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Why we need reading-glasses before a zimmer-frame

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Abstract

Accommodative loss represents the fastest human biological decrement. Why? To examine this, the shape of the eye-lens during accommodation or its relaxation is analysed with special reference to age-related changes. The capsule is viewed as a force transmitter: the distribution of forces along centripetal capsular arcs and the resulting stresses and strains are calculated. The effect of zonular 'shifts' is considered. Age-related accommodative loss can be modelled successfully if changes in mechanical properties are linked to those in lenticular and capsular shape. © 2000 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Presbyopia is the name given to the decreasing age-related ability of the eye to focus on objects at close quarters. Granted that it is of limited clinical importance because its symptoms are readily relieved with reading glasses, an inquiry into its aetiology is nonetheless of interest because the rate of decline differs from that of some 100 other human biological functions on a statistically significant basis (Pierscionek & Weale, 1995; Weale, 1995, 1997, 1998). Fig. 1 shows a comparison of the loss of accommodation with the age-variations of a number of relevant ocular attributes: the logarithms of the original data is normalised at the age of 10 years. By some measures, the accommodative amplitude diminishes 1.5–2 times faster than e.g. the bone fracture threshold, the concentration of striate myelin, the regenerative power of damaged DNA, and many others (Weale, 1995). In Fig. 1 the accommodative decline, too, is shown to be fast. This conundrum needs resolving: does presbyopia stand fundamentally apart from the rest of human biological functions, or could it be the result of a combination of factors which determine the relatively marked speed with which most of us are driven to using reading glasses?

The approach to this problem rests on biophysical considerations. Many present-day workers consider accommodation and its relaxation to be based on the capsule moulding the lens matrix which it contains. When we wish to focus on a nearby object, accommodation is effected by the contraction of the circular ciliary muscle and the consequent reduction of the tension it transmits to the capsule via the suspensory ligaments or zonule, attached to it near its equator: the lens increases its curvature, thereby increasing its optical power. When accommodation is relaxed, the ciliary muscle pulls on the suspensory ligaments of the lens with the result that the lens flattens.

Earlier experimental studies of the mechanical properties of the lenticular capsule (Fisher, 1969a) and the matrix (Fisher, 1971) were based on such considerations, but did not offer an explicit description of how the capsule is pictured to act. Indeed, explanations are frequently confined to the capsule effecting 'a flattening of the lens' without the *how?* being addressed. The problem has been ably tackled analytically (Koretz & Handelman, 1983) with an emphasis on younger eyes. As in all treatments of this complicated subject, assumptions had to be made, notably as regards the anisotropy or otherwise of the lens (Handelman & Koretz, 1982; Koretz & Handelman, 1986). *Prima facie*, the lenticular fibres join opposing sutures on either side of the equatorial plane of the lens, and are shaped like

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parts of coils with angles between their arms not far off the 45° when projected on a plane perpendicular to the equator: such an angle optimises the efficiency of an applied bending moment (Champion & Davy, 1942). One would therefore expect that lenticular elasticity measured along a radial direction would be in excess of that measured along its principal axis, and this is observed (Fisher, 1971). However, though there is a measurable difference between the variations with age of the two elasticities, it is small, and, since their relative contributions to regions other than the pole and the equator are unknown, it is here assumed that the polar component is dominant.

Additionally, it is of interest to examine the role played by the loci of insertion of the zonule: their distance from the equator has been shown to increase with age (Farnsworth & Shyne, 1979). For this reason

