2 + m_4 T^2 + m_5 T_x^2)\,\mathrm{d}l, \tag{4.2}$$

where m_4 and m_5 are also positive. If we define W as in (3.9), it follows:

$$\frac{\mathrm{d}W(t)}{\mathrm{d}t} = -\int_0^L (a_1 z_x^2 + 2a_2 u_x z_x + \mu u_x^2 - \rho_z(\dot{z})^2 - \rho_u(\dot{u})^2 + \beta_1 T_x z + \beta_2 T_x u)\,\mathrm{d}l. \tag{4.3}$$

It is clear that in this situation, it is easy to reproduce the arguments to prove the exponential stability of the solutions.

5. Isothermal problem: limiting case $\xi = 0$ and $a_2 \neq 0$

The aim of this section is to prove the exponential stability of the solutions of the isothermal problem in the particular case that $\xi = 0$ and $a_2 \neq 0$. To make the calculations easier, we assume that $L = \pi$.

The solutions in this case will be combinations of functions of the form

$$z = A\exp(\omega t)\sin nx, \qquad u = B\exp(\omega t)\sin nx. \tag{5.1}$$

Imposing (5.1) as a solution of Eqs. (2.1) and (2.2) we obtain the following homogeneous system with the unknowns A, B:

$$A\rho_z\omega^2 = -Aa_1n^2 - Ba_2n^2 - A\mu_z\omega n^2, \qquad B\rho_u\omega^2 = -Aa_2n^2 - B\mu n^2. \tag{5.2}$$

Our aim is to obtain a nontrivial solution. We impose that the determinant of the system is equal to zero. Here, ω is a solution of the equation

$$(\rho_z x^2 + a_1 n^2 + \mu_z n^2 x)(\rho_u x^2 + \mu n^2) - a_2^2 n^4 = 0. \tag{5.3}$$

We can write this equation in the form

$$\rho_u\rho_z x^4 + \mu_z\rho_u n^2 x^3 + (a_1\rho_u + \mu\rho_z)n^2x^2 + \mu_z\mu n^4 x + (a_1\mu - a_2^2)n^4 = 0. \tag{5.4}$$

In order to prove the exponential stability, it will be sufficient to prove that all the solutions of Eq. (5.4) have negative real part that is less or equal than $-\varepsilon$, where ε is a positive number. This is equivalent to prove that all the solutions of the equation

$$\rho_u\rho_z(x-\varepsilon)^4 + \mu_z\rho_u n^2(x-\varepsilon)^3 + (a_1\rho_u + \mu\rho_z)n^2(x-\varepsilon)^2 + \mu_z\mu n^4(x-\varepsilon) + (a_1\mu - a_2^2)n^4 = 0 \tag{5.5}$$

have a negative real part.

To show it, we use the Hurwitz theorem that says that the necessary and sufficient condition to guarantee that the solutions of the equation

$$l_4x^4 + l_3x^3 + l_2x^2 + l_1x + l_0 = 0 \tag{5.6}$$

have a negative real part given by

$$\Lambda_0 = l_0 > 0, \quad \Lambda_1 = l_1 > 0, \quad \Lambda_2 = \det\begin{pmatrix} l_1 & l_3 \\ l_0 & l_2 \end{pmatrix} > 0, \quad \Lambda_3 = \det\begin{pmatrix} l_1 & l_3 & 0 \\ l_0 & l_2 & l_4 \\ 0 & l_1 & l_3 \end{pmatrix} > 0 \tag{5.7}$$

and

$$\Lambda_4 = \det\begin{pmatrix} l_1 & l_3 & 0 & 0 \\ l_0 & l_2 & l_4 & 0 \\ 0 & l_1 & l_3 & 0 \\ 0 & l_0 & l_2 & l_4 \end{pmatrix} > 0. \tag{5.8}$$

In our case we can write Eq. (5.5) as

$$\begin{aligned} &\rho_u\rho_z x^4 + (\mu_z\rho_u n^2 + P_3(\varepsilon))x^3 + [(a_1\rho_u + \mu\rho_z)n^2 + P_2(\varepsilon, n^2)]x^2 \\ &\quad + [\mu_z\mu n^4 + P_1(\varepsilon, n^2)]x + (a_1\mu - a_2^2)n^4 + P_0(\varepsilon, n^4) = 0, \end{aligned} \tag{5.9}$$

where

$$P_3 = -4\rho_u\rho_z\varepsilon, \quad P_2 = 6\rho_u\rho_z\varepsilon^2 - 3\varepsilon\mu_z\rho_u n^2,$$

$$P_1 = -4\varepsilon^3\rho_u\rho_z + 3\varepsilon^2\mu_z\rho_u n^2 - 2\varepsilon(a_1\rho_u + \mu\rho_z)n^2,$$

$$P_0 = \rho_u\rho_z\varepsilon^4 - \mu_z\rho n^2\varepsilon^3 + (a_1\rho_u + \mu\rho_z)n^2\varepsilon^2 - \mu_z\mu\varepsilon n^4.$$

