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View FI*m*P miracle (by scale invariance) à la self-interaction

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A B S T R A C T

Combining feebly interacting massive particle (FIMP) dark matter (DM) with scale invariance (SI) leads to extremely light FIMP (thus the FI*m*P) with FI*m*P miracle, i.e., the mass and relic generations of FI*m*P DM share the same dynamics. In this paper we show that due to the lightness of FI*m*P, it, especially for a scalar FI*m*P, can easily accommodate large DM self-interaction. For a fermionic FI*m*P, such as the sterile neutrino, self-interaction additionally requires a mediator which is another FI*m*P, a scalar boson with mass either much lighter or heavier than the FI*m*P DM. DM self-interaction opens a new window to observe FI*m*P (miracle), which does not leave traces in the conventional DM searches. As an example, FI*m*P can account for the offsets between the centroid of DM halo and stars of galaxies recently observed in the galaxy cluster Abel 3827.

1. Introduction and motivation

The conventional weakly interacting massive particle (WIMP) paradigm for dark matter (DM) is being challenged by quite a few null results of direct and indirect DM searches. They are already probing the typical WIMP DM, and yield particularly strong bounds in the lighter DM region. On the other hand, viewing from DM relic density along, WIMP does not take advantage over FIMP, i.e., the feebly interacting massive particle [1,2]. Instead of the freeze-out dynamics of WIMP which is used to keep thermal equilibrium with the plasma, FIMP gains correct relic density $\Omega h^2 \simeq 0.1$ through the freeze-in dynamics. It never enters the plasma but still arrives $\Omega h^2 \simeq 0.1$ via slow thermal productions like thermal particles decay. An obvious merit of FIMP is that it, just as expected, leaves null results in the conventional DM detectors devised for WIMP DM. Moreover, as a competitor to the WIMP miracle, a miracle of FIMP can arise from the combination of FIMP and scale invariance (SI) [3]. This classical symmetry may provide a way to address the hierarchy problem [4].

FIMP with SI is a nontrivial combination, which gives rise to several important consequences. Immediately, quite generically FIMP must be extremely light (thus dubbed FI*m*P). The point is simple. In most of the SI schemes for generating the electroweak (EW) scale, spontaneously breaking of SI happens at the electroweak (EW) scale [5] or TeV scale [6], by means of a scalar field collectively denoted as Φ with vacuum expectation value (VEV) $u \equiv \langle \Phi \rangle \lesssim$ TeV. By virtue of SI, all particles including DM X should gain masses via coupling to scalar fields with non-vanishing VEV, for instance, to Φ. Schematically, we can write down terms for mass generation

$$\frac{1}{2}\lambda_\phi X^2 \Phi^2, \quad \frac{1}{2} y_\phi X^2 \Phi, \tag{1}$$

with X assumed to be a real scalar and Majorana fermion, respectively. Because X is a FIMP, one has $\lambda_\phi, y_\phi \ll 1$ thus very light FIMP. The ensuing important consequence is the aforementioned FI*m*P miracle, which now becomes obvious: (due to SI) the mass and relic generations of FI*m*P share the common dynamics. The final consequence is that dark parity by hand may be not necessary for FI*m*P. It can be sufficiently long-lived even without an exact protective symmetry, because its decay width is greatly suppressed by light mass and moreover feeble couplings.

FIMP barely produces observable signatures except for a decaying one [7] (but may leave hint in tensor-to-scalar ratio [8]). However, FI*m*P does. First, since FI*m*P, such as a sterile neutrino or Majoron [9], does not require a parity, it is well expected that it can decay into X-ray photon(s).[1] Second, also the core of this paper, FI*m*P is likely to have appreciable self-interaction[2] partially by virtue of its lightness; hence, that kind of FI*m*P can be probed

[1] Neutrino and photon are almost massless particles in the SM, so probably they are the only kinematically accessible final states for FI*m*P decay. Neutrino is hard to observe while X-ray line is a good observable.

[2] DM self-interaction was originally motivated to address the small scale problems [10].