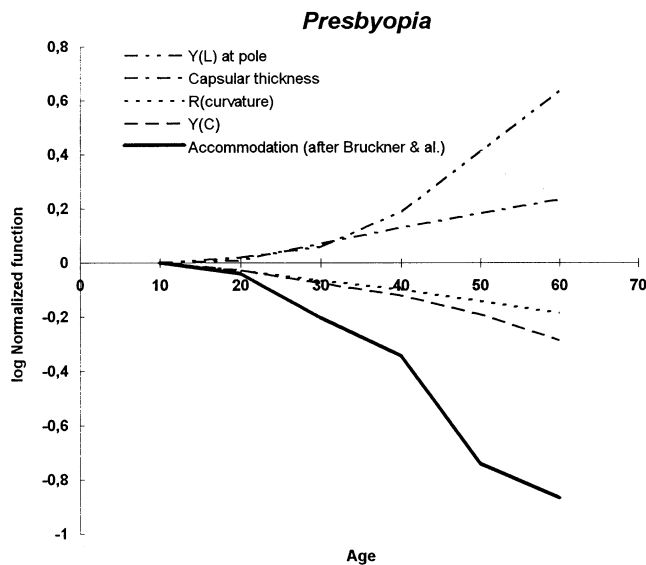


Fig. 1. Comparison of the age-related variations of physical data used in the computation of the accommodative loss with presbyopia (data due to Brückner et al., 1987; $Y(L)$ and $Y(C)$ are Young's moduli of elasticity for the lens matrix and capsule respectively. See also Table 1).

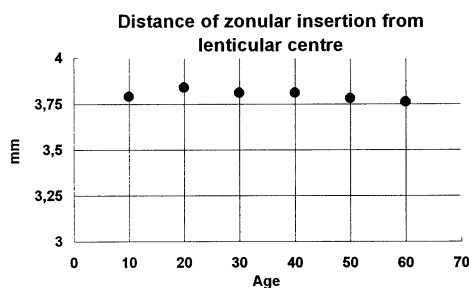


Fig. 2. The distance between the zonular insertion and the lenticular centre as determined from the difference between an exponential fitted to published data for the insertions and averaged data for the transverse lenticular diameter (Farnsworth & Shyne, 1979; Weale, 1982).

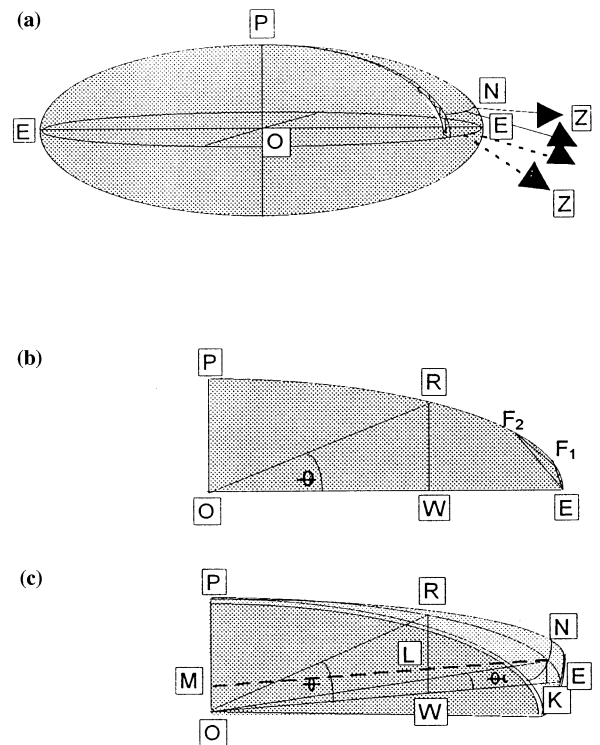


Fig. 3. (a) Illustration of two types of insertion of the zonule, namely at the equator E, and anteriorly at N. (b) A partial section through the anterior part of the lens to illustrate the computation of the force transmission from zonule to capsule. (c) Illustration of the stress/strain variation along a capsular sector.

the phenomenon has been referred to as a zonular shift. However, the notion of a shift suggests movement, and a force causing it is not easy to picture: an alternative has to be considered. To this end, an exponential was fitted to Farnsworth and Shyne's data to obtain a smooth curve. This was subtracted from a mean value of the age-related transverse lenticular diameter (cf. Weale, 1982), yielding the points shown in Fig. 2. This suggests that the distance between the circle of insertion and the lenticular centre is virtually independent of age (Weale, 1992). Thus what shifts, in fact, appears to be the lenticular equator (owing to continual growth). While it is liable to reduce the efficiency in zonular tension (Pierscionek & Weale, 1995), this has not been quantified. The object, then here is also to probe the effect which the growth of the lens beyond the ring of the zonular insertion may have on the age-related variation in the amplitude of accommodation.

