



Constraints on the parity-violating couplings of a new gauge boson

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Abstract

High-energy particle physics experiments allow for the possible existence of a new light, very weakly coupled, neutral gauge boson (the U boson). This one permits for *light* (spin- $\frac{1}{2}$ or spin-0) particles to be acceptable Dark Matter candidates, by inducing sufficient (stronger than weak) annihilation cross sections into e^+e^- . They could be responsible for the bright 511 keV γ ray line observed by INTEGRAL from the galactic bulge. Such a new interaction may have important consequences, especially at lower energies. Parity-violation atomic-physics experiments provide strong constraints on such a U boson, if its couplings to quarks and electrons violate parity. With the constraints coming from an unobserved axionlike behaviour of this particle, they favor a *pure vector* coupling of the U boson to quarks and leptons, unless the corresponding symmetry is broken sufficiently above the electroweak scale.

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1. Introduction

The $SU(3) \times SU(2) \times U(1)$ standard model gives a very good description of strong and electroweak phenomena, so that the possible existence, next to the gluons, photon, W^\pm and Z , of an additional neutral gauge boson, called here the U boson, is severely constrained. The U contributions to neutral-current amplitudes should be sufficiently small, as well as its mixing with the Z . According to the usual belief, any such new interaction must be weaker than ordinary weak interactions, or it would have been seen already.

This only applies directly, in fact, to heavy neutral gauge bosons. For *light* gauge bosons having *small* couplings to Standard Model particles, the discussion is different [1]. When the mass m_U of the exchanged boson is small (compared to the momentum transfer $\sqrt{|q^2|}$), propagator effects are important. U -induced cross sections then generally *decrease with energy*, as for electromagnetic ones, as soon as $|q^2|$ gets larger than $\approx m_U^2$ (as for Z -exchanges, above the Z mass), and may be sufficiently small, if the U couplings are small enough. In particular, the existence of a new *light* gauge boson U having couplings f to matter particles such that

$$\frac{f^2}{m_U^2} \sim \frac{g^2 + g'^2}{m_Z^2} \quad (\text{or } \sim G_F), \quad (1)$$

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for example, is not necessarily excluded by *high-energy* scattering experiments. Experiments performed at lower energies, such as those measuring parity-violation effects in atomic physics (as we shall discuss here), or neutrino scattering cross sections at lower $|q^2|$, are particularly relevant to search for such a particle, and constrain its properties [1–3].

Let us recall, however, that even if it is *very* weakly coupled, a light spin-1 U boson could still have detectable interactions. And this, even in the limit in which its couplings f to quarks and leptons would almost vanish, a very surprising result indeed (apparently)!

In fact a very light spin-1 U boson behaves in this case ($f \rightarrow$ very small, $m_U \rightarrow$ very small) very much as a quasimassless spin-0 axionlike particle, if the current to which it is coupled includes a (non-conserved) axial part.¹ This axionlike behavior then restricts rather strongly its possible existence and properties, implying that the corresponding gauge symmetry be broken at a scale at least somewhat above the electroweak scale; or even at a very high scale, according to the “invisible U -boson” mechanism [1,5].

It is also possible that the new current to which the U couples is *purely vectorial*, involving a linear combination of the conserved B , L and electromagnetic currents, as in a class of models discussed in Refs. [5, 6]. In this case there is no such axionlike behavior of the U boson. No significant extra contribution to parity-violation effects is then to be expected.

We now turn to the recent suggestion of Light Dark Matter particles. Contrasting with the heavy WIMPs, such as the neutralinos of supersymmetry, *light* (annihilating) spin- $\frac{1}{2}$ or spin-0 particles can also be acceptable Dark Matter candidates. This requires, however, that they annihilate *very* efficiently, necessitating new interactions, as induced by a light U boson [7,8]. The required annihilation cross sections (≈ 4 – 10 pb, depending on whether Dark Matter particles are self-conjugate or not), must be significantly *larger* than weak-interaction cross sections (for this energy), otherwise the relic abundance would be too large! The

U -induced Dark Matter annihilation cross section into e^+e^- ($\sigma_{\text{ann}}v_{\text{rel}}/c$) also includes, naturally, a v_{dm}^2 low-energy suppression factor (as desirable to avoid excessive γ rays from residual light Dark Matter annihilations [9]). This requirement is satisfied, in the case of a spin- $\frac{1}{2}$ Dark Matter particle *axially* coupled to the U , if this one is *vectorially* coupled to electrons [8].

