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Path integral formulation with deformed antibracket

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ABSTRACT

We propose how to incorporate the Leites–Shchepochkina–Konstein–Tyutin deformed antibracket into the quantum field–antifield formalism.

1. Introduction

The concept of deformations in the field–antifield formalism [1–3] based on a nilpotent higher-order Δ_* operator was developed in a series of papers [4–12]. Such deformations typically modify the Jacobi identity with BRST-exact terms. In contrast, in this Letter we shall only discuss *local* deformations of the antibracket with a Grassmann-even deformation parameter such that the Jacobi identity holds strongly, and without assuming an underlying Δ_* operator a priori. Recently [13–15], a non-trivially deformed antibracket

$$(f,g)_* := (f,g) + (-1)^{\varepsilon_f}\left(\frac{\kappa c(\kappa)}{1+\frac{\kappa c(\kappa)}{2}N}\Delta f\right)\cdot\left(1-\frac{N}{2}\right)g + \left(\left(1-\frac{N}{2}\right)f\right)\cdot\frac{\kappa c(\kappa)}{1+\frac{\kappa c(\kappa)}{2}N}\Delta g, \tag{1.1}$$

for functions f, g of finitely many variables z^A was constructed inside various algebras $\mathcal{A}$ (e.g., polynomial algebra, algebra of smooth functions with compact support, etc.). Here κ is a deformation parameter; $c(\kappa) = \sum_{k=0}^{\infty} c_k \kappa^k$ is an arbitrary formal power series in κ; and $N := z^A \partial/\partial z^A$ is the Euler/conformal vector field. Moreover, it was shown [14] that this deformed antibracket (1.1) is *unique* modulo trivial deformations and reparameterizations of the deformation parameter κ. Thus, it is expected to play a central rôle.

In this Letter, we propose how to incorporate the non-trivially[1] deformed antibracket (1.1) into the quantum field–antifield formalism [1–3]. Concretely, we suggest a κ-deformed odd Laplacian; quantum master action $W = S + \mathcal{O}(\hbar)$; quantum master equation; and partition function $\widetilde{\mathcal{Z}}$ such that the classical master equation is given in terms of the above κ-deformed antibracket

$$(S,S)_* = 0; \tag{1.2}$$

the classical BRST symmetry is $s = (S,\cdot)_*$; and the partition function $\widetilde{\mathcal{Z}}$ is formally independent of the gauge-fixing X.

How would a κ-deformation be realized in practice? Firstly, we stress that field theory implies infinitely many z^A-variables, so that both the Euler vector field N and the odd Laplacian Δ would need regularization. Nevertheless, it is reasonable to assume that the naive finite-dimensional N-deformation (1.1) still serves as a model of what to come in field theory. Secondly, we note that the traditional field–antifield approach [1–3] (where one starts from a classical action, which is independent of ghosts and antifields, and one introduces ghosts

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[1] A trivial deformation $(f,g)_* = T^{-1}(Tf, Tg)$ of the antibracket with $T = 1 + \mathcal{O}(\kappa)$ amounts to a trivial deformation $\Delta_* = T^{-1}\Delta T$ and $f * g = T^{-1}(Tf \cdot Tg)$ of the underlying BV algebra $(\mathcal{A}; \Delta_*; *)$.

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and antifields as generators of gauge- and BRST-symmetry, respectively) is *not* expected to produce a κ-deformation, as the antibracket traditionally remains on Darboux form. Rather, a relevant physical system should have an antisymplectic phase space built in from the beginning, like, e.g., closed string field theory [16], or generalized Poisson sigma models [17–19]. It is believed that the κ-deformation here could be caused by a choice of regularization scheme that manifestly preserves the Jacobi identity.

The new construction is motivated by two key ideas, which may be symbolized with the introduction of a Bosonic and Fermionic variable, t and θ, respectively, with collective notation $\tau := \{t;\theta\}$. Mathematically, they are, in fact, intimately tied to Lie cohomology theory. We will only here sketch the Lie cohomology argument, and defer a more detailed explanation to an accompanying paper [20]. Recall that the ambiguity/uniqueness of deformations of a Lie-bracket is measured by the second Lie cohomology group, while the first Lie cohomology group classifies outer (= non-Hamiltonian) Lie algebra derivations. Konstein and Tyutin have calculated [14] the first and second Lie cohomology group for the constant, non-degenerated antibracket $(\cdot,\cdot)$. The first Lie cohomology group is two-dimensional, and, in detail, it is generated by the odd Laplacian Δ and the affine operator $N-2$. The second Lie cohomology group is two-dimensional as well, and, in accordance with the Künneth formula, it is generated by all possible non-zero[2] cup product combinations of the first cohomology. These are $\Delta\cup(N-2)=(N-2)\cup\Delta$ and $(N-2)\cup(N-2)$, which lead to two deformed antibrackets, with an even and an odd deformation parameter, respectively, where we here will only consider the former. The first key idea is to *suspend* the algebra $\mathcal{A}$ by introducing a suspension parameter t to turn the affine operator $N-2$ into a genuine vector field $N_\tau = N + t\partial/\partial t$, which satisfies the Leibniz rule. The non-triviality of the N_τ vector field in the $\{z^A;t\}$ space means that it is *not* a Hamiltonian vector field. The second key idea is to complement the $\{z^A;t\}$ space with an antisymplectic partner θ, in such a way, that θ becomes (minus) the Hamiltonian generator for the vector field $N_\tau = -(\theta,\cdot)_\tau$, and hence, so that the vector field N_τ becomes trivial, and, in turn, it makes the corresponding $(t;\theta)$-extended deformed antibracket $(\cdot,\cdot)_{\tau*}$ trivial.