Our intention is to obtain the existence of ε uniformly for every $n \geqslant 1$. We have

$$\begin{aligned} \Lambda_0 &= (a_1\mu - a_2^2)n^4 + P_0 \geqslant (a_1\mu - a_2^2)n^4 - \rho_u\rho_z\varepsilon^4 - \mu_z\rho n^2\varepsilon^3 - (a_1\rho_u + \mu\rho_z)n^2\varepsilon^2 - \mu_z\mu\varepsilon n^4 \\ &\geqslant n^4[(a_1\mu - a_2^2) - \rho_u\rho_z\varepsilon^4 - \mu_z\rho\varepsilon^3 - (a_1\rho_u + \mu\rho_z)\varepsilon^2 - \mu_z\mu\varepsilon]. \end{aligned} \tag{5.10}$$

It is clear that we can select ε_1 such that

$$(a_1\mu - a_2^2) - \rho_u\rho_z\varepsilon_1^4 - \mu_z\rho\varepsilon_1^3 - (a_1\rho_u + \mu\rho_z)\varepsilon_1^2 - \mu_z\mu\varepsilon_1 > 0. \tag{5.11}$$

In a similar way

$$\begin{aligned} \Lambda_1 &\geqslant \mu_z\mu n^4 - 4\varepsilon^3\rho_u\rho_z - 3\varepsilon^2\mu_z\rho_u n^2 - 2\varepsilon(a_1\rho_u + \mu\rho_z)n^2 \\ &\geqslant n^4[\mu_z\mu - 4\varepsilon^3\rho_u\rho_z - 3\varepsilon^2\mu_z\rho_u - 2\varepsilon(a_1\rho_u + \mu\rho_z)]. \end{aligned} \tag{5.12}$$

We can select ε_2 such that

$$\mu_z\mu - 4\varepsilon_2^3\rho_u\rho_z - 3\varepsilon_2^2\mu_z\rho_u - 2\varepsilon_2(a_1\rho_u + \mu\rho_z) > 0. \tag{5.13}$$

Some easy calculations allow us to see that

$$\Lambda_2 = \mu^2\mu_z\rho_z n^6 + a_2^2\mu_z\rho_u n^6 - Q_2(\varepsilon, n^6), \tag{5.14}$$

where

$$Q_2 = P_1(a_1\rho_u + \mu\rho_z)n^2 + P_2\mu_z\mu n^4 + P_2P_1 - P_3P_0 - (a_1\mu - a_2^2)n^4P_3 - \mu_z\rho_u n^2 P_0.$$

We can repeat the previous argument to prove that there exists ε_3 such that $\Lambda_2 > 0$. Similar arguments give

$$\Lambda_3 = n^8\mu_z^2\rho_u^2 a_2^2 - Q_3(\varepsilon, n^8) \tag{5.15}$$

and

$$\Lambda_4 = n^8\mu_z^2\rho_u^3\rho_z a_2^2 - Q_4(\varepsilon, n^8). \tag{5.16}$$

Here, Q_3 and Q_4 are two polynomials that can be treated in a way similar to that used in the study of Λ_0, Λ_1 and Λ_2, but we do not include the total expression to save cumbersome calculations. Nevertheless, it is important to note that we can do that because $a_2 \neq 0$. The relevant thing is to see that we can repeat the previous arguments and that we can find ε sufficiently small to guarantee that conditions (5.7) and (5.8) are satisfied. Thus, for ε sufficiently small $\Lambda_i > 0$, $i = 0, \ldots, 4$, $n \geqslant 1$. Thus, the solutions of Eq. (5.4) lie on the left of the line $\mathrm{Re}\{z\} = -\varepsilon$. This implies the exponential stability of solutions.

From the analysis, we can conclude that the condition $L = \pi$ is not restrictive. We can adapt the same method in the general case. Thus, we can obtain exponential stability in the general case.

It is natural to expect exponential stability for the nonisothermal case.