2. Method

Fig. 3a–c illustrate the approach followed in the present analysis. Changes in lenticular power take place mainly by alterations in the anterior surface (Fincham, 1937; Fisher, 1971; Brown, 1974). Let the volume be

tween the equatorial plane of the lens and its anterior pole be described for simplicity's sake by a hemi-ellipsoid (Fig. 3a; but see Kasprzak, 2000). It is also convenient to represent this surface in terms of spherical (Fig. 3b), rather than Cartesian or cylindrical, co-ordinates (see Appendix A). Further, let the capsule be pictured as divided into so many narrow sectors, one of which is shown in Fig. 3c.

Now consider what happens to its shape when the ciliary muscle applies a pull via the proximal zonular insertions. The increase in tension is transmitted through the capsule — a force distributor (Koretz & Handelman, 1986) — and the capsular sector will be extended. The strain, i.e. the fractional increase in its arcuate length, increases non-uniformly: where the cross-section is large, as near the insertions, the stress or force per unit area is smaller than where the cross-section is small, i.e. more centrally. Thus the narrower the width of the sector, the greater is the radial elongation. Hence the original shape, which projects onto the equatorial plane as a sector with two rectilinear radial boundaries (Fig. 3c), is going to change into one with biconcave flanks. This will set up in the capsule — all along rings concentric with the equator — tangential elastic forces between neighbouring sectors, which peak at the centre. Within the elastic limit, the pole P can move owing to symmetry only along the principal axis (Fig. 3a). As there is a measurable component parallel to the optic axis of the lens the radial extension is consequently accompanied by an inward force which moulds the lens matrix. The assumption of a uniform capsular thickness, here made implicitly, does not seem to have been tested. Krag, Olsen and Andreassen (1997) report a non-linear relation between stress and strain, but this is a matter for a separate theoretical analysis.

Fig. 3b and c show how the capsular strain, i.e. the fractional change in the radius of curvature dR/R , can be determined when a zonular force is applied or relaxed. Note that, for a given lens, the stress differs depending on whether the insertions run along a circle passing through N or, e.g. along the equator, E (Fig. 3c). The co-ordinates of the two points are given by ON, Θ_c , and OR, Θ , respectively.

Consider two small, neighbouring co-arcuate radial arc fragments on the capsule. The magnitude and direction of the centripetal superficial force in the more peripheral one EF_1 can be represented by a vector (Fig. 3b). The same holds for the more nearly central arclet F_1F_2 . Therefore the resultant at the central end of the latter is determined by the triangle of forces, i.e. by the vector EF_2 joining the point E on the periphery to point F_2 . This applies to every point along the arc up to the pole P. If the insertion were at N (Fig. 3a and c), the resultants would have to be measured from N rather than E.

One can hence determine the components of all the forces that are parallel to the optic axis of the lens, i.e. to PO. As mentioned above, the capsular cross-section decreases systematically: both it and the vertical components of the compressing force vary as functions of the co-ordinate radius OR and the angle θ this includes with the equatorial plane. The stress is the ratio of the vertical component to the cross-sectional area at each point along PE or PN. Given this stress, multiplying it by the experimentally determined ratio of the capsular and lenticular elasticities (Fisher, 1969a, 1971), the strain at each radial point can be found; summing along each meridian, the change in curvature and, after taking into account the refractive index, that of lens power are obtained.

Note that elements which have not been shown to exhibit any significant variation with age, such as Poisson's ratio, the index of lenticular refraction and zonular tension τ (Strenk, Semmlow, Strenk, Munoz, Gronlund-Jacob & DeMarco, 1999), are unlikely to affect age-related trends, and have, therefore, been assumed to be constant and subsumed by the normalisations that follow (cf. Appendix A, Eq. (9)). For example, the physical data used in the analysis are normalised to unity at the age of 10 years (Fig. 1), which facilitates a comparison of age trends.