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through its astrophysical effects, e.g., leading to a separation between the DM halo and stars of a galaxy which is moving through a region with large DM number density. Interestingly, such a phenomenon was reported recently [11]. It was discovered in the galaxy cluster Abell 3827, within which (in the inner 10 kpc core) four elliptical galaxies were observed and their DM halos were reconstructed, finding at least one spatially offset from its stars, by a distance of $\Delta = 1.62^{+0.47}_{-0.49}$ kpc [11]. Such kind of offset can be explained by DM with self-interaction that leads to DM self-scattering rate per mass $\sigma_{\rm DM}/m_{\rm DM} \sim (1.0\text{–}1.5)\ {\rm cm}^2/{\rm g}$ [12], or in the particle physics unit:

$$\sigma_{\rm DM}/m_{\rm DM} \sim (4.7 - 7.0) \times 10^3\ {\rm GeV}^{-3}. \tag{2}$$

Noticeably, despite of a mild tension with the upper bound set by the bullet galaxy cluster, this value is also indicated to solve the small scale problems [13]. Therefore, it is of great interest to explore FI*m*P with self-interactions (see other attempts [14–16]).

In the absence of velocity dependence, Eq. (2) is saying that the mass scale involved in DM scattering, mass of a mediator or DM itself, should be far below the weak scale. Actually, for a typical WIMP, $\sigma_{\rm DM}/m$ is expected to scale as

$$\frac{\sigma_{\rm DM}}{m} \sim \frac{1}{32\pi}\frac{\lambda^2}{m^3} \simeq 10^{-10} \times \left(\frac{\lambda}{0.1}\right)^2 \left(\frac{100\ {\rm GeV}}{m}\right)^3 {\rm GeV}^{-3}, \tag{3}$$

which is about 14 orders of magnitude smaller than the tentative value given by Eq. (2). By contrast, if the dark sector mass scale m is at the MeV scale, we can easily get the correct order. But it immediately raises two questions: Is there any theoretical motivation for that light DM scale? And for that light DM how does it get correct relic density? Bare in mind that in the SM maybe only the photon and neutrino can be the final states of MeV scale DM annihilating, thus the second question concerns us.

For our FI*m*P with SI, the two questions are simultaneously addressed in a coherent way. In the light of FI*m*P miracle, interactions from Eq. (1) are supposed to freeze-in dark matter. Concretely, it is the two-body decay $\Phi \to XX$ that dominates the freeze-in processes. Here Φ is in the thermal plasma while X, assumed to have negligible initial yield, stays outside the plasma, and as a consequence there is no appreciable inverse process $XX \to \Phi$. The final yield via freeze-in is proportional to the decay width $\Gamma(\Phi \to XX)$ [2,7]:

$$Y_X(\infty) \approx \frac{45\, g_\Phi}{1.66\pi^4 g_*^S\sqrt{g_*^\rho}} \frac{\Gamma(7/2)\Gamma(5/2)}{16} \frac{M_{\rm Pl}}{m_\Phi^2}\Gamma(\Phi \to XX), \tag{4}$$

with g_Φ and m_Φ the internal degrees of freedom and mass of Φ, respectively. The gamma functions $\Gamma(7/2) = 15\sqrt{\pi}/8$ and $\Gamma(5/2) = 3\sqrt{\pi}/4$. The parameters g_*^S (g_*^ρ) are the effective number of degrees of freedom contributing to the entropy (energy) density at $T \simeq m_\Phi$. Within the SM $g_*^S \approx g_*^\rho \approx 106$. For multi mother particles contributing to freeze-in X, there is a summation over Φ. Eventually, with $Y_X(\infty)$ one can express the FI*m*P relic density as

$$\Omega_X h^2 = 2.82 \times 10^5 \left(\frac{m_{\rm DM}}{\rm MeV}\right) Y_X(\infty). \tag{5}$$

In the ideal FI*m*P miracle case, it is proportional to $\lambda_\phi^{5/2}$ and y_ϕ^3, for a real scalar and fermionic FI*m*P respectively. For the typical scale of u and m_Φ, which are not far from the TeV scale, $\Omega_X h^2 \simeq 0.1$ uniquely determines the feeble coupling λ_ϕ (y_ϕ). In the following two subsections, we will detail the scalar and fermionic FI*m*P with large self-interactions.