2. Basic setting: constant non-degenerate antibracket

Let $\mathcal{A} := C[[z]]$ be the algebra of formal power series $f = f(z)$ in $2n$ variables z^A of Grassmann parity $\varepsilon(z^A)\equiv\varepsilon_A$, equipped with a constant, non-degenerate antibracket $E^{AB}=(z^A,z^B)$ with Grassmann parity $\varepsilon(E^{AB})=\varepsilon_A+1+\varepsilon_B$ corresponding to the odd Laplacian

$$\Delta := \frac{(-1)^{\varepsilon_A}}{2}\frac{\overrightarrow{\partial^\ell}}{\partial z^A}E^{AB}\frac{\overrightarrow{\partial^\ell}}{\partial z^B}, \qquad \Delta^2=0, \qquad \varepsilon(\Delta)=1. \tag{2.1}$$

The antibracket

$$(f,g) := (-1)^{\varepsilon_f}\big[[\overrightarrow{\Delta},f],g\big]1 = -(-1)^{(\varepsilon_f+1)(\varepsilon_g+1)}(g,f), \quad f,g\in\mathcal{A}, \tag{2.2}$$

satisfies skewsymmetry (2.2), the Jacobi identity

$$\sum_{\text{cycl. } f,g,h}(-1)^{(\varepsilon_f+1)(\varepsilon_h+1)}\big(f,(g,h)\big)=0, \quad f,g,h\in\mathcal{A}, \tag{2.3}$$

and the Leibniz rule/Poisson property

$$(fg,h)=f(g,h)+(-1)^{\varepsilon_f\varepsilon_g}g(f,h), \quad f,g,h\in\mathcal{A}. \tag{2.4}$$

3. Non-trivially deformed algebra $\mathcal{A}$

We will from now on use the simplifying convention that the power series from Eq. (1.1) is $c(\kappa)=-2$. To reintroduce the whole $c(\kappa)$ series, just replace $\kappa\to-\frac{\kappa c(\kappa)}{2}$. The deformed odd Laplacian Δ_* and antibracket $(\cdot,\cdot)_*$, cf. Eq. (1.1), read

$$\Delta_* := \Delta\frac{1}{1-K} = \frac{1}{1-\kappa N}\Delta, \qquad \Delta_*^2=0, \tag{3.1}$$

$$(f,g)_* := (f,g)+(-1)^{\varepsilon}_f(\Delta_* f)\cdot(Kg)+(Kf)\cdot(\Delta_* g) \tag{3.2}$$

$$= (-1)^{\varepsilon_f}\Delta(fg)-(1-K)\big\{(-1)^{\varepsilon_f}(\Delta_* f)g+f(\Delta_* g)\big\} \tag{3.3}$$

$$= -(-1)^{(\varepsilon_f+1)(\varepsilon_g+1)}(g,f)_*, \quad f,g\in\mathcal{A}, \tag{3.4}$$

$$K := \kappa(N-2), \qquad N := z^A\frac{\overrightarrow{\partial^\ell}}{\partial z^A}, \qquad [\Delta,N]=2\Delta, \qquad \varepsilon(\kappa)=0. \tag{3.5}$$

Within the algebra $\mathcal{A}$, the deformed odd Laplacian Δ_* is characterized by nilpotency, and the property

$$\Delta_*(f,g)_* = (\Delta_* f,g)_* - (-1)^{\varepsilon_f}(f,\Delta_* g)_*, \quad f,g\in\mathcal{A}, \tag{3.6}$$

i.e., that Δ_* differentiates the deformed antibracket $(\cdot,\cdot)_*$. The standard Witten formula (2.2), cf. Ref. [21], is deformed into (3.3), which, in turn, can be used to prove the Jacobi identity (3.7) for the deformed antibracket $(\cdot,\cdot)_*$,

$$\sum_{\text{cycl. } f,g,h}(-1)^{(\varepsilon_f+1)(\varepsilon_h+1)}\big(f,(g,h)_*\big)_*=0, \quad f,g,h\in\mathcal{A}. \tag{3.7}$$

[2] The last $\Delta\cup\Delta=0$ of the $2\times2=4$ possibilities vanishes identically, because the cup product $\cup$ is (graded) commutative.

