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Multi-Echelon Supply Chain Management for Deteriorating items with Partial Backordering under Inflationary Environment

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Abstract

As the industrial environment becomes more competitive, supply chain management has become essential part of the industries. In this paper, a multi-echelon inventory model for deteriorating items with partial backlogging has been developed under inflationary environment. The optimal number of deliveries is derived with the minimal joint total cost from the integrated point of view. A numerical example is given to illustrate the model. This paper shows that the integrated approach strategy results in the lowest joint total cost as compared with the independent decision approaches.

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1. Introduction

In the past most studies in supply chain management did not consider the influence of inflation. This was due to the belief that inflation would not influence the inventory policy to any significant degree. This belief is unrealistic since the resource of an enterprise is highly correlated to the return of the investment. The concept of the inflation should be considered especially for long-term investment and forecasting. Sarker et al. [18] developed a supply chain model to determine an optimal ordering policy for deteriorating items under inflation, permissible delay of payment and allowable shortage. Moon and Lee [15] investigated the impact of inflation and unit cost. They considered the normal distribution as a production life cycle and developed a simulation model with probability distribution. Chung and Lin [2] developed inventory models with complete backlogging. Lo et al. [14] developed an

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integrated production and inventory model from the perspectives of both the manufacturer and the retailer with varying rate of deterioration, partial backordering, inflation, imperfect production process and multiple deliveries.

The research in deteriorating inventory is becoming more important. This is because in the real life, decay and deterioration occur in almost all products, such as medicine, fruit and vegetables. Deteriorating inventory models have been widely studied by several authors in recent years. Ghare and Schrader [7] were the first researchers to consider exponentially decaying inventory when the demand is constant. Covert and Philip [5] extended the model to consider deterioration with Weibull distribution. Other authors such as Elsayed and Teresi [6], Kang and Kim [12], and Heng et al. [10] continued to refine the deterioration model. Wee [20] developed a deterministic inventory model with quantity discount, pricing and partial backordering when the product in stock deteriorates with time. Papachristos and Skouri [17] generalized the work of Wee [20] and considered a model where the demand rate is a convex decreasing function of the selling price and the backlogging is a time-dependent function. Wu et al. [21] derived an optimal replenishment policy for non-instantaneous demand and partial backlogging. Lo et al. [14] developed an integrated production and inventory model from the perspectives of both the manufacturer and the retailer with varying rate of deterioration, partial backordering, inflation, imperfect production process and multiple deliveries.

Manufacturers procure raw material from suppliers and process them into finished goods, and sell the finished goods to retailer then to customers. When an item moves through more than one stage before reaching the final customer, it forms a “multi-echelon” inventory system. A large amount of researches on multi-echelon inventory control has appeared in the literature during the last decades. Clark and Scarf [3] were the first to study the two-echelon inventory model. They proved the optimality of a base stock policy for the pure serial inventory system and developed an efficient decomposing method to compute the optimal base stock ordering policy. Sherbrooke [19] considered an ordering policy of two-echelon model for warehouse and retailer. It is assumed that stockouts at the retailers are completely backlogged. Chou [1] showed that an integrated two-stage inventory model for deteriorating items with cooperative strategy results in higher profits. Though above models are useful for real situation, their models are restricted to two stage inventory systems. Therefore, several researchers extended to more general multi-echelon systems. Iida [11] considered a dynamic multi-echelon inventory model with non-stationary demands. As the environment becomes more competitive, supply chain management has become a topic of interests to many authors. In some of the literature, models have been established for optimizing supply chain integrated systems. Goyal [8] first introduced the idea of joint total cost for the buyer and the supplier. Cohen and Lee [4] developed a model for establishing a material requirement policy for all materials at every stage in the integrated supply chain system. Pake and Cohen [16] used stochastic sub-models to calculate the values of the included random variables for analyzing the integrated system. Kim and Ha [13] developed an integrated inventory model with JIT concepts and small lot size to derive the minimal joint total cost. Gyana and Bhaba [9] considered an optimal multi ordering policy for procurement of raw materials. A single manufacturing system was developed to minimize the total inventory cost for both raw materials and finished goods. Yang and Wee [23] developed an integrated deteriorating inventory model for both buyers and vendors; it can be shown that the integrated approach results in an impressive cost-reduction compared with an independent decision by the buyer. However, their approach assumed continuous delivery in order to simplify the model development. Yang and Wee [22] developed a multi-lot-size production and inventory model of deteriorating items with constant production and demand rates. Lo et al. [14] developed an integrated production and inventory model from the perspectives of both the manufacturer and the retailer with varying rate of deterioration, partial backordering, inflation, imperfect production process and multiple deliveries.