A new interaction stronger than weak interactions could seem, naively, to be ruled out experimentally. In fact, however, the U -mediated Dark-Matter/Matter interactions should be *stronger than ordinary weak interactions* but only *at lower energies*, when weak interactions are really very weak. But *weaker at higher energies*, at which they are damped by U propagator effects (for s or $|q^2| > m_U^2$), when weak-interaction cross sections, still growing with energy like s , become important. The smallness of the U couplings to ordinary matter (f), as compared to e , by several orders of magnitude, and of the resulting U amplitudes compared to electromagnetic ones, can then account for the fact that these particles have not been observed yet. The U boson, in addition, may well have dominant invisible decay modes into unobserved Dark Matter particles.

We indicated in May 2003 that a gamma ray signature from the galactic centre at low energy could be due to the existence of a light new gauge boson, inducing annihilations of Light Dark Matter particles into e^+e^- [7]. The observation, a few months later, by the satellite INTEGRAL of a bright 511 keV γ ray line from the galactic bulge [10], requiring a rather large number of annihilating positrons, may then be viewed as originating from Light Dark Matter annihilations [11]. Indeed spin-0, or as well spin- $\frac{1}{2}$ particles, could be responsible for this bright 511 keV line, which does not seem to have an easy interpretation in terms of known astrophysical processes [8,12]. One should, however, also keep in mind that Light Dark Matter particles may still exist, even if they are not responsible for this line. (And that a light U boson may be present, even if Light Dark Matter particles do not exist at all.²)

¹ This is very similar to what happens in supersymmetry/supergravity theories, in which a very light spin- $\frac{3}{2}$ gravitino does not decouple in the $\kappa \rightarrow 0$ limit, but interacts (proportionately to $\kappa/m_{3/2}$ or $1/\Lambda_{\text{ss}}^2$) like the massless spin- $\frac{1}{2}$ goldstino of global supersymmetry (a feature largely used in “GMSB” models) [4].

² In addition, a U that would be both extremely light and extremely weakly coupled would lead to a new long-range force, and to the possibility of (apparent) violations of the Equivalence Principle.

Returning to Standard Model particles, a new interaction that would be stronger than weak interactions at lower energies (at least when dealing with Light Dark Matter particle annihilations) could have important implications on ordinary physics, *especially at lower energies or momentum transfer*, even if it has no significant influence on high-energy neutral current processes.

As we shall see, parity-violation atomic-physics experiments [13] provide new strong constraints on such a gauge boson—whether light or heavy—if its couplings to quarks and electrons violate parity, then requiring that the corresponding symmetry be broken significantly above the electroweak scale.

2. The effective weak charge of a nucleus

Such models, in which the standard gauge group is extended to include $SU(3) \times SU(2) \times U(1) \times$ extra- $U(1)$ at least, have been discussed in detail. They involve an additional neutral gauge boson U (which may also be called Z' or Z''), initially associated, before gauge symmetry breaking, with the extra- $U(1)$ generator. Mixing effects with the Z , however, in general play an important role, as far as the U couplings are concerned [1,5,6]. When the extra- $U(1)$ gauge coupling constant (g'') is small compared to g and g' , the mixing angle ($\propto g''/\sqrt{g^2 + g'^2}$) turns out to be small also, and the modification to the Z weak neutral current, still given by $(J_3^\mu - \sin^2 \theta J_{\text{em}}^\mu)$ up to very small corrections ($\propto g''^2/(g^2 + g'^2)$), is in fact negligible.