Note that the deformed antibracket $(\cdot,\cdot)_*$ does *not* satisfy the Leibniz rule/Poisson property, cf. Eq. (2.4), and hence the deformed antibracket $(\cdot,\cdot)_*$ is, technically speaking, *not* an odd Poisson bracket. Therefore, the deformation and the corresponding cohomology must be treated within the framework of (infinite-dimensional, graded) Lie algebras instead of (finitely generated, graded) Poisson algebras.

4. *k*-Suspended deformed operators

Define for later convenience a k-suspended deformed odd Laplacian $\Delta_*^{(k)}$ and a (k,ℓ)-suspended deformed antibracket $(\cdot,\cdot)_*^{(k,\ell)}$,

$$\Delta_*^{(k)} := \Delta\frac{1}{1-K^{(k)}}, \qquad \big(\Delta_*^{(k)}\big)^2 = 0, \qquad K^{(k)}\Delta = \Delta K^{(k-2)}, \tag{4.1}$$

$$(f,g)_*^{(k,\ell)} := (f,g) + (-1)^{\varepsilon_f}\big(\Delta_*^{(k)}f\big)\cdot\big(K^{(\ell)}g\big) + \big(K^{(k)}f\big)\cdot\big(\Delta_*^{(\ell)}g\big) \tag{4.2}$$

$$= (-1)^{\varepsilon_f}\Delta(fg) - \big(1-K^{(k+\ell+2)}\big)\big\{(-1)^{\varepsilon_f}\big(\Delta_*^{(k)}f\big)g + f\big(\Delta_*^{(\ell)}g\big)\big\} \tag{4.3}$$

$$= -(-1)^{(\varepsilon_f+1)(\varepsilon_g+1)}(g,f)_*^{(\ell,k)}, \quad f,g\in\mathcal{A}, \tag{4.4}$$

$$K^{(k)} := \kappa N^{(k)}, \qquad N^{(k)} := N + k, \qquad N_*^{(k)} := N^{(k)}\frac{1}{1-K^{(k)}}, \qquad K_*^{(k)} := \kappa N_*^{(k)}, \tag{4.5}$$

where k,ℓ are integers. In particular, the k-suspended definitions (4.1)–(4.5) generalize definitions (3.1)–(3.5) of Section 3 in the following way,

$$\Delta_*^{(-2)} \equiv \Delta_*, \qquad (f,g)_*^{(-2,-2)} \equiv (f,g)_*, \qquad K^{(-2)} \equiv K, \qquad N^{(0)} \equiv N. \tag{4.6}$$

Eq. (4.3) is a (k,ℓ)-suspended deformed Witten formula [21]. Note also the elementary, but useful, formula

$$K^{(k+\ell)}(fg) = \big(K^{(k)}f\big)g + f\big(K^{(\ell)}g\big), \quad f\in\mathcal{A}. \tag{4.7}$$

Eqs. (4.3) and (4.7) can be used to prove the Jacobi identity

$$\sum_{\text{cycl. }(f,k),(g,\ell),(h,m)} (-1)^{(\varepsilon_f+1)(\varepsilon_h+1)}\big(f,(g,h)_*^{(\ell,m)}\big)_*^{(k,\ell+m+2)} = 0, \quad f,g,h\in\mathcal{A}, \tag{4.8}$$

and the differentiation rule

$$\Delta_*^{(k+\ell+2)}(f,g)_*^{(k,\ell)} = \big(\Delta_*^{(k)}f,g\big)_*^{(k+m,\ell)} - (-1)^{\varepsilon_f}\big(f,\Delta_*^{(\ell)}g\big)_*^{(k,m+\ell)}, \quad f,g\in\mathcal{A}. \tag{4.9}$$

5. τ-Extended algebra $\mathcal{A}_\tau$

Let us now introduce a τ-extended algebra $\mathcal{A}_\tau := C[[z;t;\theta]][\frac{1}{t}]$ of formal (lower truncated) Laurent series

$$F = \sum_{k=-M_F}^{\infty} F_{(k)}(z;\theta)t^k, \qquad F_{(k)}(z;\theta) = F_{(k|0)}(z) + \theta F_{(k|1)}(z), \tag{5.1}$$

where the lower limit $k=-M_F$ may depend on the series F, and $\tau := \{t;\theta\}$ is a collective notation for the two new variables t and θ of Grassmann parity $\varepsilon(t)=0$ and $\varepsilon(\theta)=1$, respectively. One introduces a suspension map $\lfloor\cdot\rfloor : \mathcal{A}\to\mathcal{A}_\tau$ as

$$\lfloor f\rfloor := \frac{f}{t^2}, \quad f\in\mathcal{A}. \tag{5.2}$$

The residue map $\pi:\mathcal{A}_\tau\to\mathcal{A}$ reads $\pi(F) := \oint_0 \frac{t\,dt}{2\pi i}\int d\theta\,\theta F = F_{(-2|0)}$ with Berezin integral convention $\int d\theta\,\theta = 1$. One has $\pi\circ\lfloor\cdot\rfloor = \mathrm{id}_{\mathcal{A}}$, or equivalently, $\pi\circ\lfloor f\rfloor = f$ for $f\in\mathcal{A}$.