In this paper, an integrated inventory model for deteriorating items with shortage starting from supplier in a supply chain in inflationary environment has been developed. The optimal number of deliveries has been derived with the optimal joint total cost.

2. Assumptions and Notations The following assumptions and notation are considered to develop the model:

Assumptions:
1. The demand for the item is known and it is constant.
2. The Production rate is deterministic and constant.
3. The inflation rate ‘r’ is a constant. In reality, the value of ‘r’ is very small.
4. A single supplier, single manufacturer and single buyer are considered.
5. Single item is considered.
6. Production rate is greater than the demand rate.
7. A constant deterioration rate is a fraction of the on-hand inventory, in reality, the value of deterioration rate is small. It deteriorates per unit time and there is no repair or replenishment of the deteriorated inventory during a replenishment cycle.
8. Supplier supplies limited raw materials.
9. Multiple lot-size deliveries per order are considered instead of single delivery per order.
10. The supplier delivers the same lot-size of raw materials each time to the manufacturer.
11. The manufacturer delivers the same lot-size of finished goods each time to buyer.
12. Raw material to finish goods factor is 1:1.

Notations:

\[ T \]: Order cycle
\[ n \]: Number of deliveries per order cycle \( T \)
\[ t \]: Delivery cycle
\[ n^* \]: Optimal delivery number
\[ TC \]: Total cost for buyer
\[ TCB \]: Total cost for buyer
\[ r \]: Constant rate of inflation per unit time
\[ t_i \]: Non-Manufacturing period
\[ TC_M \]: Total cost for manufacturer
\[ TC_{MW} \]: Total cost for manufacturer’s warehouse
\[ i \]: Subscript, \( i=1 \) for the case of without shortage and \( i=2 \) with shortage
\[ q_{B_i}(t') \]: Inventory level of finished goods for a buyer at time \( t' \)
\[ D \]: Demand rate of finished goods for a buyer (constant)
\[ \theta_B \]: Deterioration rate of finished goods for a buyer (constant)
\[ t_B \]: Inventory consumption period for the buyer
\[ W_{B_i} \]: Inventory quantity on hold of finished goods from time 0 to time \( t_B \)
\[ q_B \]: Maximum inventory level of finished goods per receiving for a buyer
\[ A_B \]: Ordering cost of finished goods for a buyer (constant)
\[ F_B \]: Receiving cost of finished goods for a buyer (constant)
\[ H_B \]: Holding cost of finished goods per unit per unit time for a buyer (constant)
\[ P_B \]: Cost of deteriorated unit for a buyer’s finished goods (constant)
\[ B_B \]: Backlog cost of finished goods for a buyer
\[ L_B \]: Lost sale cost of finished goods for a buyer
\[ Q_{M_i}(t') \]: Inventory level of finished goods for a manufacturer at time \( t' \)
\[ \theta_M \]: Deterioration rate of finished goods for a manufacturer (constant)
\[ W_{M_i} \]: Inventory quantity on hold of finished goods from time 0 to time \( t_M \)
\[ q_M \]: Maximum inventory level of finished goods for a manufacturer during \( t_M \)
\[ S_M \]: Set cost for a manufacturer per set up (constant)
\[ F_M \]: Delivery cost of finished goods per delivery for a manufacturer (constant)
\[ H_{MW} \]: Holding cost of finished goods per unit per unit time for a manufacturer
\[ P_M \]: Cost of deteriorated unit for a manufacturer’s finished goods (constant)
\[ B_M \]: Backlog cost of finished goods for a manufacturer
\[ L_{MW} \]: Lost sale cost of finished goods for a manufacturer
\[ Q_{MW_i}(t') \]: Inventory level of raw material for a manufacturer’s warehouse at time \( t' \)
\[ \theta_{MW} \]: Deterioration rate of manufacturer’s raw material (constant)
\[ W_{MW} \]: Inventory quantity on hold of raw materials from time 0 to time \( t_{MW} \)
\[ q_{BW} \]: Quantity of raw materials per delivery from supplier to a manufacturer’s warehouse
\[ F_{MW} \]: Receiving cost of raw material per receiving for a manufacturer
\[ H_{MW} \]: Holding cost of raw materials per unit per unit time for a manufacturer’s warehouse
\[ P_{MW} \]: Cost of deteriorated unit for a manufacturer’s raw material (constant)
\[ Q_S(t') \]: Inventory level of raw material for a supplier at time \( t' \)
\[ \theta_s \]: Deterioration rate of supplier’s raw material (constant)
\[ q_S \]: Quantity of raw materials per delivery from outside vendor to supplier
\[ W_s \]: Inventory quantity on hold of raw materials for supplier from time 0 to time \( t \)
\[ A_s \]: Ordering cost of raw material per order for a supplier
3. Formulation of Mathematical Model

