Journal of Pure and Applied Algebra 33 (1984) 209-217
North-Holland

REPRESENTATIONS OF ORDERS AND VECTOR SPACE CATEGORIES

Kenji NISHIDA
Department of Mathematics, Kitami Institute of Technology, Kitami, 090, Japan

Communicated by H. Bass
Received 31 May 1983
Revised 30 November 1983

1. Introduction, notation, and results

Let R be a complete discrete valuation ring with prime element π and residue field $\mathbf{k}$. Let Λ be an R-order in the semisimple finte-dimensional algebra over the quotient field of R and Λ_0 a hereditary R-order in the same algebra such that $\operatorname{rad}\Lambda_0 \subset \Lambda \subset \Lambda_0$. Put $A=\Lambda/\operatorname{rad}\Lambda_0$, $B=\Lambda_0/\operatorname{rad}\Lambda_0$. Then A is a $\mathbf{k}$-subalgebra of the semisimple $\mathbf{k}$-algebra B. We assume Λ to be basic, so that A, too.

In this paper, we shall study latt Λ, the category of all right Λ-lattices, through a generalized vector space category $\mathbf{K}$ associated with A and B. Recently, Green and Reiner [2] and Ringel and Roggenkamp [4] have succeeded to reduce latt Λ to a certain subcategory over an artinian $\mathbf{k}$-algebra obtained from A and B. Then it has become a problem to investigate these subcategories arising from latt Λ. Green and Reiner [2], Ringel and Roggenkamp [4,5], Roggenkamp [6] have considered this problem under some conditions. On the other hand, Ringel [3] and Simson [7] have investigated vector space categories, in particular, Simson [7] showed many useful results by using a category of socle projective modules over a right peak ring. Thus applying the results of [7] we can consider orders which include those in [2,4,5,6].

Following Simson [7, §6, B] for $K=K_1\times\cdots\times K_t$ with each K_i a division ring, a generalized vector space category $\mathbf{K}$ over K is an additive category with a faithful additive functor $|-|:\mathbf{K}\to\operatorname{mod}K$, where $\operatorname{mod}K$ is the category of all finitely generated right K-modules. The factor space category $V(\mathbf{K})$ of $\mathbf{K}$ is defined as follows. The objects of $V(\mathbf{K})$ are triples (V,X,ϕ) where $V\in\operatorname{mod}K$, $X\in\mathbf{K}$, and $\phi:|X|_K\to V_K$ is a K-homomorphism. The map from (V,X,ϕ) into (V',X',ϕ') is a pair (u,h) with $u\in\operatorname{Hom}_K(V,V')$ and $h\in\operatorname{Hom}_{\mathbf{K}}(X,X')$ such that $\phi'|h|=u\phi$. Let $V_1(\mathbf{K})$ be the full subcategory of $V(\mathbf{K})$ consisting of the objects which have no direct summands of the form $(V,0,0)$ or $(0,W,0)$ where $V\in\operatorname{mod}K$ and $W\in\mathbf{K}$.

We shall define the generalized vector space category $\mathbf{K}$ which plays a crucial role in this paper. Let $S_1,\ldots,S_t$ be the representatives of nonisomorphic simple right

B-modules. Put $K_i = \mathrm{End}_B S_i$ $(1 \le i \le t)$, $K = \prod_{i=1}^{t} K_i$, $G = \bigoplus_{i=1}^{t} S_i$. Then each K_i $(1 \le i \le t)$ is a division ring over $\mathbf{k}$ and G is a K-B-bimodule. Put $X^* = \mathrm{Hom}_K(X, K)$ for a K-module X. We put $\mathbf{K} = \{\mathrm{Hom}_A(P, {}_K G)^* \mid P$ is a finitely generated projective A-module$\}$ (cf. [3, 3.2], [8, Theorem 1.1]). Then $\mathbf{K}$ is a generalized vector space category over K. We note $\mathrm{Hom}_{\mathbf{K}}(\mathrm{Hom}_A(P, G)^*, \mathrm{Hom}_A(P', G)^*) \cong \mathrm{Hom}_A(P, P')$ by definition.

We have our main result.

Theorem 1. *There exists a representation equivalence* $\mathrm{latt}\, \Lambda \approx V_1(\mathbf{K})$.

