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Book Review

PETER J. OLVER, *Applications of Lie Groups to Differential Equations*, Springer Graduate Texts in Mathematics, 1986.

Peter Olver's book contains most of the information that, in the reviewer's opinion, should be part of the mathematical working knowledge of anyone doing substantial research involving analytical solutions of differential equations. The author himself has been a principal contributor to the mathematics presented. Graduate students in mathematical physics and applied mathematics are strongly urged to read this extremely clearly written book.

Olver makes it clear that he is dealing with applied mathematics: in the first two sentences of his preface, the words "applied" or "applications" appear three times. However, the basics are by no means neglected, thereby making the exposition very nicely self-contained. Emphasis is placed on significant applications of Lie-group methods, organized into well-chosen examples of direct physical importance that illustrate the theoretical foundations of the subject.

The book's theme—symmetry analysis of differential equations—summons the required theoretical foundations in a natural sequence, as needed for the increasingly sophisticated applications. Beginning with the basic theory of manifolds, Lie groups and Lie algebras, transformation groups, and differential forms, the book proceeds to explain prolongation theory, characteristics, the Cauchy–Kovalevskaya theorem, integrability of differential equations, jet bundles, quotient manifolds, adjoint and co-adjoint representations of Lie groups, the formal theory of the calculus of variations (including the variational complex), and the general theory of Poisson structures for Hamiltonian systems, all developed in the context of applications.

The applications themselves include numerous examples from classical mechanics, fluid dynamics, elasticity, and other areas of theoretical and mathematical physics of current interest. Olver's treatment of Hamiltonian systems is especially remarkable in that it concentrates on Poisson brackets, proceeding directly to the heart of the matter and taking the reader right to the threshold of research in the subjects of variational calculus and Hamiltonian structure, including Lie–Poisson brackets in ideal fluid dynamics and bi-Hamiltonian structures for completely integrable evolutionary systems.

Olver's book is an authoritative, clearly written, scholarly, and complete treatment of Lie-group methods for differential equations at the intermediate graduate student level in applied mathematics or mathematical physics. It achieves its purpose of providing the reader with an excellent preparation for further research in this field. I expect it to become a classic text in this field.

Among the helpful pedagogical features of the book are the notes at the end of each chapter and at the beginning of the book conveying to the reader the congenial feeling that the author is really trying to communicate with him as thoughtfully and clearly as possible, without unnecessary diversions. These notes also convey something of especially great value to students, namely, a sense of the history and development of the mathematical ideas underlying Lie-group methods. Thus, not only does the text explain the material clearly, but—much in the same spirit as some of Garrett Birkhoff's writings—the notes trace the mathematical interconnections of the basic ideas being discussed, including their historical motivations and the ongoing reaffirmations and rediscoveries of these ideas in new applications.

Besides the notes, the exercises for each chapter are an integral part of the pedagogical exposition, occasionally providing significant extensions of the material in the text, or giving a

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flavor of current research in the field. The *three* indices—of authors, subjects, and nomenclature—not only help to orient the student, but make the book useful as a reference for researchers as well.

If I were going to teach Lie-group methods for differential equations in a graduate course in mathematics of physics, I would use Peter Olver's book. Olver's book pays much more attention to the basic mathematical ideas than Bluman and Cole's book [1], and is much more congenially written than Ovsiannikov's book [2]. Moreover, Olver's book treats a much broader range of topics and material than do either of these books.

References

1. G. W. Bluman and J. D. Cole, "Similarity Methods for Differential Equations," Appl. Math. Sci., No. 13, Springer-Verlag, New York, 1974.
2. L. V. Ovsiannikov, "Group Analysis of Differential Equations," Academic Press, New York, 1982.

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