The current to which the U boson couples, however, is significantly affected by the mixing, acquiring, in addition to the initial extra- $U(1)$ term, a new contribution proportional to $(J_3^\mu - \sin^2 \theta J_{\text{em}}^\mu)$. The resulting U -current includes in general a *vector* part which appears as a linear combination of the conserved B , L and electromagnetic currents, as well as an *axial* part (which may, however, not be present at all, depending on the models considered). In particular, in a class of simple one-Higgs-doublet models the quark-and-lepton contribution to the U current turns out to be *purely vectorial* [5,6]—which also provides the desired v_{dm}^2 factor in the annihilation cross section of spin- $\frac{1}{2}$ light Dark Matter particles [8].

We now proceed with the phenomenological analysis, expressing the relevant couplings in the Lagrangian density as follows:

$$\begin{aligned} \mathcal{L} = & -e A_\mu J_{\text{em}}^\mu \\ & - Z_\mu \sqrt{g^2 + g'^2} (J_3^\mu - \sin^2 \theta J_{\text{em}}^\mu) \\ & - U_\mu \sum_{f=l,q} \bar{f} \gamma^\mu (f_{Vf} - \gamma_5 f_{Af}) f. \end{aligned} \quad (2)$$

The left- and right-handed projectors are $\mathcal{P}_L = \frac{1-\gamma_5}{2}$, $\mathcal{P}_R = \frac{1+\gamma_5}{2}$ (so that a left-handed U -current would correspond to $f_V = f_A$), with $\gamma_5 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$, and the metric $(+ - - -)$. The relevant terms in the Z weak neutral current are given by

$$\begin{aligned} J_Z^\mu = & J_3^\mu - \sin^2 \theta J_{\text{em}}^\mu \\ = & \frac{1}{4} \bar{e} \gamma^\mu \gamma_5 e + \left(-\frac{1}{4} + s^2 \right) \bar{e} \gamma^\mu e \\ & - \frac{1}{4} \bar{u} \gamma^\mu \gamma_5 u + \left(\frac{1}{4} - \frac{2}{3} s^2 \right) \bar{u} \gamma^\mu u \\ & + \frac{1}{4} \bar{d} \gamma^\mu \gamma_5 d + \left(-\frac{1}{4} + \frac{1}{3} s^2 \right) \bar{d} \gamma^\mu d, \end{aligned} \quad (3)$$

with $s^2 = \sin^2 \theta = g'^2/(g^2 + g'^2)$, θ being the electroweak mixing angle.³ The vector part of the Z weak neutral current is associated with the Z (vectorial) weak charge, which reads, as far as the quark contribution is concerned,^{4,5}

³ We disregard here, as explained earlier, the very small influence of Z - U mixing effects on the Z current.

⁴ This may also be obtained from the quark vector couplings to the Z as

$$\begin{aligned} Q_Z = & (2Z + N) \left(\frac{1}{4} - \frac{2}{3} s^2 \right) + (Z + 2N) \left(-\frac{1}{4} + \frac{1}{3} s^2 \right) \\ \equiv & \frac{1}{4} [Z(1 - 4s^2) - N] = \frac{1}{4} Q_W(Z, N)_{\text{SM}}. \end{aligned}$$

⁵ More generally, $Q_Z = \frac{1}{2} T_{3(L+R)} - \sin^2 \theta Q$ may be rewritten, using $T_{3(L+R)} = Q - \frac{1}{2}(B - L)$, as the conserved charge

$$Q_Z = -\frac{1}{4}(B - L) + \left(\frac{1}{2} - \sin^2 \theta \right) Q,$$

leading to a Standard Model “weak charge” $-(B - L) + (2 - 4 \sin^2 \theta) Q$, identical to (5) in the case of a nucleus.

$$\begin{aligned}
Q_Z &= (T_{3L})_V - \sin^2 \theta Q = \frac{T_{3L} + T_{3R}}{2} - \sin^2 \theta Q \\
&= \frac{Z - N}{4} - \sin^2 \theta Z = \frac{1}{4} Q_{\text{weak}}(Z, N). \quad (4)
\end{aligned}$$

The quantity

$$Q_W(Z, N) = Z(1 - 4 \sin^2 \theta) - N \approx -N \quad (5)$$

to which it is proportional, is usually referred to as the “weak charge” of a nucleus of Z protons and N neutrons, and governs the parity-violation effects in atomic physics we are interested in [14,15].