Let

$$C = \begin{pmatrix} A & B \\ 0 & B \end{pmatrix}$$

be a generalized triangular matrix $\mathbf{k}$-algebra. Let $\mathscr{C}$ be a full subcategory of $\mathrm{mod}\, C$ consisting of the modules $X = (P_A, V_B, \phi)$ such that P_A is finitely generated projective, $\ker \phi \subset \mathrm{rad}\, P_A$, and $\mathrm{Im}\, \phi \cdot B = V$, where we identify $X \in \mathrm{mod}\, C$ with a triple (P_A, V_B, ϕ), $P \in \mathrm{mod}\, A$, $V \in \mathrm{mod}\, B$, $\phi \in \mathrm{Hom}_A(P, V)$. In [2, 4] the following was proved.

Theorem 2. *There exists a representation equivalence* $\mathrm{latt}\, \Lambda \approx \mathscr{C}$.

Indeed, we shall show the following.

Theorem 3. *There exists a functor* $\Phi : \mathrm{mod}\, C \to V(\mathbf{K})$ *which induces a representation equivalence* $\mathscr{C} \approx V_1(\mathbf{K})$.

Thus Theorem 1 follows from Theorems 2 and 3.

Let $M_1, \ldots, M_n$ be all pairwise nonisomorphic indecomposable objects in $\mathbf{K}$. Then we can associate to $\mathbf{K}$ the right peaks $\mathbf{k}$-algebra, that is, a finite-dimensional $\mathbf{k}$-algebra which has a projective right socle

$$C_{\mathbf{K}} = \begin{pmatrix} E & {}_E M_K \\ 0 & K \end{pmatrix}, \quad \text{where } M = \bigoplus_{i=1}^{n} |M_i|,\ E = \mathrm{End}_{\mathbf{K}} M.$$

Combining Theorem 1 with [7, Theorem 3.3] we get.

Theorem 4. *There exists a representation equivalence* $\mathrm{latt}\, \Lambda \approx \mathrm{mod}_{\mathrm{sp}}^0\, C_{\mathbf{K}}$, *where* $\mathrm{mod}_{\mathrm{sp}}^0\, C_{\mathbf{K}}$ *is the full subcategory of* $\mathrm{mod}\, C_{\mathbf{K}}$ *whose modules have a projective socle and no direct summand of the form* $(0, K_i, 0)$.

In what follows, the notation provided above is preserved.

2. Proof of Theorems 3 and 4

We construct the functor $\Phi: \mathrm{mod}\, C \to V(\mathbf{K})$. Let $X=(U_A, V_B, \phi) \in \mathrm{mod}\, C$ with $\phi \in \mathrm{Hom}_A(U, V)$. Let $p: P \to U$ be a projective cover of U_A and δ the composition

$$\mathrm{Hom}(V,G) \xrightarrow{\mathrm{Hom}(\phi, G)} \mathrm{Hom}(U,G) \xrightarrow{\mathrm{Hom}(p, G)} \mathrm{Hom}(P,G).$$

Then $\delta^*: \mathrm{Hom}(P,G)^* \to \mathrm{Hom}(V,G)^*$ is a K-homomorphism of right K-modules. Let $\Phi(X)=(\mathrm{Hom}_B(V,G)^*, \mathrm{Hom}_A(P,G)^*, \delta^*) \in V(\mathbf{K})$. Defining $\Phi(u)$ naturally for $u \in \mathrm{Hom}_C(X, X')$ we get a functor $\Phi: \mathrm{mod}\, C \to V(\mathbf{K})$. Let $Y=(V'_K, \mathrm{Hom}(P,G)^*, \psi_1) \in V(\mathbf{K})$, $\psi=\psi_1^*$, $V'^*=\prod_{i=1}^t K_i^{\alpha_i}$. Put $\psi=(\psi_i)_{1\le i\le t}$ with $\psi_i: {}_{K_i}K_i^{\alpha_i} \to {}_{K_i}\mathrm{Hom}_A(P, S_i)$ and each $\psi_i=(\psi_{ij})_{1\le j\le \alpha_i}$ with $\psi_{ij}: {}_{K_i}K_i \to \mathrm{Hom}_A(P, S_i)$ for $1\le i\le t$. Let $\phi_{ij}=\psi_{ij}(1)$ $(1\le i\le t;\ 1\le j\le \alpha_i)$, $\phi_i=(\phi_{ij})_{1\le j\le\alpha_i}$ $(1\le i\le t)$ and $\phi=(\phi_i)_{1\le i\le t}$. Then $\phi \in \mathrm{Hom}_A(P, V)$ where $V=\bigoplus_{i=1}^t S_i^{\alpha_i}$ and we can prove $X=(P, V, \phi) \in \mathrm{mod}\, C$ and $\Phi(X)=Y$. It is clear that Φ induces a representation equivalence $\mathscr{C} \approx V_1(\mathbf{K})$.