6. τ-Extended antisymplectic structure

Define generalized Darboux[3] coordinates $\{z_0^A;t_0;t_0^*\}$ as

$$z_0^A := \frac{z^A}{t}, \qquad t_0 := \ln(t), \qquad t_0^* := \theta, \tag{6.1}$$

with inverse transformation

$$z^A = e^{t_0}z_0^A, \qquad t = e^{t_0}, \qquad \theta = t_0^*. \tag{6.2}$$

The Berezin volume densities for the generalized Darboux and original coordinates are chosen as

$$\rho_0 := 1, \qquad \rho_\tau := \frac{\rho_0}{J} = \frac{1}{t}, \qquad J := \operatorname{sdet}\frac{\partial\{z^A;t;\theta\}}{\partial\{z_0^A;t_0;t_0^*\}} = t. \tag{6.3}$$

[3] *Generalized Darboux coordinates* are coordinates in which the (odd) Poisson bi-vector is constant, cf. Eq. (6.10).

The algebra $\mathcal{A}_\tau$ is equipped with the second-order odd Laplacian[4]

$$\Delta_\tau := \frac{(-1)^{\varepsilon_A}}{2}\frac{\overrightarrow{\partial^\ell}}{\partial z_0^A}E^{AB}\frac{\overrightarrow{\partial^\ell}}{\partial z_0^B} + \frac{\overrightarrow{\partial^\ell}}{\partial t_0}\frac{\overrightarrow{\partial^\ell}}{\partial t_0^*} = t^2\Delta + N_\tau\frac{\overrightarrow{\partial^\ell}}{\partial\theta}, \qquad \Delta_\tau^2 = 0, \tag{6.4}$$

$$N_\tau := N + t\frac{\overrightarrow{\partial^\ell}}{\partial t} = -(\theta,\cdot)_\tau, \qquad [N_\tau,\Delta_\tau] = 0, \tag{6.5}$$

$$(F,G)_\tau := (-1)^{\varepsilon_F}\big[[\overrightarrow{\Delta}_\tau,F],G\big]1, \tag{6.6}$$

such that the suspension map $\lfloor\cdot\rfloor$ intertwines between an operation and its τ-extended counterpart,

$$\Delta_\tau\lfloor f\rfloor = \Delta f, \qquad N_\tau\lfloor f\rfloor = \lfloor (N-2)f\rfloor, \quad f\in\mathcal{A}, \tag{6.7}$$

$$\big(\lfloor f\rfloor,\lfloor g\rfloor\big)_\tau = \lfloor (f,g)\rfloor, \qquad \big(f,\lfloor g\rfloor\big)_\tau = (f,g), \quad f,g\in\mathcal{A}. \tag{6.8}$$

The non-vanishing antibrackets $(\cdot,\cdot)_\tau$ of the fundamental variables $\{z^A;t;\theta\}$ read

$$\big(z^A,z^B\big)_\tau = t^2E^{AB}, \qquad \big(z^A,\theta\big)_\tau = z^A, \qquad (t,\theta)_\tau = t, \tag{6.9}$$

or in terms of generalized Darboux coordinates $\{z_0^A;t_0;t_0^*\}$,

$$\big(z_0^A,z_0^B\big)_\tau = E^{AB}, \qquad \big(t_0,t_0^*\big)_\tau = 1. \tag{6.10}$$

7. Trivially deformed τ-extended odd Poisson algebra $\mathcal{A}_\tau$

Define a trivially deformed odd Laplacian

$$\Delta_{\tau*} := \Delta_\tau\frac{1}{1-K_\tau} = T^{-1}\Delta_\tau T, \qquad \Delta_{\tau*}^2 = 0, \tag{7.1}$$

$$K_\tau := \kappa N_\tau, \qquad [K_\tau,\Delta_\tau] = 0, \tag{7.2}$$

cf. Appendix A, where T is the trivialization map in the τ-extended algebra $\mathcal{A}_\tau$,

$$T := 1+\kappa\theta\Delta_{\tau*}, \qquad T^{-1} := 1-\kappa\theta\Delta_\tau, \qquad T^{-1}T = 1 = TT^{-1}, \tag{7.3}$$

cf. Appendix B, so that in the suspended sector,

$$\Delta_{\tau*}\lfloor f\rfloor = \Delta_* f, \qquad K_\tau\lfloor f\rfloor = \lfloor Kf\rfloor, \quad f\in\mathcal{A}. \tag{7.4}$$

If one expands with respect to the t variable, one gets

$$\Delta_{\tau*}F = \sum_k\big(t^2\Delta_*^{(k)}F_{(k)} + N_*^{(k)}F_{(k|1)}\big)t^k, \quad F\in\mathcal{A}_\tau, \tag{7.5}$$

$$K_\tau F = \sum_k\big(K^{(k)}F_{(k)}\big)t^k, \quad F\in\mathcal{A}_\tau. \tag{7.6}$$