The corresponding effective Lagrangian density involves the products of the (Z and U) axial currents of the electron by the vector neutral currents of the quarks (i.e., ultimately the vector currents associated with protons and neutrons). It may be written, in the local limit approximation (assuming m_U^2 somewhat larger than the relevant $|q^2|$, cf. Section 4) as:

$$\begin{aligned}
-\mathcal{L}_{\text{eff}} &= \frac{g^2 + g'^2}{m_Z^2} \frac{1}{4} \bar{e} \gamma_\mu \gamma_5 e \\
&\times \left[\left(\frac{1}{4} - \frac{2}{3} s^2 \right) \bar{u} \gamma^\mu u + \left(-\frac{1}{4} + \frac{1}{3} s^2 \right) \bar{d} \gamma^\mu d \right] \\
&- \frac{f_{Ae}}{m_U^2} \bar{e} \gamma_\mu \gamma_5 e [f_{Vu} \bar{u} \gamma^\mu u + f_{Vd} \bar{d} \gamma^\mu d]. \quad (6)
\end{aligned}$$

The quark (or proton and neutron) contribution to the charge Q_U associated with the vector part of the U current reads

$$\begin{aligned}
Q_U &= (2f_{Vu} + f_{Vd})Z + (f_{Vu} + 2f_{Vd})N \\
&= 3f_{Vq}^{\text{eff}}(Z + N) = 3f_{Vq}^{\text{eff}}A. \quad (7)
\end{aligned}$$

This proportionality to the total number of nucleons A holds only, strictly speaking, when the U has equal vector couplings to the u and d quarks, $f_{Vu} = f_{Vd}$. If not, we can still use Eq. (7) as defining the average effective vector coupling f_{Vq}^{eff} of the U boson to a quark, within the nucleus considered.

The effective Lagrangian density (6) responsible for atomic parity-violation effects leads to the parity-violating Hamiltonian density for the electron field, in the vicinity of the nucleus:⁶

$$\begin{aligned}
\mathcal{H}_{\text{eff}} &= e^\dagger(\vec{r}) \gamma_5 e(\vec{r}) \\
&\times \left[\frac{g^2 + g'^2}{16m_Z^2} [Z(1 - 4s^2) - N] \right. \\
&\quad \left. - \frac{f_{Ae} f_{Vq}^{\text{eff}}}{m_U^2} 3(Z + N) \right] \delta(\vec{r}) \\
&= e^\dagger(\vec{r}) \gamma_5 e(\vec{r}) \frac{G_F}{2\sqrt{2}} Q_W^{\text{eff}}(Z, N) \delta(\vec{r}). \quad (8)
\end{aligned}$$

In the non-relativistic limit (with small components expressed as $\simeq \vec{\sigma} \cdot \vec{p} / (2m_e)$ acting on the electron wave-function), this turns into the parity-violating Hamiltonian for an atomic electron,

$$H_{\text{eff}} = \frac{G_F}{2\sqrt{2}} \frac{\vec{\sigma} \cdot \vec{p} \delta(\vec{r}) + \delta(\vec{r}) \vec{\sigma} \cdot \vec{p}}{2m_e} Q_W^{\text{eff}}(Z, N), \quad (9)$$

\vec{p} being the electron momentum operator.

This Hamiltonian is expressed in terms of an “effective weak-charge” of the nucleus, which includes, in addition to the standard contribution $Q_W(Z, N)_{\text{SM}}$ (given by Eqs. (4), (5), plus radiative correction terms), an additional U contribution, in the case of a parity-violating U current:

$$\begin{aligned}
Q_W^{\text{eff}}(Z, N) &= Q_W(Z, N)_{\text{SM}} \\
&- \frac{2\sqrt{2}}{G_F} \frac{f_{Ae} f_{Vq}^{\text{eff}}}{m_U^2} 3(Z + N). \quad (10)
\end{aligned}$$