2. The scalar FI*m*P

A scalar FI*m*P $X = S$ can be easily realized in the scale invariant SM where only the Higgs doublet H radiatively obtains VEV.[3] The resulting model embodies the ideal FI*m*P miracle. The relevant part of the model is very simple (see relevant studies of real scalar as a FI*m*P without SI [19]):

$$\frac{\lambda_{sh}}{2}S^2|H|^2 + \frac{\lambda}{4}S^4. \tag{6}$$

Note that here S is automatically stable as a consequence of SI [17]. This model leads to $u = v = 246$ GeV. After EW spontaneously breaking (EWSB), the first term solely determines DM mass and relic density. The FI*m*P mass is $m_S = \sqrt{\lambda_{sh}/2}\,v$. As long as DM is lighter below $m_h/2$, freeze-in via the Higgs portal will be dominated by the Higgs decay instead of pair production with Higgs in the s-channel. The reason is ascribed to the fact that, the former involves a smaller number of couplings and is not more Boltzmann suppressed than the annihilation processes [18]. The decay width of $h \to SS$ is

$$\Gamma(h \to SS) = \frac{1}{32\pi}\frac{\lambda_{sh}^2 v^2}{m_h}. \tag{7}$$

Then, in terms of Eq. (4) the relic density is estimated to be

$$\Omega_X h^2 \simeq 0.12 \times \left(\frac{\lambda_{sh}}{10^{-10.5}}\right)^{5/2} \left(\frac{v/m_h}{2.0}\right)^3 \left(\frac{10^3}{g_*^S\sqrt{g_*^\rho}}\right). \tag{8}$$

Equating $\Omega_X h^2$ with 0.11 one can fix the unique free parameter λ_{sh} and hence the mass of FI*m*P, $m_S = 1.0$ MeV. Actually, this result was already obtained in the Appendix of our earlier work [3].

In most cases, the coupling constant λ plays no roles in DM phenomenologies. However, this ignored parameter can readily generate a large DM self-interaction in the FI*m*P scenario under consideration. The resulting FI*m*P self-scattering rate in per unit DM mass is

$$\frac{\sigma_{\rm DM}}{m_S} = \frac{1}{128\pi}\frac{\lambda^2}{m_S^3} \simeq 7.9 \times 10^3 \left(\frac{\rm MeV}{m_S}\right)^3 \left(\frac{\lambda}{0.1}\right)^2 {\rm GeV}^{-3}. \tag{9}$$

The scattering is from the contact interaction of S, so the scattering rate only involves the DM scale. Given a light DM scale, the self-scattering rate easily becomes large as long as λ is not very small. This generic advantage of light scalar dark matters, which always allow a quartic self-coupling to generate significant self-interactions, has already been utilized in the early studies [21]. We would like to stress again, not only the light DM scale but also correct DM relic density, which is fairly problematic for the MeV scale DM, are naturally and coherently achieved here by virtue of the FI*m*P miracle.

3. The self-interacting fermionic FI*m*P

Although this example does not give an ideal FI*m*P miracle, it takes a theoretical advantage, i.e., a Majorana FI*m*P DM candidate $X = N$ is naturally predicted rather than introduced in the very low scale seesaw mechanism [22]. The scale invariant version of this model shows several merits [3]. Scalar singlets S_i with non-vanishing VEVs are necessary ingredients of the model, to generate Majorana mass for N. These singlets are also badly needed to implement hidden SI spontaneously breaking. At the same time, they

[3] It is well known that this model fails in triggering successful electroweak spontaneously breaking and then modifications are indispensable. But this is not of our concern here [5,6,20]. Our discussion is particularly suited for the modification where additional bosonic states are introduced to overcome the top quark.

alleviate the serious relic density problem of sterile neutrino DM through the freezing-in mechanism, admitting the FI*m*P miracle (not ideal, see reasons later).

Before heading toward the explanation to the not ideal miracle, here we introduce the Lagrangian and report a few relevant conclusions. We refer to Ref. [3] for more details. Without imposing any symmetry by hand, the relevant Lagrangian takes a form of

$$-\mathcal{L}_N = V(S_i, H) + y_N \bar{l} H N + \frac{\lambda_i}{2} S_i N^2, \tag{10}$$

$V(S_i, H)$ is a generic scalar potential for the singlets and Higgs doublet, and the concrete form containing two singlets can be found in Ref. [3]. In practice, at least two singlets (here we use the minimal number) are needed to trigger EWSB and moreover accommodate a quite SM-like Higgs boson near 125 GeV [23]. For later use, we denote these two singlets as σ and J. Now, there are three Higgs bosons from the Higgs sector. The additional two Higgs bosons are singlet-like and labeled as H_2 and $\mathcal{P}$ (in the mass eigenstates), with the latter (also the lighter) one being the pseudo Goldstone boson of SI spontaneously breaking.