It will be seen from Eqs. (4) and (6) that the stresses calculated for the pole of the lens assume very large values. For this reason variations with age were determined separately for $\theta = 90^\circ$ and for summed values based on θ s smaller than this in order to check on the presence of any possible discontinuity. It was assumed that the lenticular strain results from the summed action at each point along the arcs shown in Fig. 3b and c. Accordingly the sums for each age of the data calculated for different values of θ (Fig. 3a and c) between its lowest value and 80° were used in the determination of the strains (and hence dF , the amplitude of accommodation). Note that there were two lower limiting angles depending on whether the zonule was deemed to be inserted at the equator E ($\Theta = 0^\circ$) or at N ($\Theta_c = 20^\circ$; data marked F&S). It was reasonable to assume that if there was no significant difference between the results for the age variation at 90° on the one hand and analogous values obtained from the sums for values from the lower limit to 80° on the other, the analysis was valid. The relevant calculation is set out in Appendix A. Comparisons with published data were based on least sums of squares of differences between normalised calculated and experimental results.

3. Results

Fig. 4 shows the data obtained by Brückner, Batschelet and Hugenschmidt (1987) and by Hamasaki,

Ong and Marg (1956) under stringent conditions, found to be representative also of earlier data (cf. survey in Weale, 1990). They are compared with calculated values for the amplitude of accommodation dF for the four parameters set out above, namely the polar and non-polar strains for the two types of insertion shown in Fig. 3a.

Table 2 shows the residues for comparisons with the data due to Brückner et al. (1987): the figures represent root-mean-square values. As mentioned above, the sums of squares of the differences between Brückner et al.'s and the indicated functions were calculated; the tabulated values result from the square root of these sums when divided by six (the number of age groups

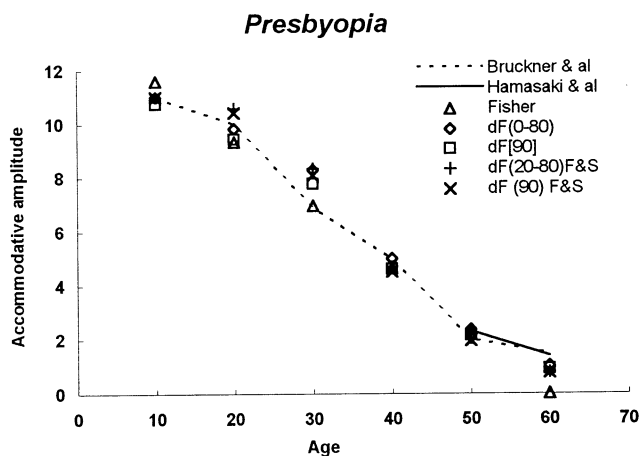


Fig. 4. A comparison between predicted and observed values (Brückner et al., 1987) for the amplitude of accommodation dF (Eq. (12)). $dF(0-80)$ and $dF(90)$ represent data for the non-polar and polar regions, respectively, zonal attachment being assumed to be equatorial; $dF(20-80)F&S$ and $dF(90)F&S$ represent the analogous values for the zonal attachment hypothesised at N in Fig. 3).

Table 1
Variables used in the calculation of the loss in accommodative amplitude dF^a

Age	10	20	30	40	50	60
A	6.2	5.8	5.2	4.7	4	3.2
B	0.7	0.73	0.8	1.08	1.8	3
C	8.96	7.95	6.5	4.35	2.22	1.07
D	11.7	11.9	13.8	15.8	17.8	20
E	4.05	4.16	4.20	4.29	4.37	4.48
F	0.95	1.07	1.19	1.34	1.54	1.79
G	17.2	16.1	14.9	13.7	12.4	11.2
H	19.091	15.014	12.417	10.222	8.052	6.25

^a Column 1 codes for the references at the bottom of the table. Lines A, B, D and G are experimental data, the others are derivations. A, Young's modulus of the capsule $Y(C)$ (Fisher, 1969a); B, Young's modulus of the lens matrix $Y(L)$ (Fisher, 1971); C, the value of ϵ (Eq. (9)); D, capsular thickness (Fisher, 1969a); E, minor radius of ellipsoid (Fig. 3) (cf. Weale, 1982); F, major radius of ellipsoid (cf. Weale, 1982); G, radius of curvature (Brown, 1974); H, k^2 , where $k = F/E$.