This applies even if the vector coupling of the U differs for the u and d quarks, the effective quark vector coupling f_{Vq}^{eff} being defined from Eq. (7) by

$$f_{Vq}^{\text{eff}} = \frac{f_{Vu}(2Z + N) + f_{Vd}(Z + 2N)}{3(Z + N)}. \quad (11)$$

3. Expression in terms of the symmetry-breaking scale

Eq. (10), namely,

$$\Delta Q_W^{\text{eff}}(Z, N) = -\frac{2\sqrt{2}}{G_F} \frac{f_{Ae} f_{Vq}^{\text{eff}}}{m_U^2} 3(Z + N), \quad (12)$$

may be identified with the one given in [2],

$$\Delta Q_W = r^2 c_\varphi 3(Z + N), \quad (13)$$

⁶ Finite-size effects of the nucleus may be taken into account by replacing $\delta(\vec{r})$ by the nuclear density $\rho_n(\vec{r})$, normalized to unity

(assuming here for simplicity that the p and n densities have the same radial behaviour).

in a simple situation with a universal axial contribution to the U current. The axial and vector couplings are then parametrized, in terms of the extra- $U(1)$ gauge coupling g'' ,⁷ as

$$\begin{cases} -f_A = \frac{g''}{4} = 2^{-3/4} G_F^{1/2} m_U r \\ \quad \simeq 2 \times 10^{-6} m_U (\text{MeV}) r, \\ f_V = \frac{g''}{4} c_\varphi = 2^{-3/4} G_F^{1/2} m_U r c_\varphi \\ \quad \simeq 2 \times 10^{-6} m_U (\text{MeV}) r c_\varphi. \end{cases} \quad (14)$$

$r \leq 1$ (here simply defined by $\frac{g''}{m_U} = \frac{g}{m_W} r$) is a dimensionless parameter related to the extra- $U(1)$ symmetry-breaking scale, and c_φ (initially denoted $\cos \varphi$, although not necessarily smaller than 1 in modulus) measures the magnitude of the quark vector coupling relatively to the axial one. The parity-violating U -exchange amplitudes are proportional to the Fermi-like constant

$$-\frac{f_A f_V}{m_U^2} = \frac{g''^2}{16m_U^2} c_\varphi = \frac{G_F}{2\sqrt{2}} r^2 c_\varphi, \quad (15)$$

allowing the identification of expressions (12) and (13) of ΔQ_W .

The parameter $r \leq 1$ represents more generally, in such models, the scale at which the extra- $U(1)$ symmetry gets spontaneously broken (to which it is, roughly, inversely proportional) [1,5]. In a class of models $r = 1$ would correspond to an extra $U(1)$ broken “at the electroweak scale” by two Higgs doublets only ($\langle \varphi_d^0 \rangle = v_1/\sqrt{2}$ and $\langle \varphi_u^0 \rangle = v_2/\sqrt{2}$); this, however, is excluded experimentally as the light U would then behave very much as a standard axion (with presumably, in the present case, invisible decay modes into Light Dark Matter particles dominating over the visible ones into e^+e^-).

r is smaller than one, if an extra Higgs singlet provides an additional contribution to the U mass, $r < 1$ measuring the amount by which the extra- $U(1)$ symmetry gets broken “above the electroweak scale” or “at

a large scale” through this (large) extra singlet v.e.v.⁸ This can make the physical effects of the U boson essentially invisible in particle physics, very much as for an axion, according to the “invisible U boson” (or similar “invisible axion”) mechanism [1,5].

Reexpressing ΔQ_W as in (13) allows us to compare directly the extra amount of parity-violation due to the U boson to the Z contribution, in terms of $r \leq 1$. The U contribution (for parity-violating couplings to quarks and electrons) would be roughly of the same order as the standard one (and then excessively large), if the extra- $U(1)$ were broken at a scale comparable to the electroweak scale.