Although Theorem 4 follows from Theorem 1 and [7, Theorme 3.3], we add some explanations here. Especially, we replace $C_{\mathbf{K}}$ by a ring obtained directly from A and B and construct a functor $\Psi: \mathrm{mod}\, C \to \mathrm{mod}\, C_{\mathbf{K}}$ which induces a representation equivalence $\mathscr{C} \approx \mathrm{mod}_{\mathrm{sp}}^0\, C_{\mathbf{K}}$. According to [7], we call a ring R' a right peaks ring if R' is semiperfect with essential and projective right socle. By [7, Proposition 2.2] a right peaks ring has a triangular form

$$\begin{pmatrix} T & {}_TN_F \\ 0 & F \end{pmatrix},$$

where $F=\prod F_i$ is a product of finite division rings F_i and a T-F-bimodule N is T-faithful and finitely generated over F. Let $\mathrm{mod}_{\mathrm{sp}}\, R'$ be the full subcategory of $\mathrm{mod}\, R'$ consisting of modules having a projective socle. $\mathrm{mod}_{\mathrm{sp}}^0\, R'$ is the full subcategory of $\mathrm{mod}_{\mathrm{sp}}\, R'$ whose modules have no direct summand of the form $(0, F_i, 0)$. For R' a right peaks algebra, $\mathrm{mod}_{\mathrm{sp}}\, R'$ was investigated in [7], and also in [5] for R' hereditary.

Returning to our case,

$$C_{\mathbf{K}}=\begin{pmatrix} E & {}_EM_K \\ 0 & K \end{pmatrix}$$

is a right peaks ring induced by a generalized vector space category $\mathbf{K}$, where $M=|M_1\oplus\cdots\oplus M_n|$ and $E=\mathrm{End}_{\mathbf{K}}\, M$. Decompose $A=P_1\oplus\cdots\oplus P_n$ where the P_i are indecomposable projective right A-modules. Then since $\{M_1, \ldots, M_n\}=\{\mathrm{Hom}_A(P_1, G)^*, \ldots, \mathrm{Hom}_A(P_n, G)^*\}$, we have

$$E \cong \mathrm{End}_{\mathbf{K}}\left(\bigoplus_{i=1}^n \mathrm{Hom}_A(P_i, G)^*\right) \cong \mathrm{End}_A\left(\bigoplus_{i=1}^n P_i\right) \cong A$$

and

$$M \cong \bigoplus_{i=1}^n \mathrm{Hom}_A(P_i, G)^* \cong D\left(\mathrm{Hom}_A\left(\bigoplus_{i=1}^n P_i, G\right)\right) \cong DG$$

where D is the duality $D(-)=\mathrm{Hom}_{\mathbf{k}}(-,\mathbf{k})$. Thus we have

$$C_{\mathbf{K}}\cong\begin{pmatrix} A & DG \\ 0 & K \end{pmatrix}.$$

Define a functor $H: V(\mathbf{K})\to\mathrm{mod}_{\mathrm{sp}}\, C_{\mathbf{K}}$ as follows (cf. [7]). For

$$X=(V_K, \mathrm{Hom}_A(P,G)^*, \phi)\in V(\mathbf{K}),$$

put ϕ' the image of ϕ under an isomorphism

$$\begin{aligned}\mathrm{Hom}_K(\mathrm{Hom}_A(P,G)^*, V)&\cong\mathrm{Hom}_K(DV,\mathrm{Hom}_A(P,G)\\ &\cong\mathrm{Hom}_A(P,\mathrm{Hom}_K(DV,G))\\ &\cong\mathrm{Hom}_A(P,\mathrm{Hom}_K(DG,V)).\end{aligned}$$

Let $H(X)=(Q,V,t)$ where $Q=\mathrm{Im}\,\phi'$ and t is the map adjoint to the inclusion $Q\hookrightarrow\mathrm{Hom}_K(DG,V)$. Next, for $X=(U_A,V_B,\phi)\in\mathrm{mod}\, C$, we put $\Psi(X)=(U_A, V\otimes_B DG_K, \phi)\in\mathrm{mod}\, C_{\mathbf{K}}$ where ψ is the composition

$$U\otimes_A DG\cong U\otimes_A B\otimes_B DG\xrightarrow{\phi\otimes 1} V\otimes_B DG.$$

Ψ is a functor $\mathrm{mod}\, C\to\mathrm{mod}\, C_{\mathbf{K}}$. Then the following is easily proved.