Define a trivially deformed antibracket

$$(F,G)_{\tau*} := T^{-1}(TF,TG)_\tau = (F,G)_\tau + (-1)^{\varepsilon_F}(\Delta_{\tau*}F)\cdot K_\tau G + (K_\tau F)\cdot\Delta_{\tau*}G \tag{7.7}$$

$$= (-1)^{\varepsilon_F}\Delta_\tau(FG) - (1-K_\tau)\big\{(-1)^{\varepsilon_F}(\Delta_{\tau*}F)G + F\Delta_{\tau*}G\big\} \tag{7.8}$$

$$= -(-1)^{(\varepsilon_F+1)(\varepsilon_G+1)}(G,F)_{\tau*}, \quad F,G\in\mathcal{A}_\tau, \tag{7.9}$$

cf. Appendix C, so that in the suspended sector,

$$\big(\lfloor f\rfloor,\lfloor g\rfloor\big)_{\tau*} = \big\lfloor (f,g)_*\big\rfloor, \quad f,g\in\mathcal{A}. \tag{7.10}$$

If one expands with respect to the t variable, one gets

$$(F,G)_{\tau*} = \sum_{k,\ell}\left(t^2(F_{(k)},G_{(\ell)})_*^{(k,\ell)} + (-1)^{\varepsilon_F}\left(\frac{1}{1-K^{(k)}}F_{(k|1)}\right)\cdot N^{(\ell)}G_{(\ell)} + \big(N^{(k)}F_{(k)}\big)\cdot\frac{1}{1-K^{(\ell)}}G_{(\ell|1)}\right)t^{k+\ell}, \quad F,G\in\mathcal{A}_\tau. \tag{7.11}$$

The trivially deformed antibracket $(\cdot,\cdot)_{\tau*}$ satisfies the Jacobi identity,

$$\sum_{\text{cycl. } F,G,H}(-1)^{(\varepsilon_F+1)(\varepsilon_H+1)}\big(F,(G,H)_{\tau*}\big)_{\tau*} = 0, \quad F,G,H\in\mathcal{A}_\tau. \tag{7.12}$$

[4] Theoretically, the parameter t serves as a *unit* of suspension. In practice, it may be more convenient to expand in terms of its square $t_2 := t^2$, so that $\lfloor f\rfloor := f/t_2$; $N_\tau := N + 2t_2\partial/\partial t_2$; $\Delta_\tau := t_2\Delta + N_\tau\partial/\partial\theta$; etc.

Eq. (7.10) therefore gives an alternative derivation of the Jacobi identity (3.7). Define a trivial associative and commutative *star product* as

$$F * G = T^{-1}(TF \cdot TG) = FG - (-1)^{\varepsilon_F}\kappa\theta(F,G)_{\tau *}, \quad F,G \in \mathcal{A}_\tau, \qquad \varepsilon(*) = 0, \tag{7.13}$$

cf. Appendix D, so that in the suspended sector,

$$\lfloor f \rfloor * \lfloor g \rfloor = \lfloor \lfloor fg \rfloor \rfloor - (-1)^{\varepsilon_f}\kappa\theta\lfloor (f,g)_* \rfloor, \quad f,g \in \mathcal{A}. \tag{7.14}$$

The trivially deformed Witten formula [21] reads

$$(F,G)_{\tau *} = (-1)^{\varepsilon_F}\Delta_{\tau *}(F * G) - (-1)^{\varepsilon_F}(\Delta_{\tau *}F) * G - F * \Delta_{\tau *}G, \quad F,G \in \mathcal{A}_\tau. \tag{7.15}$$

The Leibniz rule/Poisson property reads

$$(F * G, H)_{\tau *} = F * (G,H)_{\tau *} + (-1)^{\varepsilon_F\varepsilon_G} G * (F,H)_{\tau *}, \quad F,G,H \in \mathcal{A}_\tau. \tag{7.16}$$

The Getzler identity [22] for the BV algebra $(\mathcal{A}_\tau; \Delta_{\tau *}; *)$ reads

$$\begin{aligned} 0 &= \Delta_{\tau *}(F * G * H) - \Delta_{\tau *}(F * G) * H - (-1)^{\varepsilon_F} F * \Delta_{\tau *}(G * H) - (-1)^{\varepsilon_G\varepsilon_H}\Delta_{\tau *}(F * H) * G \\ &\quad + (\Delta_{\tau *}F) * G * H + (-1)^{\varepsilon_F} F * (\Delta_{\tau *}G) * H + (-1)^{\varepsilon_F+\varepsilon_G} F * G * \Delta_{\tau *}H, \quad F,G,H \in \mathcal{A}_\tau, \end{aligned} \tag{7.17}$$

which encodes the vanishing of higher antibrackets [7,8,23]. The *star exponential* is defined as

$$\begin{aligned} e_*^B &:= 1 + B + \frac{1}{2}B * B + \frac{1}{3!}B * B * B + \frac{1}{4!}B * B * B * B + \cdots = T^{-1}e^{(TB)} \\ &= e^B\left(1 - \frac{1}{2}\kappa\theta(B,B)_{\tau *}\right) = e^{B - \frac{1}{2}\kappa\theta(B,B)_{\tau *}}, \quad B \in \mathcal{A}_\tau, \qquad \varepsilon(B) = 0, \end{aligned} \tag{7.18}$$