Effect of locus of zonular insertion

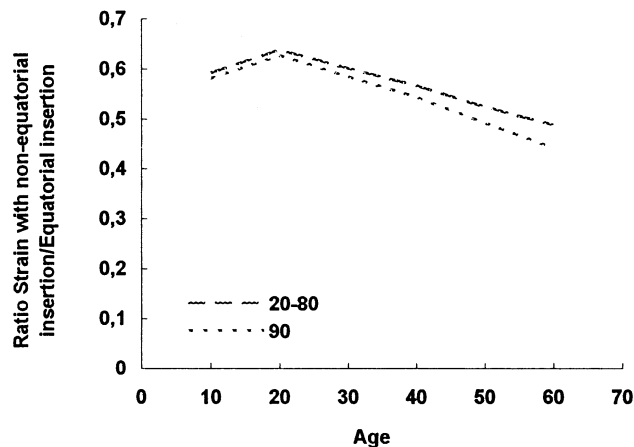


Fig. 5. A comparison between the predicted age-related values of lenticular strain for non-equatorial and equatorial insertions with polar and non-polar strains as a parameter.

10–60 years). There is a marginally better fit with both sets of polar data, perhaps because Fisher's polar elasticities are specially applicable to this region. There is, however, not much to choose between the different variations with age though they all provide a somewhat better fit than do Fisher's (1973) data.

The effect of the locus of the zonular insertion on accommodative decline (Fig. 5) is expressed as the ratio of the strains summed over the capsule for the actual ring of insertion ($\Theta_c = 20^\circ$) to a hypothetical one at the lenticular equator ($\Theta = 0^\circ$). Once again there is support for the view that the polar and non-polar variations of the stress with age are similar. With the insertions kept at a distance from the lenticular centre of -3.75 mm — a radius corresponding to that of the infant lens (Weale, 1982) — there is an approximate halving in strain of what it would be if the insertions remained fixed to the equator.

4. Discussion

I suggested almost 40 years ago that, contrary to what had been believed till then, both the lens capsule and the matrix were elastic, and that the lens shape results from a balance between capsular and lenticular elasticities (Weale, 1963), not quantified till Fisher (1969a,b, 1971) undertook extensive experimental studies, recently elaborated by Krag et al. (1997). These several workers used refined analytical techniques which are capable of yielding an insight into the underlying processes far more readily than is possible from a bulk approach (cf. Glasser & Campbell, 1999). The earlier workers demonstrated that presbyopia was inexplicable in terms of just a single decremental factor. They showed also that the age-long and still persistent

explanation of presbyopia solely in terms of a hardening of the lens is unsustainable: while the accommodative loss is the result of continuous lenticular changes, the elasticity of the lens rises only later in this continuous process. Indeed, even Glasser and Campbell's bulk experiments show that there is no statistically significant resistance of the human lens to compression till after the age of 40 years, when most of the accommodative amplitude available in our childhood has been lost.