4. Propagator effects for a very light U boson

In addition, if the U is light enough (i.e., as compared to the typical $\sqrt{|q^2|}$ in the experiment considered), one can no longer use for its propagator the local limit approximation. One writes instead,

$$\frac{f_{Ae} f_{Vq}^{\text{eff}}}{m_U^2 - q^2} = \frac{f_{Ae} f_{Vq}^{\text{eff}}}{m_U^2} \frac{m_U^2}{m_U^2 - q^2}, \quad (16)$$

which leads to a corrective factor

$$\frac{m_U^2}{m_U^2 - q^2} = \frac{m_U^2}{m_U^2 + \bar{q}^2} \simeq \begin{cases} 0 & \text{for } m_U^2 \ll \bar{q}^2, \\ 1 & \text{for } m_U^2 \gg \bar{q}^2, \end{cases} \quad (17)$$

as compared to a calculation that would have been performed in the local limit approximation.

Expression (16) is associated with a Yukawa-like (or Coulomb-like, if the U is massless) parity-violating potential, i.e.,

$$\begin{aligned} & \frac{f_{Ae} f_{Vq}^{\text{eff}}}{m_U^2 - q^2} \\ & \longleftrightarrow f_{Ae} f_{Vq}^{\text{eff}} \frac{e^{-m_U |\vec{r}|}}{4\pi |\vec{r}|} = \frac{f_{Ae} f_{Vq}^{\text{eff}} m_U^2 e^{-m_U |\vec{r}|}}{m_U^2 4\pi |\vec{r}|}, \end{aligned} \quad (18)$$

where

$$\frac{m_U^2 e^{-m_U |\vec{r}|}}{4\pi |\vec{r}|} (= “\delta_{m_U}”(\vec{r})) \xrightarrow{m_U \rightarrow \infty} \delta(\vec{r}), \quad (19)$$

⁷ The couplings to the left-handed and right-handed fermion fields were expressed as $-\frac{g''}{4}(1 - c_\varphi)$, $\frac{g''}{4}(1 + c_\varphi)$, respectively, which corresponds to a vector coupling $f_V = \frac{g''}{4} c_\varphi$, and an axial coupling $f_A = -\frac{g''}{4}$.

⁸ In particular, if we define $v_2/v_1 = 1/x = \tan \beta$, the axial coupling f_{Ae} is given, after Z - U mixing effects, by $-f_{Ae} = 2^{-3/4} G_F^{1/2} m_U r/x$.

in the case of a sufficiently “heavy” U (typically $m_U \gtrsim 100 \text{ MeV}/c^2$ as we shall see), leading to the parity-violating Hamiltonian (9) (with $\delta(\vec{r})$ replaced by the normalized nuclear density $\rho_n(\vec{r})$, if the nucleus is not taken as pointlike).

For a too light U boson, however, the local limit approximation is not valid, and the new contribution ΔQ_W^{eff} as given by (12) should be multiplied by a correction factor $K(m_U)$ obtained by replacing $\delta(\vec{r})$ in (8) or (9) by the appropriate Yukawa distribution $\delta_{m_U}(\vec{r})$ of Eq.(19), which extends over a range $\approx \hbar/(m_U c)$. The normalized nuclear density $\rho_n(\vec{r}_n)$ (which may be approximated by a $\delta(\vec{r}_n)$ distribution although it extends over a radius $R_n \simeq r_0 A^{1/3} \simeq 6$ Fermi for Cs) gets replaced by its convolution product with the $\delta_{m_U}(\vec{r} - \vec{r}_n)$ Yukawa distribution, corresponding to the exchange of a very light U between an electron at \vec{r} and the nucleus at \vec{r}_n .

This can be expressed through a corrective factor

$$K(m_U) = \frac{\langle H_{\text{eff}}(m_U) \rangle}{\langle H_{\text{eff}}(m_U \rightarrow \infty) \rangle} \simeq \frac{\int \langle e^\dagger(\vec{r}) \gamma_5 e(\vec{r}) \rangle \rho_n(\vec{r}_n) \delta_{m_U}(\vec{r} - \vec{r}_n) d^3\vec{r} d^3\vec{r}_n}{\int \langle e^\dagger(\vec{r}) \gamma_5 e(\vec{r}) \rangle \rho_n(\vec{r}) d^3\vec{r}} \quad (20)$$

evaluated in [3], and given numerically in Table 1.