In this paper, an integrated inventory model for deteriorating items among a supplier, manufacturer and a buyer, which form a supply chain, is developed. The inventory model for each partner in the supply chain is developed. The supplier has limited raw materials, which results in that the supplier is not able to meet the manufacturer’s demand. In the supply pipeline, this shortage affects the supply from manufacture to the buyer. In this paper, two inventory model have been discussed; the first is without shortages (subscript i=1) presented in Fig.1 with dotted lines and the second is with shortages (subscript i=2) presented in Fig.1 with solid lines. The order cycle ‘T’ is defined as the time between two orders, and the delivery cycle ‘t’ is defined as the time between two deliveries. In an order cycle, there are multiple delivery ‘n’, then the delivery cycle can be found ‘t=T/n’.

3.1 Inventory Model of a Buyer’s Finished Goods

Finished goods inventory level consumption at any time t’, Q_{Bi}(t’). Inventory level decreases due to satisfy the demand and goods deterioration. Mathematically, this situation can be represents by the following differential equation:

\[
\frac{dQ_{Bi}(t)}{dt} = -D - \theta_B Q_{Bi}(t') \quad 0 \leq t' \leq t_{Bi}
\]  

As shown in Fig.1, the inventory level conditions for buyer are:

\[
Q_{Bi}(0) = q_{Bi}, \quad Q_{Bi}(t_{Bi}) = 0
\]

Where \( t_{Bi} \) is equal to the delivery cycle ‘t’, there is no shortage in this case. When there is a shortage, the inventory level is zero at \( t_{B2} \), which is earlier than ‘t’. On solving equation (1) and using the boundary condition \( Q_{Bi}(t_{Bi}) = 0 \), we get

\[
Q_{Bi}(t') = \frac{D}{\theta_B} \left( e^{\theta_B(t_{Bi}-t')} - 1 \right) \quad \ldots (2)
\]

We have \( q_{Bi} = Q_{Bi}(0) \)

\[
\text{Equation (2) } \Rightarrow q_{Bi} = \frac{D}{\theta_B} \left( e^{\theta_B t_{Bi}} - 1 \right) \quad \ldots (3)
\]

The inventory quantity of finished goods for the buyer from time 0 to \( t_{Bi} \) is:

\[
W_{Bi} = \int_0^{t_{Bi}} Q_{Bi}(t') dt' = \int_0^{t_{Bi}} \frac{D}{\theta_B} \left( e^{\theta_B(t_{Bi}-t')} - 1 \right) dt' = \frac{q_{Bi} e^{\theta_B t_{Bi}} - q_{Bi}}{\theta_B} \quad \ldots (4)
\]

Cost of Placing Order = \( A_B(0) + A_B(t) + A_B(2t) + \cdots + A_B((n-1)t) = A_B \left( \frac{e^{\theta_B t_{Bi}} - 1}{e^{\theta_B t_{Bi}} - 1} \right) \) Cost of receiving of finished goods from manufacture = \( F_B(0) + F_B(t) + F_B(2t) + \cdots + F_B((n-1)t) = \left( \frac{e^{\theta_B t_{Bi}} - 1}{e^{\theta_B t_{Bi}} - 1} \right) \)

Holding Cost = \( H_B(0) + H_B(t) + H_B(2t) + \cdots + H_B((n-1)t) \) \( W_{Bi} = H_B W_{Bi} \left( \frac{e^{\theta_B t_{Bi}} - 1}{e^{\theta_B t_{Bi}} - 1} \right) \)

Deterioration Cost = \( P_B(0) + P_B(t) + P_B(2t) + \cdots + P_B((n-1)t) \) \( (q_{Bi} - D t_{Bi}) = P_B(q_{Bi} - D t_{Bi}) \left( \frac{e^{\theta_B t_{Bi}} - 1}{e^{\theta_B t_{Bi}} - 1} \right) \)

Backlog Cost = \( B_B(0) + B_B(t) + B_B(2t) + \cdots + B_B((n-1)t) \) \( B_B(q_{B1} - q_{B2}) \) \( B_B(q_{B1} - q_{B2}) \left( \frac{e^{\theta_B t_{Bi}} - 1}{e^{\theta_B t_{Bi}} - 1} \right) \)