The present model provides a somewhat better fit than Fisher's (1973) presumably because he did not use values for the lenticular radius of curvature measured *in vivo* (cf. Table 2). The accommodative decline appears to be a function of the age-related changes of Young's modulus of the capsule and the lens matrix, of the capsular thickness, and of the change in the magnitude of the polar radius of curvature (Brown, 1974). The decrease in the magnitude of this quantity disguises a circumstance which appears to be unrelated to the elastic properties of the lens (cf. below), but, as indicated earlier (Fisher, 1969b), impedes accommodation. As k , the ratio of the principal lenticular axes OE/OP, diminishes with age (cf. Table 1), the force $\tau(v)$ (Eq. (6)) diminishes, thereby rendering ineffectual the ciliary muscle (Strenk et al., 1999) to control the power of the lens. These authors found that the *in vivo* ciliary circle diminishes with age (cf. Fuchs, 1928; Stieve, 1949); but this is more likely to be a consequence than a cause of presbyopia (Weale, 1982) because unused muscles tend to diminish in size owing to disuse atrophy. The structural change in the ciliary muscle notwithstanding Strenk et al. found that its pulling power only slightly diminished with age: in the present study it was assumed to be constant.

It may be mentioned in passing that the accommodative decline is faster in warm than in temperate or cold climates. The physical properties of the lenses may differ. As suggested earlier (Weale, 1992), one cannot

rule out the possibility that the zonular tension τ (Eq. (5)) is subject to a relative ageing factor by virtue of the pupil being more constricted in the tropics than in temperate zones.

The link between the reduction in capsular elasticity and the decrease in the polar radius of curvature is understandable. The lens, deprived of both zonule and capsule, has a smaller radius of curvature than *in situ* (but see Glasser & Campbell, 1999). Like the cornea, it has preserved the transparency originally characteristic of all embryonic tissues. In addition, if not controlled by the elastic capsule, it tends to revert to its erstwhile more nearly spherical shape. This appears to be true also in its more advanced years. Now changes in lenticular refraction occur mainly in the cortex so that it is not surprising that the nuclear hardening, such as it is, fails to play the role conveniently attributed to the whole of the lens not only in the past but also more recently (Glasser & Campbell, 1999). If the nuclear hardness extended to the cortex, the continuous reduction in the polar radius of curvature (Brown, 1974) would be hard to explain.

Three other circumstances are worth noting in this connection. The age-related loss of NIMR signal strength is much more marked in the nucleus than in the cortex (Moffat, Landman, Truscott, Sweeney & Pope, 1999), suggesting a significant difference in changes in hydration between the two lenticular components: the cortical decline is much the weaker of the two. Furthermore, Kasprzak's (2000) model for the lens shows that the iso-indicial surface at the centre of the nucleus appears to be spherical, and that the nucleus is consequently difficult to mould (Fisher, 1969b). Also it has been known since the days of Newton that the density of a substance increases with [the square of] the refractive index. This lends further support to the view that the cortex and nucleus should not be treated as one when their mechanical properties are under consideration.

The tendency to myopia implicit in the above model has been something of a bone of contention (Saunders, 1986; Brown, Phelps, Koretz & Bron, 1999), which can, however, be resolved by Pierscionek's (1993) observations on age-related changes in the lenticular refractive index (cf. Kasprzak, 2000).

Attention is drawn in passing to the apparent coincidence of the radius of zonular insertion with that of the infant lens. It leads to the interesting questions of why, at the age of ~ 1 year, i.e. when accommodation becomes active, the rate of growth of the lens appears to slow down. As mentioned earlier, the insertion phenomenon is referred to as a shift (Farnsworth & Shyne, 1979), no explanation having been given of any hypothetical forces that might cause it. Fig. 2 overcomes this difficulty, and implies also that zonular thickening may counteract the loss in elasticity, keeping the business

Table 2
Minimised dioptric differences between the functions shown in column 1 and Brückner et al.'s results^a

Function	RMS value (dioptries)
Fisher	0.304
d \mathcal{F} (0–80)	0.235
d \mathcal{F} (90)	0.214
d \mathcal{F} (20–80)F&S	0.246
d \mathcal{F} (90)F&S	0.235

^a All the data listed were normalised at Brückner et al.'s value for the age of 10 years, namely 11 dioptries. Then the squares of the differences between each function and Brückner et al.'s results were calculated for each age, their sums minimised, and divided by six to yield the root-mean-square values shown. This provided the best fit. d \mathcal{F} represents the calculated decline in accommodation (cf. Eq. (12)), the figures in brackets the angular range they refer to, and F&S mark the results for non-equatorial insertions.

part of the lens more or less constant in size. Nonetheless, as Fig. 5 suggests, the result of this is that the stress exerted by the capsule decreases relatively to what it would be if the attachment were equatorial. The effect is 35–40% in the young, and progresses somewhat after the age of 20 years.