The mass $m_U \simeq 2.4 \text{ MeV}/c^2$, for which $K(m_U) = 1/2$, defines the typical momentum transfer associated with cesium parity-violation experiments. The corresponding $\hbar/(m_U c)$ is $\simeq 80$ Fermi: the electron involved in the parity-violating transition of the cesium atom “feels” in fact the new U -mediated interaction, even if it is relatively long-ranged, essentially in the vicinity of the nucleus, where the screening of the Coulomb potential of the nucleus by the core electrons can be neglected. One has therefore, ultimately,

$$\Delta Q_W^{\text{eff}}(Z, N) = -\frac{2\sqrt{2} f_{Ae} f_{Vq}^{\text{eff}}}{G_F m_U^2} 3(Z + N) K(m_U). \quad (21)$$

For $m_U < 100 \text{ MeV}/c^2$ the presence of the factor K weakens the expressions of the limits that would otherwise be obtained from (12), especially in the case of a very light gauge boson.⁹ Still they remain of the same

order as obtained from a local limit approximation, for $m_U \gtrsim$ a few MeV/c^2 , as can be seen from Table 1.

5. Experimental limits on $f_{Ae} f_{Vq}$

From the present comparison between experimental results on $Q_W(Z, N)$ for cesium [13], which rely on atomic physics calculations [15], and theoretical predictions from standard model estimates [16],

$$\begin{cases} Q_{W \text{ exp}} = -72.74 \text{ (29)}_{\text{exp}} \text{ (36)}_{\text{theor}}, \\ Q_{W \text{ SM}} = -73.19 \pm 0.13, \end{cases} \quad (22)$$

one gets

$$\Delta Q_W = Q_{W \text{ exp}} - Q_{W \text{ SM}} = 0.45 \pm 0.48, \quad (23)$$

which corresponds to an uncertainty of less than 1%, i.e. (working conservatively at a pseudo “ 2σ ” level)

$$-0.51 < \Delta Q_W < 1.41. \quad (24)$$

For cesium ($A = 133$), using $G_F/(2\sqrt{2}3A) \simeq 1.03 \times 10^{-14} \text{ MeV}^{-2}$ and expression (12) of ΔQ_W (for $m_U \gtrsim 100 \text{ MeV}/c^2$), we get the constraint

$$-0.53 \times 10^{-14} \text{ MeV}^{-2} < \frac{-f_{Ae} f_{Vq}}{m_U^2} < 1.46 \times 10^{-14} \text{ MeV}^{-2}, \quad (25)$$

or, approximately,

$$-0.5 \times 10^{-3} G_F < \frac{-f_{Ae} f_{Vq}}{m_U^2} < 1.3 \times 10^{-3} G_F. \quad (26)$$

For a lighter U the limits get divided by the corrective factor $K(m_U)$ of Table 1 (i.e., approximately doubled, for $m_U \simeq$ a few MeV/c^2 ’s).

This analysis applies to heavy as well as to light U ’s. In the first case it can constrain a U (unmixed with the Z , with couplings $\sim g, g'$ or e) to be heavier than several hundred GeV/c^2 or even more, depending on its couplings. As a toy-model illustration, an extra Z or U boson that would have the same couplings as the Z would lead directly to a negative contribution $\Delta Q_W \simeq Q_{W \text{ SM}} (m_Z/m_{\text{extra}Z})^2$ [17]. Assuming for simplicity that no other contribution has to

⁹ Furthermore, in the limit of a very light U (i.e., for $m_U \ll m_e \alpha \simeq 4 \text{ keV}/c^2$ so that $\hbar/(m_U c) \gg \hbar/(m_e c \alpha) \simeq 0.5 \times 10^{-8} \text{ cm}$),

$K(m_U) \propto m_U^2$ (as one can see from (17)), and the limits may then be expressed as $|f_{Ae} f_{Vq}^{\text{eff}}| < \dots$, instead of $|f_{Ae} f_{Vq}^{\text{eff}}|/m_U^2 < \dots$.