Lost Sale Cost = \( (L_B(0) + L_B(t) + L_B(2t) + \cdots + L_B((n-1)t)) \) \( (1 - \delta)(q_{B1} - q_{B2}) = L_B((1 - \delta)(q_{B1} - q_{B2}) \left( \frac{e^{\theta_B t_{Bi}} - 1}{e^{\theta_B t_{Bi}} - 1} \right) \)
The total cost for finished goods for the buyer per unit of time can be expressed as the sum of the order cost, receiving cost, holding cost, deterioration cost, backlog cost and lost sale cost.

\[ T_{C_{BI}} = \frac{1}{t}(A_B + F_B + H_B W_{BI} + P_B (q_{BI} - D t_{BI}) + B_B (\delta(q_{B1} - q_{B2})) + L_B ((1 - \delta)(q_{B1} - q_{B2}))(\frac{e^{rt}-1}{r t-1}) \] … (5)

\[ Q_S(t') \]

The model without shortage

Supplier [\[ \text{Raw Materials} \]]

The model with shortage

\[ Q_{MW}(t') \]

Manufacturer’s [\[ \text{Raw Materials} \]]

\[ Q_{Pi}(t') \]

Manufacturer [\[ \text{Finished Goods} \]]

\[ Q_{Bi}(t') \]

Buyer [\[ \text{Finished Goods} \]]

Fig.1 Inventory Level of Supply Chain

Where \((q_{BI} - D t_{BI})\) is the number of deteriorated units, and \(\delta(q_{B1} - q_{B2})\) is the number of backlog units. The last two terms applies only for the case with shortages.

3.2 Inventory Model of a Manufacture’s Finished Goods

The finished goods inventory level for the manufacturer is accumulated for manufacturing but consumed due to deterioration. Mathematically, this situation can be represents by the following differential equation:

\[ \frac{dQ_{Mi}(t')}{dt'} = P - \theta_M Q_{Mi}(t') \quad 0 \leq t' \leq t_{Mi} \] … (6)

As shown in Fig.1, the inventory level conditions for buyer are:

\[ Q_{Mi}(0) = 0, \quad Q_{Mi}(t_{Mi}) = q_{Mi} \]

Where \(t_{Mi}\) is the production period, \(t_{Mi}\) (without shortage) is greater than \(t_{Mi}\) (with shortage), and \(t_0 = t - t_{Mi}\) is the non-production period. On solving equation (6) and using the boundary condition \(Q_{Mi}(0) = 0\), we get
\[
Q_{MI}(t) = \frac{P}{\theta_M} \left(1 - e^{-\theta_M t} \right) 
\]

We have \( q_{MI} = Q_{MI}(t_{MI}). \) Equation (7) \( \Rightarrow \) \( q_{MI} = \frac{P}{\theta_M} \left(1 - e^{-\theta_M t_{MI}} \right) = q_{BI} \) \( \hspace{1cm} \ldots (8) \)

Thus, one can derive period \( t_{MI} \) from equation (8). Thus \( t_{MI} = -\frac{\log((P-q_{BI}/\theta_M)/P)}{\theta_M} \) \( \hspace{1cm} \ldots (9) \)

The inventory quantity of finished goods on hold for a manufacture from time 0 to \( t_{MI} \) is:

\[
W_{MI} = \int_0^{t_{MI}} Q_{MI}(t) \, dt = \int_0^{t_{MI}} \frac{P}{\theta_M} \left(1 - e^{-\theta_M t} \right) \, dt = \frac{P t_{MI} - q_{MI}}{\theta_M} = \frac{P t_{MI} - q_{BI}}{\theta_M} \hspace{1cm} \ldots (10)
\]

Set-up Cost = \( S_M (0) + S_M (t) + S_M (2t) + \ldots \) \( \ldots \) \( \ldots + S_M (n-1)t \) = \( S_M \left( \frac{n^{\ln t - 1}}{e^{\ln t - 1}} \right) \)

Delivery cost of finished goods from manufacture = \( F_M (0) + F_M (t) + F_M (2t) + \ldots + F_M ((n-1)t) = F_M \left( \frac{n^{\ln t - 1}}{e^{\ln t - 1}} \right) \)

Holding Cost = \( (H_M (0) + H_M (t) + H_M (2t) + \ldots + H_M ((n-1)t)) \) \( \hspace{1cm} \) \( W_{MI} = H_M W_{MI} \left( \frac{n^{\ln t - 1}}{e^{\ln t - 1}} \right) \)

Deterioration Cost = \( (P_M (0) + P_M (t) + P_M (2t) + \ldots + P_M ((n-1)t)) \) \( (P t_{MI} - q_{MI}) = P_M (P t_{MI} - q_{MI}) \left( \frac{n^{\ln t - 1}}{e^{\ln t - 1}} \right) \)