Lemma. *We have* $\Psi=H\Phi$.

By [7, Theorem 3.3] H induces a representation equivalence $V_1(\mathbf{K})\approx\mathrm{mod}^0_{\mathrm{sp}}\, C_{\mathbf{K}}$, and then Ψ induces a representation equivalence $\mathscr{C}\approx\mathrm{mod}^0_{\mathrm{sp}}\, C_{\mathbf{K}}$.

3. Concluding remarks

Let $\mathbf{k}'$ be a commutative field and

$$C'=\begin{pmatrix} T & {}_TM_F \\ 0 & F \end{pmatrix}$$

a right peaks $\mathbf{k}'$-algebra, where $F=\prod_{i=1}^{t} F_i$ with each F_i a finite-dimensional division algebra over $\mathbf{k}'$ and M is a T-F-bimodule finite-dimensional over $\mathbf{k}'$ and a faithful left T-module. Put $B=\mathrm{End}_F M$. Then there exists a $\mathbf{k}'$-algebra monomorphism $\sigma: T\to B$ and a hereditary R'-order Γ and an R'-order Λ such that $\mathrm{rad}\,\Gamma\subset\Lambda\subset\Gamma$, $\Lambda/\mathrm{rad}\,\Gamma\cong T$, $\Gamma/\mathrm{rad}\,\Gamma\cong B$ where R' is a complete discrete valuation ring (cf. proof of [6, (1.11) Theorem II]). Thus we have $\mathrm{latt}\,\Lambda\approx\mathrm{mod}^0_{\mathrm{sp}}\, C'$ by Theorem 4.

Example 1. (cf. [7, Example]). Let

$$C=\begin{pmatrix} \mathbf{k} & \mathbf{k} & \mathbf{k}^2 \\ \mathbf{k} & \mathbf{k} & \mathbf{k}^2 \\ 0 & 0 & \mathbf{k} \end{pmatrix}=\begin{pmatrix} F_1 & {}_1M_2 & {}_1M_3 \\ {}_2M_1 & F_2 & {}_2M_3 \\ 0 & 0 & F_3 \end{pmatrix}.$$

The multiplications in C are induced from the following.

$$_1M_2 \otimes {}_2M_3 \to {}_1M_3, \quad x\otimes(y,z)\mapsto(xy,0),$$

$$_2M_1 \otimes {}_1M_3 \to {}_2M_3, \quad x\otimes(y,z)\mapsto(0,xz),$$

$$_1M_2 \otimes {}_2M_1 \to F_1, \quad x\otimes y\mapsto 0,$$

$$_2M_1 \otimes {}_1M_2 \to F_2, \quad x\otimes y\mapsto 0, \quad x,y,z\in \mathbf{k}.$$

We have $B=\mathrm{End}_{\mathbf{k}}({}_1M_3\oplus{}_2M_3)\cong(\mathbf{k})_4$ and $\sigma: A\to B$ is

$$\begin{pmatrix} x & y \\ z & w \end{pmatrix} \to \begin{bmatrix} x & 0 & 0 & 0 \\ 0 & x & 0 & z \\ y & 0 & w & 0 \\ 0 & 0 & 0 & w \end{bmatrix},$$

so that

$$A=\begin{bmatrix} \mathbf{k} & 0 & 0 & 0 \\ 0 & \mathbf{k} & 0 & \mathbf{k} \\ \mathbf{k} & 0 & \mathbf{k} & 0 \\ 0 & 0 & 0 & \mathbf{k} \end{bmatrix}. \qquad \text{Hence } \Lambda=\begin{bmatrix} R & \pi & \pi & \pi \\ \pi & R & \pi & R \\ R & \pi & R & \pi \\ \pi & \pi & \pi & R \end{bmatrix}$$

where we use the same π to indicate the ideal πR and latt $\Lambda\approx \mathrm{mod}^0_{\mathrm{sp}}\, C$ is of infinite type by [7,8].