cf. Appendix E. The star exponential satisfies

$$e_*^{-B} * e_*^B = 1, \qquad e_*^{-B} * \left(\Delta_{\tau *}e_*^B\right) = (\Delta_{\tau *}B) + \frac{1}{2}(B,B)_{\tau *}, \qquad \delta e_*^B = e_*^B * \delta B, \tag{7.19}$$

$$e_*^{B+B'} = e_*^B * e_*^{B'}, \qquad B, B' \in \mathcal{A}_\tau, \qquad \varepsilon(B) = 0 = \varepsilon\left(B'\right). \tag{7.20}$$

If we want to stress the deformation parameter κ, we write a subindex "(κ)", i.e.,

$$T \equiv T_{(\kappa)}, \qquad \Delta_{\tau *} \equiv \Delta_{\tau *(\kappa)}, \qquad (\cdot,\cdot)_{\tau *} \equiv (\cdot,\cdot)_{\tau *(\kappa)}, \qquad F * G \equiv F *_{(\kappa)} G, \qquad e_*^B \equiv e_{*(\kappa)}^B. \tag{7.21}$$

8. Deformed quantum master equations

We will here for simplicity use the strong first-level[5] W–X-formalism, which consists of gauge-generating and gauge-fixing actions, W and X [24–28,8,29,30]. In the τ-extended case, we adorn the two actions with tildes. The two quantum master equations are

$$\Delta_{\tau *(\kappa)} e_{*(\kappa)}^{\frac{i}{\hbar}\widetilde{W}} = 0, \qquad \Delta_{\tau *(-\kappa)} e_{*(-\kappa)}^{\frac{i}{\hbar}\widetilde{X}} = 0, \qquad \widetilde{W}, \widetilde{X} \in \mathcal{A}_\tau, \qquad \varepsilon(\widetilde{W}) = 0 = \varepsilon(\widetilde{X}), \tag{8.1}$$

or equivalently,

$$\frac{1}{2}(\widetilde{W}, \widetilde{W})_{\tau *(\kappa)} = i\hbar\Delta_{\tau *(\kappa)}\widetilde{W}, \qquad \frac{1}{2}(\widetilde{X}, \widetilde{X})_{\tau *(-\kappa)} = i\hbar\Delta_{\tau *(-\kappa)}\widetilde{X}. \tag{8.2}$$

From now on, it is implicitly assumed that the star deformations in the $\widetilde{W}$- and $\widetilde{X}$-sector refer to the deformation parameter κ and $-\kappa$, respectively, to avoid clutter. Consider first the $\widetilde{W}$ action. Let us mention that $\widetilde{W}$ satisfies the κ-deformed quantum master equation if and only if $T\widetilde{W}$ satisfies the undeformed quantum master equation. If one expands the quantum master equation for $\widetilde{W} = \sum_{k=-\infty}^{\infty} \widetilde{W}_{(k)} t^k$ with respect to the t variable, one gets

$$\frac{1}{2}\sum_{\ell=-\infty}^{\infty}\left(\widetilde{W}_{(\ell)}, \widetilde{W}_{(k-\ell)}\right)_*^{(\ell,k-\ell)} + \sum_{\ell=-\infty}^{\infty} N^{(\ell)}\widetilde{W}_{(\ell)} \cdot \frac{1}{1 - K^{(k-\ell+2)}}\widetilde{W}_{(k-\ell+2|1)} = i\hbar\Delta_*^{(k)}\widetilde{W}_{(k)} + i\hbar N_*^{(k+2)}\widetilde{W}_{(k+2|1)}. \tag{8.3}$$

We next identify the component $\widetilde{W}_{(-2|0)} = S$ with the proper[6] classical action S from Eq. (1.2). To have the classical master equation (1.2) within the t-hierarchy (8.3), the Laurent series $\widetilde{W}$ must truncate from below as

[5] The strong first-level gauge-fixing action $\widetilde{X}$ also depends on first-level Lagrange multipliers $\{\lambda^{\widetilde{\alpha}}\} = \{\lambda^\alpha; \lambda_\theta\}$, and is capable of incorporating all Abelian gauge-fixing constraints $(G_{\widetilde{\alpha}}, G_{\widetilde{\beta}})_\tau = 0$. For non-Abelian gauge-fixing constraints, it is necessary to add weak terms in the quantum master equation [27], or still better, to go to the second-level formalism, which introduces antifields $\lambda^*_{\widetilde{\alpha}}$ for the first-level Lagrange multipliers; second-level Lagrange multipliers $\lambda^{\widetilde{\alpha}}_{(2)}$; odd Laplacian $\Delta_{[1]\tau *} = \Delta_{\tau *} + (-1)^{\varepsilon_{\widetilde{\alpha}}}\partial/\partial\lambda^{\widetilde{\alpha}}\partial/\partial\lambda^*_{\widetilde{\alpha}}$; and action $\widetilde{W}_{[2]} = \lambda^*_{\widetilde{\alpha}}\lambda^{\widetilde{\alpha}}_{(2)} + \widetilde{W}$.