Backlog Cost = \( (B_M (0) + B_M (t) + B_M (2t) + \ldots + B_M ((n-1)t)) \) \( \delta (q_{MI} - q_{M2}) = B_M (\delta (q_{MI} - q_{M2})) \left( \frac{n^{\ln t - 1}}{e^{\ln t - 1}} \right) \)

Lost Sales Cost = \( (L_M (0) + L_M (t) + L_M (2t) + \ldots + L_M ((n-1)t)) \) \( (1 - \delta) (q_{M2} - q_{M3}) = L_M ((1 - \delta) (q_{M2} - q_{M3})) \left( \frac{n^{\ln t - 1}}{e^{\ln t - 1}} \right) \)

The total cost for finished goods for the manufacturer per unit of time can be expressed as the sum of the set-up cost, delivering cost, holding cost, deterioration cost, backlog cost and lost sale cost.

\[
TC_{MI} = \frac{(S_M + F_M + H_M W_{MI} + P_M (P t_{MI} - q_{MI}) + B_M (\delta (q_{MI} - q_{M2}) + L_M ((1 - \delta) (q_{M2} - q_{M3}))) \left( \frac{n^{\ln t - 1}}{e^{\ln t - 1}} \right))}{t} \hspace{1cm} \ldots (11)
\]

Where \( (P t_{MI} - q_{MI}) \) is the number of deteriorated units, and \( \delta (q_{MI} - q_{M2}) \) is the number of backlog units. The last two terms applies only for the case with shortages.

3.3 Inventory Model of a Manufacturer’s Raw Materials:

The raw material inventory level for the manufacturer is decreases due to manufacturing of finished goods and due to deterioration. Mathematically, this situation can be represented by the following differential equation:

\[
\frac{dQ_{MRI}(t)}{dt} = -P - \theta_{MR} Q_{MRI}(t) \hspace{1cm} 0 \leq t' \leq t_{MI} \hspace{1cm} \ldots (12)
\]

As shown in Fig.1, the inventory level conditions for buyer are:

\[
Q_{MRI}(0) = q_{MRI}, \hspace{1cm} Q_{MRI}(t_{MI}) = 0
\]

On solving equation (12) and using the boundary condition \( Q_{MRI}(t_{MI}) = 0, \) we get

\[
Q_{MRI}(t) = \frac{P}{\theta_{MR}} \left( e^{\theta_{MR} (t_{MI} - t) - 1} \right) \hspace{1cm} \ldots (13)
\]

We have \( q_{MI} = Q_{MI}(0). \) Equation (13) \( \Rightarrow q_{MRI} = \frac{P}{\theta_{MR}} \left( e^{\theta_{MR} t_{MI}} - 1 \right) \hspace{1cm} \ldots (14) \)

The inventory quantity of raw materials on hold for a manufacture from time 0 to \( t_{MI} \) is:

\[
W_{MRI} = \int_0^{t_{MI}} Q_{MRI}(t) \, dt = \int_0^{t_{MI}} \frac{P}{\theta_{MR}} \left( e^{\theta_{MR} (t_{MI} - t) - 1} \right) \, dt = \frac{q_{MRI} - P t_{MI}}{\theta_{MR}} \hspace{1cm} \ldots (15)
\]

Receiving cost of raw materials from supplier = \( F_{MR} (0) + F_{MR} (t) + F_{MR} (2t) + \ldots + F_{MR} ((n-1)t) = F_{MR} \left( \frac{n^{\ln t - 1}}{e^{\ln t - 1}} \right) \)

Holding Cost = \( (H_{MR} (0) + H_{MR} (t) + H_{MR} (2t) + \ldots + H_{MR} ((n-1)t)) W_{MRI} = H_{MR} W_{MRI} \left( \frac{n^{\ln t - 1}}{e^{\ln t - 1}} \right) \)

Deterioration Cost = \( (P_{MR} (0) + P_{MR} (t) + P_{MR} (2t) + \ldots + P_{MR} ((n-1)t)) (q_{MRI} - P t_{MI}) = P_{MR} (q_{MRI} - P t_{MI}) \left( \frac{n^{\ln t - 1}}{e^{\ln t - 1}} \right) \)
The total cost for raw materials for the manufacturer per unit of time can be expressed as the sum of receiving cost, holding cost, deterioration cost, backlog cost and lost sale cost.

\[ TC_{MRI} = \frac{1}{T} (F_{MR} + H_{MR}W_{MRI} + P_{MR}(q_{MRI} - P_tM_t)) \left( e^{r_{t-1}} \right) \]

\[ \cdots (16) \]