In [7] Simson defines the differentiation algorithm for a right peak ring. The differentiation algorithm includes the reduction technique used in [5]. The details about the differentiation algorithm for a right peak ring are seen in [1,7], so that we only provide here a sketch of it and the succeeding example will be available for the reader's understanding. Let $P_1,\dots,P_n$ be all pairwise nonisomorphic indecomposable right C-modules, ${}_iM_j=\mathrm{Hom}_C(P_j,P_i)$, and $F_i=\mathrm{End}\,P_i$. Assume C to be Schurian, that is, all F_i are division rings.

Definition. A pair, P_a of indecomposable projective C-modules with $a\neq s$ is said to be smooth if P_a is simple, and

(1) $\dim_{F_s}({}_sM_a)=\dim({}_sM_a)_{F_a}=1$,

(2) $c_{tsa}: {}_tM_s\otimes{}_sM_a\to{}_tM_a$, $f\otimes g\mapsto fg$, is surjective provided $c_{tsa}\neq 0$,

(3) there is no $j\neq s,a$ with ${}_sM_j\neq 0$ and ${}_jM_a\neq 0$,

(4) for $s^\wedge=\{j \mid {}_jM_s\neq 0$ or ${}_jM_b\neq 0$ for a simple projective module P_b with $b\neq a\}$ and $s^{\wedge c}=\{1,\dots,n\}-s^\wedge$, a right peak $\mathbf{k}$-algebra $T=\mathrm{End}_C(\bigoplus_{j\in s^{\wedge c}} P_j)$ is sp-representation finite, that is, $\mathrm{mod}_{\mathrm{sp}}\, T$ is of finite representation type.

If P_s, P_a is a smooth pair then we can define a differentiation $C_{s,a}$ of C with respect to a pair of points s,a and the following holds.

Theorem 5 [7, Theorem 5.2 and §6, B]. *$C_{s,a}$ is also a right peaks $\mathbf{k}$-algebra and* *there exists a functor* $\phi_s : \text{mod}_{\text{sp}}\, C \to \text{mod}_{\text{sp}}\, C_{s,a}$ *which induces a category equivalence*

$$\text{mod}_{\text{sp}}\, C/[\text{mod}_{\text{sp}}\, T] \approx \text{mod}_{\text{sp}}\, C_{s,a}$$

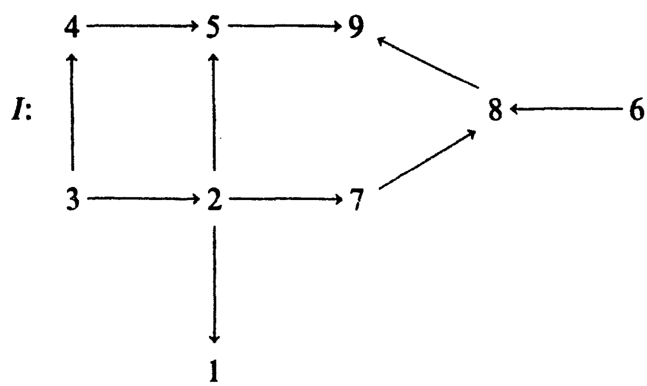
and a representation equivalence

$$\text{mod}^{s}_{\text{sp}}\, C \approx \text{mod}_{\text{sp}}\, C_{s,a},$$

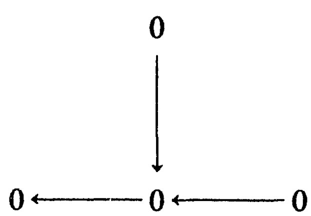
where $\text{mod}^{s}_{\text{sp}}\, C$ *is the full subcategory of* $\text{mod}_{\text{sp}}\, C$ *consisting of modules* X *such that* $\text{Hom}_C(P_s, X) \neq 0$ *or* $\text{Hom}_C(P_b, X) \neq 0$ *for a simple projective module with* $P_b \neq P_a$.