[6] An action is called *proper* (with respect to a set of antisymplectic variables) if its corresponding Hessian has rank equal to half the number of variables at the stationary surface, see e.g., Ref. [31].

$$\widetilde{W} = \sum_{k=-2}^{\infty} \widetilde{W}_{(k|0)} t^k + \theta \sum_{k=1}^{\infty} \widetilde{W}_{(k|1)} t^k. \tag{8.4}$$

The minimal Ansatz for the gauge-generating and gauge-fixing actions, $\widetilde{W}$ and $\widetilde{X}$, reads[7]

$$\widetilde{W} = \frac{1}{t^2} W\left(z; \hbar t^2; \kappa\right) = \lfloor S \rfloor + \hbar M_1 + \mathcal{O}\left(\hbar^2 t^2\right), \qquad \frac{\partial \widetilde{W}}{\partial \theta} = 0, \tag{8.5}$$

$$\widetilde{X} = X\left(\frac{z}{t}; \lambda; \hbar\right) + i\hbar\theta\lambda_\theta = X(z_0; \lambda; \hbar) + i\hbar t_0^* \lambda_0, \qquad N_\tau \widetilde{X} = 0, \tag{8.6}$$

where $\lambda_\theta \equiv \lambda_0$ is a Fermionic first-level Lagrange multiplier to gauge-fix the θ variable, and where

$$W = W\left(z; \hbar t^2; \kappa\right) = S + \sum_{k=1}^{\infty} \left(t^2 \hbar\right)^k M_k, \qquad S = S(z; \kappa), \qquad M_k = M_k(z; \kappa) \quad \text{for } k \geqslant 1. \tag{8.7}$$

In t-components, the minimal Ansatz (8.5) for $\widetilde{W}$ reads

$$\widetilde{W}_{(-2)} = S, \qquad \widetilde{W}_{(2k-2)} = \hbar^k M_k \quad \text{for } k \geqslant 1, \qquad \widetilde{W}_{(-2k)} = 0 \quad \text{for } k \geqslant 2, \qquad \widetilde{W}_{(2k+1)} = 0. \tag{8.8}$$

The quantum hierarchy (8.3) for $\widetilde{W}$ becomes

$$(S, S)_* = 0, \qquad (M_1, S)_*^{(0,-2)} = i\Delta_* S, \tag{8.9}$$

$$(M_k, S)_*^{(2k-2,-2)} = i\Delta_*^{(2k-4)} M_{k-1} - \frac{1}{2} \sum_{\ell=1}^{k-1} (M_\ell, M_{k-\ell})_*^{(2\ell-2,2k-2\ell-2)} \quad \text{for } k \geqslant 2. \tag{8.10}$$

The hierarchy (8.9)–(8.10) successively determines S and M_k for $k \geqslant 1$. The untilded gauge-fixing action X satisfies an ordinary quantum master equation

$$\Delta e^{\frac{i}{\hbar} X} = 0 \quad \Leftrightarrow \quad \frac{1}{2}(X, X) = i\hbar \Delta X, \tag{8.11}$$

which is undeformed in the deformation parameter $-\kappa$.

9. Deformed path integral

The first-level *path integral measure* is

$$d\mu = \rho_\tau \, dt \, d\theta \, d\lambda_\theta \, [dz][d\lambda] = \rho_0 \, dt_0 \, dt_0^* \, d\lambda_0 \, [dz_0][d\lambda], \tag{9.1}$$

cf. Eq. (6.3). The *transposed operator* A^T of an operator A is defined via [8]

$$\int d\mu \left(A^T F\right) \cdot G = (-1)^{\varepsilon_A \varepsilon_F} \int d\mu \, F \cdot (AG), \tag{9.2}$$

where F, G are two arbitrary functions. The transposed odd Laplacians and transposed Euler vector fields are

$$\Delta^T = \Delta, \qquad N^T = -N, \qquad \Delta_\tau^T = \Delta_\tau, \qquad N_\tau^T = -N_\tau, \qquad \Delta_{\tau*(\kappa)}^T = \Delta_{\tau*(-\kappa)}. \tag{9.3}$$

The first-level *path integral* $\widetilde{\mathcal{Z}}$ in the τ-extended antisymplectic phase space is defined as

$$\widetilde{\mathcal{Z}} = \int d\mu \, e_{*(\kappa)}^{\frac{i}{\hbar}\widetilde{W}} \cdot e_{*(-\kappa)}^{\frac{i}{\hbar}\widetilde{X}} = \int d\mu \, e^{\frac{i}{\hbar}\widetilde{A}}, \tag{9.4}$$

where the total first-level action $\widetilde{A}$ is

$$\widetilde{A} = \widetilde{W} - \frac{i\kappa\theta}{2\hbar}(\widetilde{W}, \widetilde{W})_{\tau*(\kappa)} + \widetilde{X} + \frac{i\kappa\theta}{2\hbar}(\widetilde{X}, \widetilde{X})_{\tau*(-\kappa)} = \widetilde{W} + \kappa\theta\Delta_{\tau*(\kappa)}\widetilde{W} + \widetilde{X} - \kappa\theta\Delta_{\tau*(-\kappa)}\widetilde{X} = T_{(\kappa)}\widetilde{W} + T_{(-\kappa)}\widetilde{X}. \tag{9.5}$$

Note that the total action $\widetilde{A}$ does not contain inverse powers of $\hbar$ due to the quantum master equations (8.2) for $\widetilde{W}$ and $\widetilde{X}$.