### 3.4 Inventory Model of Supplier’s Raw Materials

In the present model, the supplier satisfies the manufacturer’s warehouse during the period ‘t’. The supplier’s raw materials inventory level is shown in Fig.1. Assume that the supplier’s opening inventory is \( q_{SI} \), ending inventory after time ‘t’ is \( q_{MRI} \), then, according to Ghare and Schrader [7], one can derive the following equation:

\[ q_{SI} = \frac{q_{MRI}}{(1 - \theta_s)^t} \]

\[ \cdots (17) \]

In general, the supplier’s raw materials inventory level at time ‘t’, \( Q_{SI}(t) \) can be expressed as \( Q_{SI}(t) = q_{SI}(1 - \theta_s)^t \). The raw materials inventory quantity on hold for supplier from 0 to time ‘t’, \( W_{SI} \), can be derived as:

\[ W_{SI} = \int_0^t Q_{SI}(t) dt = \frac{q_{MRI} - q_{SI}}{\log(1 - \theta_s)} \]

\[ \cdots (18) \]

**Ordering Cost**

\[ A_S(0) + A_S(t) + A_S(2t) + \cdots + A_S((n-1)t) = A_S \left( e^{r_{t-1}} \right) \]

**Delivery cost of raw materials**

\[ F_S(0) + F_S(t) + F_S(2t) + \cdots + F_S((n-1)t) = F_S \left( e^{r_{t-1}} \right) \]

**Holding Cost**

\[ H_S(0) + H_S(t) + H_S(2t) + \cdots + H_S((n-1)t) W_{SI} = H_S W_{SI} \left( e^{r_{t-1}} \right) \]

**Deterioration Cost**

\[ P_S(0) + P_S(t) + P_S(2t) + \cdots + P_S((n-1)t) (q_{SI} - q_{MRI}) = P_S(q_{SI} - q_{MRI}) \left( e^{r_{t-1}} \right) \]

**Backlog Cost**

\[ B_S(0) + B_S(t) + B_S(2t) + \cdots + B_S((n-1)t) (\delta(q_{SI} - q_{S2})) = B_S(\delta(q_{SI} - q_{S2})) \left( e^{r_{t-1}} \right) \]

**Lost Sale Cost**

\[ L_S(0) + L_S(t) + L_S(2t) + \cdots + L_S((n-1)t) (1 - \delta)(q_{S1} - q_{S2}) = L_S((1 - \delta)(q_{S1} - q_{S2})) \left( e^{r_{t-1}} \right) \]

The total cost for raw materials for the supplier per unit of time can be expressed as the sum of the ordering cost, delivering cost, holding cost, deterioration cost, backlog cost and lost sale cost.

\[ TC_{SI} = \frac{1}{T} \left( A_S + F_S + H_S W_{SI} + P_S(q_{SI} - q_{MRI}) + B_S(\delta(q_{S1} - q_{S2})) + L_S((1 - \delta)(q_{S1} - q_{S2})) \right) \left( e^{r_{t-1}} \right) \]

\[ \cdots (19) \]

Where \( q_{SI} - q_{MRI} \) is the number of deteriorated units, and \( \delta(q_{S1} - q_{S2}) \) is the number of backlog units. The last two terms apply only for the case with shortages.

### 3.5 Integrated Inventory Model

The integrated total cost for the buyer, manufacturer and supplier, TC, is the sum of equation (5), (11), (16) and (19), i.e.,

\[ TC = TC_{BI} + TC_{MI} + TC_{MRI} + TC_{SI} \]

\[ \cdots (20) \]

### 4. Solution Procedure

Now, we describe the solution procedure to find the optimal value of ‘n’ in different case.

#### 4.1 Case of without Shortages

For the case without shortages, the solution procedure is as follows:

1. Given \( t (= T/n) \), \( t_m (= t) \) can be obtained then \( q_{BI} \) can be found from equation (3) for given \( D \) and \( \theta_B \), and \( W_{BI} \) can be found from equation (4).
2. \( TC_{B1} \) can be obtained from equation (5) without the last two terms for given \( A_B, F_B, H_B, P_B, r \).
3. Because \( q_{BI} = q_{MI} \), from equation (9), \( t_1 \) can be found, then \( q_{M1} \) and \( W_{M1} \) can be obtained for \( P \) and \( \theta_M \).
4. \( TC_{M1} \) can be obtained from equation (11) without the last two terms for given \( A_M, F_M, H_M, P_M, r \).
5. \( q_{MRI} \) and \( W_{MRI} \) can be obtained from equation (14) and (15), respectively, for given \( \theta_{MR} \).
6. \( TC_{MRI} \) can be obtained from equation (16) for given \( F_{MR}, H_{MR}, P_{MR}, r \).
7. \( q_{S1} \) and \( W_{S1} \) can be obtained from equation (17) and (18), respectively, for given \( \theta_S \).
8. \( TC_{SI} \) can be obtained from equation (19) without the last two terms for given \( A_S, F_S, H_S, P_S, r \).
9. \( TC_1 \) can be obtained from equation (20).
10. Start with \( n=1 \), go through Steps (1-9).
11. \( n=n+1 \) repeats steps (1-9) until the minimal \( TC_1 \) is found, then the optimal delivery number \( n^* \) can be obtained. Where the optimal \( n^* \) must satisfy the condition:
\[
TC_1(n^* - 1) \geq TC_1(n^*) \leq TC_1(n^* + 1)
\]
Let the supplier’s total raw material order quantity, which satisfies the manufacturer’s warehouse demand form 0 to \( T \), be
\[
Q_{S1} = n q_{S1}
\]  
(21)
If the maximal receiving quantity \( X \) from the vendor to the supplier form time 0 to \( T \) is less than \( Q_{S2} = X \), then there is a shortage occurring along the supply chain.