The FI*m*P miracle is not as ideal as that of the scalar FI*m*P, mainly owing to the presence of multi singlets with VEVs. They provide multi sources for generations of FI*m*P mass and relic, whose numerical correlation hence is weakened. Even then, the miracle still holds in the sense of order of magnitude. In light of Ref. [3], decays $H_a \to NN$ freeze-in DM and the relic density can be parameterized as

$$\Omega_{\rm DM}h^2 = 0.11 \times \sum_{H_a=\mathcal{P},H_2} \left(\frac{f_{H_a}^2}{1.0}\right)\left(\frac{m_{\rm DM}}{0.1\ {\rm MeV}}\right)^3 \left(\frac{10\ {\rm TeV}}{v_J}\right)^2 \times \left(\frac{1000\ {\rm GeV}}{m_{H_a}}\right)\left(\frac{10^3}{g_*^S\sqrt{g_*^\rho}}\right), \tag{11}$$

where $m_{\rm DM} = M_N = \sum_i \lambda_i \langle S_i\rangle$ and v_J is the VEV of singlet J. Compared to Eq. (8), the extra parameters f_{H_a} (also the undetermined masses m_{H_a}) manifest the deviation from the ideal FI*m*P miracle. They are model dependent, on the patters of singlets VEVs and as well their coupling to N. But in most cases they are order one numbers. Given that the singlets VEVs are not far above the TeV scale, the fermionic FI*m*P is favored to be much lighter than the scalar FI*m*P. Typically it is around the sub-MeV scale or even below. The reason is nothing but that the fermion mass is proportional to the coupling constant y_ϕ instead of its square root like the scalar FI*m*P. To maintain the coldness of the FI*m*P DM, we take a conservative value $M_N = 0.1$ MeV in this paper. Probably, it can be lowered down substantially, on account of a mildly colder DM spectrum from freeze-in [3,24].[4]

Now we turn our attention to self-interaction of N. Unlike the scalar FI*m*P, large self-interactions are not a built-in part of the fermionic FI*m*P. It calls for a light mediator, either a vector or scalar boson [25]. Viewing from our Lagrangian Eq. (10), a light scalar boson, denoted as S_0, is a natural choice. S_0 is also a FI*m*P, but it has a sizable coupling to N via the Yukawa coupling $\mathcal{L}_{S_0} \supset -\lambda S_0 \bar{N} N$ (in four-component). With it, N can scatter with each other through $s/t/u$-channel exchanging S_0. To get a sufficiently large scattering rate, two options are of interest here. One is a heavy S_0 with mass m_{S_0} much larger than M_N and the other one is the opposite. In what follows we discuss them case by case.

3.1. Dark force

If S_0 is very light, it becomes a dark force mediator and DM self-scattering, in the non-relativistic limit, is described by the following attractive Yukawa potential

$$V = -\frac{\alpha_\lambda}{r} e^{-m_{S_0} r}. \tag{12}$$

with $\alpha_\lambda \equiv \lambda^2/4\pi$. In different parameter space spanned by (α_d, M_N, m_{S_0}), the potential may induce different velocity-dependent DM self-scattering, and we refer to Ref. [13] for a comprehensive discussion. Here we focus on the simplest case, i.e., $\alpha_\lambda M_N/m_{S_0} \ll 1$ such that the Born approximation holds. Then, the perturbative computation in α_λ from V leads to [13]

$$\sigma_T^{\rm Born} = \frac{8\pi\alpha_\lambda^2}{M_N^2 v^4}\left[\log\left(1+\xi_v^2\right) - \frac{\xi_v^2}{1+\xi_v^2}\right], \tag{13}$$

with $\xi_v \equiv M_N v/m_{S_0}$. For a small M_N/m_{S_0} such that $\xi_v \ll 1$ one actually gets the velocity-independent approximation $\sigma_T^{\rm Born} \approx 4\pi\alpha_\lambda^2 M_N^2/m_{S_0}^4$. If the self-interaction could leave observable effect at the cluster scale without spoiling the small scale structures, we should work in this limit.[5] As an estimation, we typically need parameters as

$$\frac{\sigma_T^{\rm Born}}{M_N} = 7.9\times 10^3 \left(\frac{M_N}{0.1\ {\rm MeV}}\right)\left(\frac{0.002\ {\rm MeV}}{m_{S_0}}\right)^4 \times \left(\frac{\alpha_\lambda}{10^{-8}}\right)^2 {\rm GeV}^{-3}. \tag{14}$$

