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Modified Vogel's Approximation Method for the Unbalanced Transportation Problem

NAGRAJ BALAKRISHNAN

A.B. Freeman School of Business, Tulane University

Abstract. Shimshak et al. [2] and Goyal [3] describe modifications to Vogel's Approximation Method for obtaining initial solutions to the unbalanced transportation problem. In this paper, we propose a further modification that could result in better initial solutions.

INTRODUCTION

The most common method used to determine efficient initial solutions for solving the transportation problem (using a modified version of the simplex method) is Vogel's Approximation Method (VAM) [1]. The method involves calculating the penalty (difference between the lowest cost and the second-lowest cost) for each row and column of the cost-matrix, and then assigning the maximum number of units possible to the least-cost cell in the row or column with the largest penalty. In the case of unbalanced transportation problems (i.e., problems where the total supply does not equal the total demand), the transportation simplex method necessitates the creation of a dummy row or column to make the problem balanced. The traditional solution approach assigns zero values to the costs of transporting goods to or from these dummies.

The drawback with this approach is that VAM will usually allocate items to the dummy cells before the other cells in the table. This initial solution may therefore not be very efficient for unbalanced problems. Shimshak et al. [2] propose a modification (SVAM) which ignores any penalty that involves a dummy row or column. For example, if there is a dummy column in the cost-matrix, the penalties are ignored not only for the dummy column, but also for all the rows since the calculation of row-penalties involves the dummy column.

Goyal [3] suggests another modification (GVAM) where the cost of transporting goods to or from a dummy point is set equal to the highest transportation cost in the problem, rather than to zero. Goyal illustrates his procedure using only one example for which VAM, SVAM, and GVAM result in initial solutions with objectives of 1745, 1695, and 1665 respectively. Additional examples, however, show that GVAM does not always perform better than SVAM.

PROPOSED MODIFICATION

In SVAM, only those penalties that do not involve the dummy row or column are computed. Thus if there is a dummy column, the penalties are not computed for that column and for all the rows. This approach, however, has the drawback that the row-penalties are essentially ignored and therefore the cost matrix is not exploited to the fullest extent possible. To avoid this, we propose the following modifications:

(1) Compute all the column penalties as before, except for the dummy column.
(2) For the rows, compute the penalties by calculating the difference between the lowest cost and the second-lowest cost, *ignoring the dummy column.*

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The penalties are now used as usual in the VAM. Each time the penalties have to be recomputed, we use our modified procedure. To illustrate this procedure, we consider the *same example used by Goyal to describe his method.* The solutions obtained by VAM, SVAM, GVAM, and by our proposed method are shown below. The number in the (i, j)th cell represents the cost of shipping one unit from i to j. The numbers in parentheses represent the actual amount shipped on the various routes due to the current solution.

Initial Solution by Vogel's Approximation Method

	1	2	3	Dummy	Supply
A	6	10 (40)	14 (10)	0	50
B	12 (30)	19	21 (20)	0	50
C	15	14	17 (25)	0 (25)	50
Demand	30	40	55	25	

The total cost resulting from this initial solution is 1745.

Initial Solution by Shimshak et al.'s Modified Method

	1	2	3	Dummy	Supply
A	6 (30)	10 (20)	14	0	50
B	12	19	21 (25)	0 (25)	50
C	15	14 (20)	17 (30)	0	50
Demand	30	40	55	25	

The total cost resulting from this initial solution is 1695.

Initial Solution by Goyal's Modified Method

	1	2	3	Dummy	Supply
A	6	10 (40)	14 (10)	21	50
B	12 (30)	19	21	21 (20)	50
C	15	14	17 (45)	21 (5)	50
Demand	30	40	55	25	

The total cost resulting from this initial solution is 1665.

Initial Solution by Proposed Method

	1	2	3	Dummy	Supply
A	6 (5)	10 (40)	14 (5)	0	50
B	12 (25)	19	21	0 (25)	50
C	15	14	17 (50)	0	50
Demand	30	40	55	25	

The total cost of this initial solution is 1650, which is in fact optimal.

To illustrate our procedure further, the initial solutions obtained by VAM, SVAM, GVAM, and our procedure on ten randomly generated unbalanced transportation problems are shown in the following table. In each case, the cost coefficients are uniformly distributed between 1 and 20, while the supply and demand values range uniformly between 1 and 50. The complete details of the ten problems are not shown here for space considerations, and are available from the author.

INITIAL SOLUTIONS OBTAINED BY ALL PROCEDURES

Problem	Rows	Columns	VAM	SVAM	GVAM	Proposed	Optimal
1	3	3	459	426	459	423*	423
2	3	3	366*	614	366*	366*	366
3	3	3	615	524*	560	542	524
4	3	3	178*	311	196	178*	178
5	3	3	300	172*	214	202	172
6	4	4	236	223*	299	223*	223
7	4	4	353*	455	473	353*	348
8	4	4	877	763	786	761*	753
9	4	4	570	485*	546	485*	485
10	4	4	252	226*	228	252	226

* indicates best initial solution.

As seen from the results, our modified approach also does not guarantee to perform better than the other procedures for all unbalanced transportation problems. For the ten problems shown here, our procedure yielded the best initial solution (or tied for best with another procedure) in seven cases. SVAM yielded the best solution in three cases with our procedure providing the second-best solution in two of those cases. However we must note that for any given problem, it is difficult to predict a priori which of the procedures will result in the best solution. In any case, since VAM, SVAM, GVAM, and our procedure all involve extremely simple operations, it should be possible to easily perform all procedures on any problem quickly, and then pick the best solution.

CONCLUSIONS

By using the proposed modification, it may be possible to obtain more efficient initial solutions for the unbalanced transportation problem.

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A.B. Freeman School of Business, Tulane University, New Orleans, LA 70118, U.S.A.