5. Numerical Illustration
The preceding theory can be illustrated by considering the numerical example, whose parameters are as follows:
Buyer’s Parameters: \( D=12000\) units per week, \( A_B=\$300\) per order, \( F_B=\$25\) per receiving, \( H_B=\$15\) per unit per week, \( P_B=\$110\) per unit, \( L_B=\$55\) per unit, \( \theta_B =0.08\)
Manufacturer’s Parameters: For Finished Goods: \( P=24000\) units per week, \( S_M=\$500\) per set-up, \( F_M=\$25\) per delivery, \( H_M=\$12\) per unit per week, \( P_M=\$90\) per unit, \( L_M=\$45\) per unit, \( \theta_M =0.095\)
For Raw Materials: \( F_{MR}=\$20\) per receiving, \( H_{MR}=\$10\) per unit per week, \( P_{MR}=\$90\) per unit, \( \theta_{MR} =0.09\)
Supplier Parameters: \( X=12000\) per \( T \), \( A_S=\$250\) per order, \( F_S=\$125\) per receiving, \( H_S=\$8\) per unit per week, \( P_S=\$75\) per unit, \( L_M=\$37.5\) per unit, \( \theta_S =0.1\)

Table-1: Optimal Total Cost without Shortage from Various Viewpoints
<table>
<thead>
<tr>
<th>Viewpoint</th>
<th>( n )</th>
<th>( TC_{B1} )</th>
<th>( TC_{M1}+TC_{MR1} )</th>
<th>( TC_{S1} )</th>
<th>( TC_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buyer</td>
<td>21</td>
<td>13830</td>
<td>25592</td>
<td>38</td>
<td>39461</td>
</tr>
<tr>
<td>Manufacturer</td>
<td>25</td>
<td>13956</td>
<td>25359</td>
<td>29</td>
<td>39347</td>
</tr>
<tr>
<td>Supplier</td>
<td>30</td>
<td>14726</td>
<td>26008</td>
<td>19</td>
<td>40754</td>
</tr>
<tr>
<td>Integrated</td>
<td>23</td>
<td>13889</td>
<td>25383</td>
<td>31</td>
<td>39304</td>
</tr>
</tbody>
</table>

Table-1 show the total cost with respect to each optimum for different viewpoints for the case of without shortage. The optimal solution from buyer’s view shows that the manufacturer and the supplier incur an increase in the total cost by \$233 and \$19 compared with their optimum solution. Total cost of the integrated system is increased by \$157. Under the optimum solution from the manufacturer’s view, the buyer and supplier incur an increase in total cost of \$126 and \$10 respectively. Under the optimum solution from the supplier’s view, the buyer and manufacturer incur an increase in total cost of \$896 and \$649 respectively. If we adopt the integrated viewpoint, the buyer, manufacturer and supplier incur an increase in the total cost of \$59, \$24 and \$12 respectively.

Table-2: Optimal Total Cost with Shortage from Various Viewpoints
<table>
<thead>
<tr>
<th>Viewpoint</th>
<th>( TC_{B1} )</th>
<th>( TC_{M1}+TC_{MR1} )</th>
<th>( TC_{S1} )</th>
<th>( TC_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buyer</td>
<td>921686</td>
<td>755777</td>
<td>2978.93</td>
<td>1.68044x10^6</td>
</tr>
<tr>
<td>Manufacturer</td>
<td>921921</td>
<td>755827</td>
<td>3250.23</td>
<td>1.681 x10^6</td>
</tr>
<tr>
<td>Supplier</td>
<td>921635</td>
<td>755891</td>
<td>2734.81</td>
<td>1.68026x10^6</td>
</tr>
<tr>
<td>Integrated</td>
<td>921686</td>
<td>755777</td>
<td>2978.93</td>
<td>1.68044x10^6</td>
</tr>
</tbody>
</table>

Table-2 shows the optimal inventory cost with respect to different point of view when there are shortages.
Fig.2, Fig.3, Fig.4 and Fig.5 shows the effect of partial backlogging rate on buyer’s inventory cost, manufacturer’s inventory cost, supplier’s inventory cost and on supply chain inventory cost.