On this view, the stability of the insertions in relation to the lenticular centre would barely support the opinion that ‘the onset and progressive development of presbyopia...’ is ‘partially attributed to the change in the geometry of the lens suspensory apparatus...’ (Farnsworth & Shyne, 1979), though the resulting change in the *direction* of the suspensory ligaments may play a role (Pierscionek & Weale, 1995).

The reason for the evolution of the insertion phenomenon is open to speculation. The equatorial region is the main site of the proliferative nuclei of active fibres. By virtue of the proximal insertions being fixed on the capsule the equatorial regions would be sheltered from mechanical stresses every time the lens accommodates or relaxes accommodation. This might help to explain the apparent coincidence of the age of the lens when it starts outgrowing the circle of zonular insertion and the onset (Fisher, 1969b) of active accommodation in the infant eye.

Finally, if the model is valid, the conundrum of the relatively rapid accommodative decline, mentioned in Section 1, would appear to be resolved. Fig. 1 shows that presbyopia is not paralleled by any decline in the relevant properties of the eye. However, once combined as shown in Appendix A, a reasonable match with experimental measurements can be achieved (Fig. 4) It is, therefore, admissible to believe on the basis of current evidence that presbyopia is a multifactorial phenomenon. None of the component factors shows a decline faster than other human biological attributes with decrements of 1% or less p.a.: it therefore follows that the rate of development of presbyopia. fits into the broad picture of human senescence (Weale, 1995, 1997), once the peculiar combination of the factors contributing to it is taken into account.

Appendix A. Calculation of the age-related trends of factors contributing to presbyopia

The first objective is to determine the radius of curvature \mathcal{R} at P, the anterior pole of the lens, because this provides a measure of the lenticular power. We proceed in four stages.

A.1. (a) The system of co-ordinates

The Cartesian expression for an ellipse (Fig. 3) is

$$(x/OE)^2 + (y/OP)^2 = 1 \tag{1}$$

where OP and OE are the major and minor axes respectively. Let $OE/OP = k$ and call θ the angle OR encloses with OE, which is a radius in the equatorial plane. Similarly θ_c is the angle enclosed by ON and OE. The appropriate substitutions for x, y in Eq. (1) yield

$$OR/OP = \text{sqrt}[1 + (k^2 - 1) \sin^2 \theta] \tag{2a}$$

Similarly

$$ON/OP = \text{sqrt}[1 + (k^2 - 1) \sin^2 \theta_c] \tag{2b}$$

Thus whereas Eq. (2a) is applied to all angles between 0 and 90°, Eq. (2b) operates only from 20° because the zonule is pictured as inserted at the equator E in the former case, but at N in the latter (Fig. 3a and c).

A.2. (b) Basic force relations

Assume that the suspensory ligaments are effectively inserted along a circle, perpendicular to the plane of the paper. Its radius is MN. Let N be given by the ordinates ON, θ_c , and those of R by OR, θ . It follows from the geometry of Fig. 3c that the ratio of the component force, parallel to the lenticular axis, to the force acting along the capsular arc NR

$$\tau(v)/\tau = LR/NR = (OR \sin \theta - ON \sin \theta_c)/NR \tag{3}$$

because the situation is analogous to the general case depicted in Fig. 3b, and $LR = NL - NR$.

A.3. (c) Calculation of stresses

The age-related thickness of the capsule is t . According to Fig. 3c, to a first approximation, the capsular cross-sectional area α at any level R is

$$\alpha = 2\pi t OR \cos \theta \tag{4}$$

because the capsular circle passing through R has a circumference equal to $2\pi OR \cos \theta$, and the capsular sheet passing through R is equal to the product of this expression and t .