Corollary. *C is sp-representation finite if and only if the finite iterated differentiation of C is sp-representation finite.*

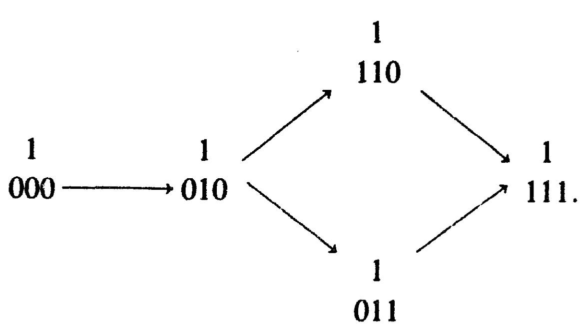
Example 2. Let C be the path algebra of the bounded quiver



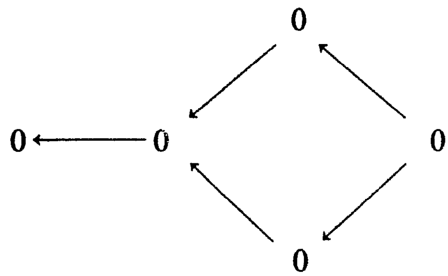
with commuting cycles. Let P_i be an indecomposable projective module corresponding to each $i \in I$ and $s=5$, $a=9$. We have $s^{\wedge} = \{1, \dots, 5\}$ and $s^{\wedge c} = \{6, \dots, 9\}$. T is the path algebra of the quiver



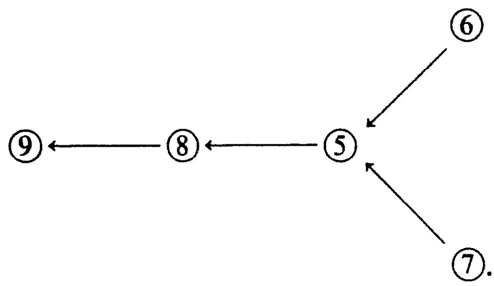
and the Auslander–Reiten quiver of $\text{mod}_{\text{sp}}\, T$ is



Put $P_i^T = \mathrm{Hom}_C(\bigoplus_{j \in s^{\wedge c}} P_j, P_i)$ $(i = 6, \ldots, 9)$, $E = E_T(P_9^T)$ an injective envelope of a simple projective T-module P_9^T. Then P_i^T $(i = 6, \ldots, 9)$, E are all pairwise nonisomorphic indecomposable modules in $\mathrm{mod}_{\mathrm{sp}}\, T$. Let $L = \bigoplus_{i=6}^{9} P_i^T \oplus E$, and $\Gamma = \mathrm{End}_T L$. Γ is the path algebra of the bounden quiver



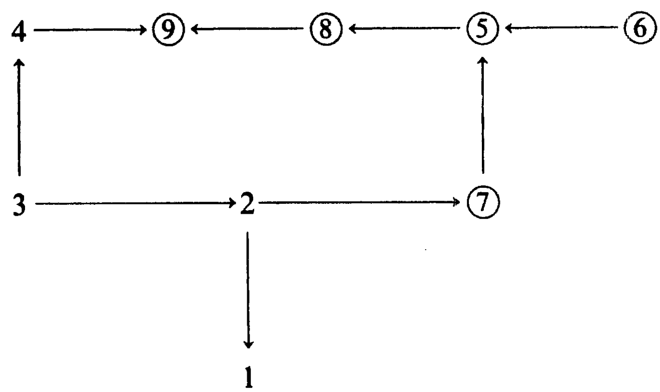
with a commuting cycle. Let T^s be the path algebra of the quiver



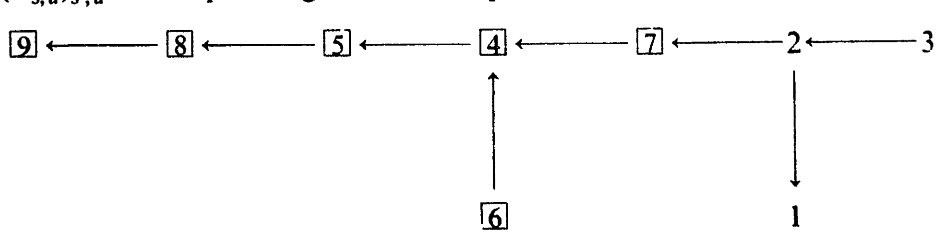
Then there exists a functor $\tau : \mathrm{mod}_{\mathrm{sp}}\, T \to \mathrm{mod}_{\mathrm{sp}}\, T_s$ and, by definition,

$$C_{s,a} = \begin{pmatrix} \bar{S} & {}_{\bar{S}}\tau(M)_{T_s} \\ 0 & T_s \end{pmatrix} \quad \text{where } \bar{S} = \mathrm{End}_C\Big(\bigoplus_{j \in s^{\wedge} - \{s\}} P_j\Big).$$