Table-3: Stockout Quantity of Different Player with respect to No. of Delivery

<table>
<thead>
<tr>
<th>n</th>
<th>q_{B1}</th>
<th>q_{M1}</th>
<th>q_{S1}</th>
<th>q_{B2}</th>
<th>q_{M2}</th>
<th>q_{S2}</th>
<th>Buyer’s Stockout</th>
<th>Manufacturer’s Stockout</th>
<th>Supplier’s Stockout</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4053</td>
<td>4053</td>
<td>645</td>
<td>383</td>
<td>383</td>
<td>484</td>
<td>3670</td>
<td>3670</td>
<td>161</td>
</tr>
<tr>
<td>5</td>
<td>2419</td>
<td>2419</td>
<td>601</td>
<td>233</td>
<td>233</td>
<td>447</td>
<td>2186</td>
<td>2186</td>
<td>154</td>
</tr>
<tr>
<td>7</td>
<td>1724</td>
<td>1724</td>
<td>569</td>
<td>168</td>
<td>168</td>
<td>437</td>
<td>1556</td>
<td>1556</td>
<td>132</td>
</tr>
<tr>
<td>9</td>
<td>1339</td>
<td>1339</td>
<td>512</td>
<td>131</td>
<td>131</td>
<td>400</td>
<td>1208</td>
<td>1208</td>
<td>112</td>
</tr>
<tr>
<td>11</td>
<td>1094</td>
<td>1094</td>
<td>498</td>
<td>107</td>
<td>107</td>
<td>397</td>
<td>987</td>
<td>987</td>
<td>101</td>
</tr>
<tr>
<td>13</td>
<td>925</td>
<td>925</td>
<td>423</td>
<td>91</td>
<td>91</td>
<td>334</td>
<td>834</td>
<td>834</td>
<td>89</td>
</tr>
</tbody>
</table>

Above table reflect the relationship between the number of deliveries and stockout unit of buyer, manufacturer and supplier. The stockout units become lower when the number of deliveries is higher.

Table-4: Comparison Between without Shortage and With Shortage Model’s

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Model without Shortage (i=1)</th>
<th>Model with Shortage (i=2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>23</td>
<td>10</td>
</tr>
<tr>
<td>TC_{B1}</td>
<td>13889</td>
<td>913548</td>
</tr>
<tr>
<td>TC_{M1} + TC_{MR1}</td>
<td>25383</td>
<td>749588</td>
</tr>
<tr>
<td>TC_{S1}</td>
<td>31</td>
<td>2354.56</td>
</tr>
<tr>
<td>TC_{i}</td>
<td>39304</td>
<td>1.66549\times10^6</td>
</tr>
</tbody>
</table>

Table-4 shows two optimal integrated cases. The first is without shortages and the second one is with shortages. All inventory cost for the case with shortages is higher than that without shortages.

Fig. 6 and Fig.7 shows the concavity nature of the total cost function of supply chain with respect to the number of delivery.

6. Conclusion

The present paper incorporates some realistic features that are likely to be associated some types of inventory. These features include deterioration, inflation and partial backlogging. First, deterioration of many items during storage period is a real fact. Next, from a financial point of view, inventory represents a capital investment and must compete with other assets because of a firm’s limited capital funds. Hence, the effect of inflation on the supply chain system cannot be ignored. Finally, in real-life situations shortages at the end of different players of supply chain is very common. As due stiff competition every customers have lot of option to satisfy his/her demands. So partial backlogging is very realistic phenomenon in inventory modelling. As a result, this paper investigated inventory models for deteriorating items in an integrated supply chain considering a shortage starting from the supplier in an inflationary environment. The main purpose of supply chain management is to achieve the optimization of global supply chain. Due to the different conditions of supply chain, an appropriate inventory control is essential. This paper has been proved that the optimal supply chain works out in the integrated view, not in just one particular viewpoint. In this paper, a comparison between models with shortage and without shortage is made. The result shows that the model without shortage can obtain the lowest joint cost.
Reference:
1. Chou TH. Integrated two-stage inventory model for deteriorating items. Master’s Thesis, Chung Yuan Christian University 2000; Taiwan, ROC.