It gives $\xi_v = 50v$, which is indeed a small number for the typical velocity $v < 10^{-3}$.

We make a comment on the fate of S_0. It is at the keV scale, and is assumed to gain mass as the scalar FI*m*P in Eq. (6). At leading order, it can decay into a pair of neutrinos, induced by the tiny active-sterile neutrino mixing, $\lesssim 10^{-10}$. The resulting decay lifetime $\gtrsim 10^{24}$ s is much longer than the cosmological timescale, so it survives as a relic today. But its energy fraction is negligible due to its lightness and small yield during freeze-in.

3.2. Four-fermion interaction

The other option is a heavy S_0. It leads to contact four-fermion interaction, but here we make a direct calculation of the self-scattering cross section without integrating out S_0. The scattering process receives all $s/t/u$-channel contributions, and from them we get the following scattering rate per DM mass

$$\frac{\sigma_{\rm DM}}{M_N} = \frac{3\lambda^4}{8\pi}\frac{M_N}{m_{S_0}^4} \simeq 6.0\times 10^3 \left(\frac{M_N}{0.1\ {\rm MeV}}\right)\left(\frac{\rm MeV}{m_{S_0}}\right)^4 \left(\frac{\lambda}{0.15}\right)^4 {\rm GeV}^{-3}. \tag{15}$$

S_0 is not favored to be much heavier than the MeV scale due to two reasons. One is that λ will become accordingly large, even larger than order 1, which is unpleasant at low energy. The other one is to prevent it from spoiling the FI*m*P miracle. Here S_0 is similar to the frozen-in scalar considered in Ref. [26], because itself is a FI*m*P and moreover could produce N via decay $S_0 \to NN$. Therefore, the heavier S_0 means the larger yield of S_0 thus larger contribution to N production. Let us estimate this contribution using Eq. (8) which just parameterizes the relic density of FI*m*P S

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[4] Despite beyond the scope of this paper, it is of interest to investigate the cosmological implications of warm DM with self-interaction. As shown here, a fermionic FI*m*P with miracle tends to be a warm DM.

[5] Otherwise, self-scattering is over enhanced in the small scale system like dwarf galaxy with characteristic $v \sim 10$ km/s but is insufficient in the large scale system like cluster where $v \sim 1000$ km/s.

with mass 1 MeV. Roughly, one can get the N relic density inherited from S_0 decay by multiplying Eq. (8) a factor $M_N/m_{S_0} \sim 0.1$. Thus, this contribution to the final relic density of N is subdominant and the FImP miracle is not significantly affected. However, it will quickly dominate over the direct freeze-in production of N as S_0 becomes heavier.[6]

4. Conclusion

FI*m*P is a necessary result after the combination between FI*m*P and SI, which further creates a FI*m*P miracle. We show that a large DM self-interaction can be easily accommodated for FI*m*P due to its lightness. This is particularly true for a scalar FI*m*P which always has a quartic self-coupling. While for a fermionic FI*m*P one has to introduce a mediator which is another FI*m*P, a scalar boson with mass either much lighter or heavier than the FI*m*P DM. DM self-scattering opens a new window to observe FI*m*P (miracle), which does not leave traces in the conventional DM searches. For instance, they are potential to explain the recently observed DM self-interaction in the galaxy cluster Abel 3827.

In the late stage of this paper, we found that Ref. [1] basically already studied the model of SM extended by a real scalar FI*m*P, aiming at solving small scale problems using the self-interaction of FI*m*P; moreover, it pointed out that the MeV mass scale is consistent a model with zero bare Higgs mass, which is nothing but the classical scale invariance in our FI*m*P framework.

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