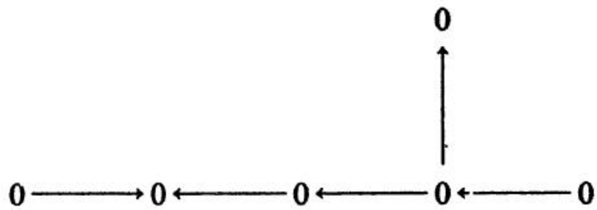
$C_{s,a}$ is the path algebra of the bounden quiver



with a commuting cycle. Next we put $s' = 4$, $a' = ⑨$ and do the differentiation of $C_{s,a}$. Then $(C_{s,a})_{s',a'}$ is the path algebra of the quiver



Further, we do for $s''=\boxed{8}$, $a''=\boxed{9}$, and so on. Finally, deleting $\boxed{9}$, $\boxed{8}$, $\boxed{5}$, we reach the path algebra of the quiver D_6:



which is of finite type. Thus C is sp-representation finite by the Corollary. Since

$$C=\begin{array}{c} \begin{matrix} 3 & 2 & 4 & 5 & 6 & 7 & 8 & 1 & 9 \end{matrix} \\ \left(\begin{matrix} \mathbf{k} & \mathbf{k} & \mathbf{k} & \mathbf{k} & 0 & \mathbf{k} & \mathbf{k} & \mathbf{k} & \mathbf{k} \\ & \mathbf{k} & 0 & \mathbf{k} & 0 & \mathbf{k} & \mathbf{k} & \mathbf{k} & \mathbf{k} \\ & & \mathbf{k} & \mathbf{k} & 0 & 0 & 0 & 0 & \mathbf{k} \\ & & & \mathbf{k} & 0 & 0 & 0 & 0 & \mathbf{k} \\ & & & & \mathbf{k} & 0 & \mathbf{k} & 0 & \mathbf{k} \\ & & & & & \mathbf{k} & \mathbf{k} & 0 & \mathbf{k} \\ & & & & & & \mathbf{k} & 0 & \mathbf{k} \\ & & & & & & & \mathbf{k} & 0 \\ & \mathbf{0} & & & & & & & \mathbf{k} \end{matrix}\right) \end{array},$$

latt Λ is of finite type for the order

$$\Lambda=\left(\begin{matrix} R & R & R & R & \pi & R & R & R & R \\ & R & \pi & R & \pi & R & R & R & R \\ & & R & R & \pi & \pi & \pi & \pi & R \\ & & & R & \pi & \pi & \pi & \pi & R \\ & & & & R & \pi & R & \pi & R \\ & & & & & R & R & \pi & R \\ & & & & & & R & \pi & R \\ & & & & & & & R & \pi \\ & \pi & & & & & & & R \end{matrix}\right).$$

Acknowledgement

The author expresses his hearty thanks to Professor D. Simson for his many suggestions and encouragements.

References

[1] R. Bautista and D. Simson, Torsionless modules over 1-Gorenstein l-hereditary artinian rings, Comm. in Algebra, to appear.

[2] E.L. Green and I. Reiner, Integral representations and diagrams, Michigan Math. J. 25 (1978) 53-84.

[3] C.M. Ringel, Report on the Brauer-Thrall conjectures, in: Representation Theory I, Lecture Notes in Math. 831 (Springer, Berlin, 1980) 104-136.

[4] C.M. Ringel and K.W. Roggenkamp, Diagrammatic methods in the representation theory of orders, J. Algebra 60 (1979) 11-42.

[5] C.M. Ringel and K.W. Roggenkamp, Socle-determined categories of representations of artinian hereditary tensor algebras, J. Algebra 64 (1980) 249-269.

[6] K.W. Roggenkamp, Indecomposable representations of orders of global dimension two, J. Algebra 64 (1980) 230-248.

[7] D. Simson, Vector space categories, right peak rings and their socle projective modules, to appear.

[8] D. Simson, On methods for the computation of indecomposable modules over artinian rings, in: 28th Reports of Symposiums on Algebra: "Ring Theory and Algebraic Geometry", 26-29 July 1982, University of Chiba (Japan), 143-170.