Now

$$(NR)^2 = (RW - NK)^2 - (ON \cos \theta_c - OR \cos \theta)^2$$

i.e.

$$NR = \text{sqrt}\{OR^2 + ON^2 - 2OR ON \cos(\theta - \theta_c)\} \tag{5}$$

Since stress σ is defined by the ratio of force to the area which it is acting over, it follows from Eq. (3)

$$\begin{aligned} \sigma &= -\tau(OR \sin \theta - ON \sin \theta_c)/(2\pi t OR NR \cos \theta) \\ &= -\tau\{(\tan \theta)/2\pi NR t\} \\ &\quad \times \text{sqrt}\{(\sec^2 \theta + k^2 - 1)/(\sec^2 \theta_c + k^2 - 1)\} \end{aligned} \tag{6}$$

After substituting for NR from Eq. (5) in Eq. (6)

$$\begin{aligned}
\sigma = & -\tau \sqrt{1 + (k^2 - 1) \sin^2 \theta} \\
& \times \{(\sin \theta) \sqrt{1 + (k^2 - 1) \sin^2 \theta_c} \\
& - (\sin \theta_c) \sqrt{1 + (k^2 - 1) \sin^2 \theta}\} / (2\pi t \text{ OE } \cos \theta) \\
& \times \sqrt{2 + (k^2 - 1)(\sin^2 \theta_c + \sin^2 \theta)} \\
& - (2 \cos(\theta - \theta_c)) \\
& \times \sqrt{(1 + (k^2 - 1)\sin^2 \theta)(1 + (k^2 - 1)\sin^2 \theta_c)} \}
\end{aligned} \tag{7}$$

The first expression for $\cos \theta$ on the right-hand side of Eq. (7) leads to σ assuming very much larger values for $\theta = 90^\circ$ than for smaller angles. For this reason the *changes with age* were calculated separately for this point, the results showing only small variations from values determined for the ranges of $0-80^\circ$, and $20-80^\circ$, applying to the zonular insertions at E and N, respectively (Fig. 3c). This is shown in Figs. 4 and 5.

We may note in parentheses that k can be expressed in terms of other attributes of the lens. Applying the differential calculus to Eq. (1), it can be shown that

$$k = \mathcal{R}^{0.75} (2\pi/3 V_{0.5})^{0.25} \tag{8}$$

\mathcal{R} is the radius of curvature at P, and $V_{0.5}$ is the half-volume of the notional ellipsoid, and a function of OE and OP. As might be expected, only shape factors of the lens are involved in the value of σ/τ .

A.4. (d) Calculation of strains and changes in lenticular power

Since Young's modulus is defined by $Y = \sigma/\delta$, where δ is the strain, i.e. the fractional change in dimensions, δ is given by σ/Y . $Y(C)$ is the modulus of the capsule. But, as the capsule acts against the elastic lens, δ is proportional to $\varepsilon = Y(C)/Y(L)$, where $Y(L)$ is Young's modulus of the lens matrix (Fisher 1969b, 1971). The change in dimension to be determined is that in the radius of curvature \mathcal{R} . Thus

$$\delta = \varepsilon/\sigma \tag{9}$$

Since the dioptric power of the lens

$$\mathcal{F} = 100\mu/\mathcal{R} \tag{10}$$

the change $d\mathcal{F}$ in the dioptric power of the lens is given by

$$d\mathcal{F} = -100\mu d\mathcal{R}/\mathcal{R}^2 \tag{11}$$

for \mathcal{R} in centimetres, or

$$d\mathcal{F} = \text{constant } \varepsilon(1/\mathcal{R})/\sigma \tag{12}$$

Note that, disregarding the constant, the first term represents mechanical attributes, the second the lenticular radius of curvature, and the third a combination of the lenticular radius of curvature and capsular thick-

ness. As the present study is concerned with changes due to age, all the functions used for rate comparisons were normalised at age 10 years; the constant, therefore, does not need to be evaluated.

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