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Appendix

The aim of this appendix is to obtain numerical values for the parameters m_i, M_i and N_i. From equality (3.4) and Poincaré's inequality, we have

$$\begin{aligned}
\frac{\mathrm{d}E_1}{\mathrm{d}t} &\leqslant -\int_0^L \left(\xi(\dot z)^2 + \xi(\dot u)^2 - 2\xi\dot u\dot z + \frac{\mu_z\pi^2}{2L^2}(\dot z)^2 + \frac{\mu_z}{2}(\dot z_x)^2\right)\mathrm{d}l \\
&= -\int_0^L \left(\left(\xi + \frac{\mu_z\pi^2}{2L^2}\right)(\dot z)^2 + \xi(\dot u)^2 - 2\xi\dot u\dot z + \frac{\mu_z}{2}(\dot z_x)^2\right)\mathrm{d}l \\
&= -\int_0^L \left(\left(\sqrt{\xi + \frac{\mu_z\pi^2}{4L^2}}\,\dot z - \frac{\xi\dot u}{\sqrt{\xi + (\mu_z\pi^2/4L^2)}}\right)^2 + \frac{\mu_z\pi^2}{4L^2}(\dot z)^2\right. \\
&\quad \left. + \left(\xi - \frac{\xi^2}{\xi + \frac{\mu_z\pi^2}{4L^2}}\right)(\dot u)^2 + \frac{\mu_z}{2}(\dot z_x)^2\right)\mathrm{d}l
\end{aligned}$$

$$\leqslant -\int_0^L \left(\frac{\mu_z \pi^2}{4L^2} (\dot{z})^2 + \left(\xi - \frac{\xi^2}{\xi + (\mu_z \pi^2/4L^2)} \right) (\dot{u})^2 + \frac{\mu_z}{2} (\dot{z}_x)^2 \right) \mathrm{d}l.$$

Thus, we can take

$$m_1 = \frac{\mu_z \pi^2}{4L^2}, \qquad m_2 = \xi - \frac{\xi^2}{\xi + (\mu_z \pi^2/4L^2)}, \qquad m_3 = \frac{\mu_z}{2}.$$

In order to find some values for the parameters N_i, it is convenient to bound $|W|$ by means of E_1. We have

$$|W| \leqslant \int_0^L \left(\frac{1}{2} \rho_z z^2 + \frac{1}{2} \rho_z (\dot{z})^2 + \frac{1}{2} \rho_u u^2 + \frac{1}{2} \rho_u (\dot{u})^2 + \xi z^2 + \xi u^2 + \frac{1}{2} \mu_z z_x^2 \right) \mathrm{d}l$$

$$\leqslant \int_0^L \left(\frac{L^2}{2\pi^2} \rho_z z_x^2 + \frac{1}{2} \rho_z (\dot{z})^2 + \frac{L^2}{2\pi^2} \rho_u u_x^2 + \frac{1}{2} \rho_u (\dot{u})^2 + \frac{L^2}{\pi^2} \xi z_x^2 + \frac{L^2}{\pi^2} \xi u_x^2 + \frac{1}{2} \mu_z z_x^2 \right) \mathrm{d}l$$

$$\leqslant \max \left(1, m^{-1} \left(\rho_z \frac{L^2}{\pi^2} + 2\xi \frac{L^2}{\pi^2} + \mu_z \right), m^{-1} \left(\rho_u \frac{L^2}{\pi^2} + 2\xi \frac{L^2}{\pi^2} \right) \right) E_1(t),$$

where m is the smallest eigenvalue of the matrix (3.2). If we take

$$N_1 = 2 \max \left(1, m^{-1} \left(\rho_z \frac{L^2}{\pi^2} + 2\xi \frac{L^2}{\pi^2} + \mu_z \right), m^{-1} \left(\rho_u \frac{L^2}{\pi^2} + 2\xi \frac{L^2}{\pi^2} \right) \right),$$

we can take

$$N_2 = N_1/2, \qquad N_3 = 3N_1/2.$$

To obtain the values for the parameters M_1 and M_2, we can consider

$$M_1 \frac{\mathrm{d}E_1}{\mathrm{d}t} + \frac{\mathrm{d}W}{\mathrm{d}t} \leqslant -\int_0^L (a_1 z_x^2 + 2a_2 u_x z_x + \mu u_x^2 + m_3 M_1 (\dot{z}_x)^2 + (M_1 m_1 - \rho_z)(\dot{z})^2 + (M_1 m_2 - \rho_u)(\dot{u})^2) \, \mathrm{d}l.$$

Thus, we can take

$$M_1 = \max \left(\frac{\rho_z}{m_1}, \frac{\rho_u}{m_2} \right) + 1$$

and

$$M_2 = \min \left(2, \frac{2(M_1 m_1 - \rho_z)}{\rho_z}, \frac{2(M_1 m_2 - \rho_u)}{\rho_u} \right).$$

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