Table 1

Atomic factor $K(M_U)$ giving the correction to the weak charge $\Delta Q_W(m_U)$ for the cesium atom

m_U (MeV/ c^2)	0.1	0.37	0.5	1	2.4	5	10	20	50	100
Corr. factor $K(m_U)$	0.025	0.15	0.20	0.33	0.5	0.63	0.74	0.83	0.93	0.98

be considered, it would have to verify, approximately, $|\Delta Q_W| < 0.55$. The new gauge boson should then be at least 11.5 times heavier than the Z , i.e.:

$$m_{\text{extra } Z} > 1.05 \text{ TeV}/c^2, \quad (27)$$

which is above present direct collider bounds [18].

For a light U on the other hand, the constraint (cf. (26)) adds to those already obtained from low-energy ν - e scattering cross sections, e.g., for m_U larger than a few MeV/ c^2 ,s,

$$\frac{|f_{V\nu} f_{Ve}|}{m_U^2} \lesssim G_F \quad (28)$$

and anomalous magnetic moments of charged leptons [1,2,7,8]. The latter constraints, however, should be considered with appropriate care, especially in the case of parity-violating couplings, due to the possibility of cancellations between (positive) vector contributions and (negative) axial ones.^{10,11}

More significant in fact are the limits from the non-observation of an axionlike particle, which severely constrain an axial contribution in the quark U current,

¹⁰ While the $g-2$ constraints on the vector and axial couplings to the electron, and vector coupling to the muon, are in general not so restrictive (e.g., for a U somewhat heavier than e but lighter than μ , $f_{Ve} \lesssim 2 \times 10^{-4} m_U$ (MeV), $f_{Ae} \lesssim 0.6 \times 10^{-4} m_U$ (MeV), $f_{V\mu} \lesssim 6 \times 10^{-4}$), the one for an axial coupling to the muons ($f_{A\mu} \lesssim 3 \times 10^{-6} m_U$ (MeV), i.e., $f_{A\mu}^2/m_U^2 < G_F$) is more severe, in connection with an axionlike behavior of the U boson in this case.

¹¹ A light U could also be detected through a bremsstrahlung from an electron, in electron beam dump experiments, as for an axion decaying into e^+e^- (but with a production cross section behaving differently, for a U having vector or pseudovector couplings); this may constrain the U to be heavier than \sim a few to 10 MeV, depending on the size of its couplings [19]. However, a relatively light U responsible for Light Dark Matter annihilations at the appropriate rate tends to be much more strongly coupled to Dark Matter (c_U) than to ordinary matter (f), possibly by several orders of magnitude. Invisible U decays into Dark Matter particles would both decrease significantly the U lifetime and make its visible decays into e^+e^- very rare, or even practically negligible; no such limits may then be obtained in this way.

requiring typically

$$\frac{f_{Aq}^2}{m_U^2} < \frac{1}{10} G_F, \quad (29)$$

from ψ or $\Upsilon \rightarrow \gamma + U$ decays, or even

$$\frac{f_{As}^2}{m_U^2} \lesssim \frac{1}{300} G_F, \quad (30)$$

from $K^+ \rightarrow \pi^+ U$ decays [1,5,8,20]. If such an axial contribution is actually present, the extra- $U(1)$ symmetry should then be broken sufficiently above the electroweak scale—a conclusion reenforced here, in the case of a U boson inducing atomic physics parity-violation effects, constrained to be very small. This illustrates, also, how parity-violation atomic physics experiments can give very valuable informations, complementing those obtained from particle physics.

6. Conclusion

This analysis of parity-violation effects in atomic physics (which also applies to heavy bosons), combined, in the case of a light U , with earlier constraints on a possible axionlike behavior of this particle, favors a situation in which the quark and lepton contribution to the U current is *purely vectorial*, as in a class of models discussed in [5,6]. Otherwise the scale at which the extra- $U(1)$ symmetry is broken should be larger than the electroweak scale, by about one order of magnitude at least; the coupling of the U to a Light Dark Matter particle would then have to be further increased, to compensate for its smaller couplings to ordinary particles.

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