[7] Note that while the leading term $\lfloor S \rfloor$ in the $\widetilde{W}$ action is proper in the original antisymplectic phase space $\{z^A\}$, it is in general *not* proper in the τ-extended antisymplectic phase space $\{z^A; t; \theta\}$. Thus if one would like to treat the t variable perturbatively, it is necessary to include t-dependent classical ($=\hbar$-independent) terms in the $\widetilde{W}$ action, which necessarily must violate the minimal Ansatz (8.5). We analyze here the minimal Ansatz (8.5) for simplicity, as the Ansatz is consistent with the quantum master equation (8.2), but with the caveat that t may acquire a non-perturbative status.

10. Independence of gauge-fixing $\widetilde{X}$

The quantum BRST operator for $\widetilde{X}$ is defined as

$$(\sigma_{\widetilde{X}*}F) := \frac{\hbar}{i} e_*^{-\frac{i}{\hbar}\widetilde{X}} * \Delta_{\tau*}\big(e_*^{\frac{i}{\hbar}\widetilde{X}} * F\big) - \frac{\hbar}{i} e_*^{-\frac{i}{\hbar}\widetilde{X}} * \big(\Delta_{\tau*} e_*^{\frac{i}{\hbar}\widetilde{X}}\big) * F = \frac{\hbar}{i}(\Delta_{\tau*}F) + (\widetilde{X}, F)_{\tau*}, \quad F \in \mathcal{A}_\tau, \qquad \sigma_{\widetilde{X}*}^2 = 0. \tag{10.1}$$

Since the $\sigma_{\widetilde{X}*}$ operator is nilpotent, one may argue on general grounds that an arbitrary infinitesimal variation $\delta\widetilde{X}$ of the action $\widetilde{X}$ should be BRST exact,

$$(\sigma_{\widetilde{X}*}\delta\widetilde{X}) = 0, \qquad \delta\widetilde{X} = (\sigma_{\widetilde{X}*}\delta\Psi), \tag{10.2}$$

for some infinitesimal Fermion $\delta\Psi$, or equivalently,

$$\frac{i}{\hbar} e_*^{\frac{i}{\hbar}\widetilde{X}} * \delta\widetilde{X} = \delta e_*^{\frac{i}{\hbar}\widetilde{X}} = \Delta_{\tau*}\big(e_*^{\frac{i}{\hbar}\widetilde{X}} * \delta\Psi\big) - \big(\Delta_{\tau*} e_*^{\frac{i}{\hbar}\widetilde{X}}\big) * \delta\Psi. \tag{10.3}$$

By using properties (9.3) of transposed operators, and the quantum master equations (8.1), one may deduce that the $\widetilde{\mathcal{Z}}$ partition function (9.4) is independent of the gauge-fixing $\widetilde{X}$.

$$\delta\widetilde{\mathcal{Z}} = \int d\mu\, e_{*(\kappa)}^{\frac{i}{\hbar}\widetilde{W}} \cdot \delta e_{*(-\kappa)}^{\frac{i}{\hbar}\widetilde{X}} = \int d\mu\, e_{*(\kappa)}^{\frac{i}{\hbar}\widetilde{W}} \cdot \Delta_{\tau*(-\kappa)}\big(e_{*(-\kappa)}^{\frac{i}{\hbar}\widetilde{X}} *_{(-\kappa)} \delta\Psi\big) = \int d\mu\, \big(\Delta_{\tau*(\kappa)} e_{*(\kappa)}^{\frac{i}{\hbar}\widetilde{W}}\big) \cdot \big(e_{*(-\kappa)}^{\frac{i}{\hbar}\widetilde{X}} *_{(-\kappa)} \delta\Psi\big) = 0. \tag{10.4}$$

11. Integrating out the τ-extended sector

One can always integrate out the new variable $\theta \equiv t_0^*$. The boundary condition (8.6) creates a delta-function

$$\int d\lambda_\theta\, e^{\frac{i}{\hbar}\cdot i\hbar\theta\lambda_\theta} = \int d\lambda_\theta\, e^{\lambda_\theta\theta} = \delta(\theta), \tag{11.1}$$

and therefore one implements the condition $\theta = 0$. The other new variable $t_0 \equiv \ln(t)$ is a Schwinger proper time variable in a world-line formalism [32]. Let us for simplicity use Darboux coordinates $\{z_0^A; t_0; t_0^*\} = \{\phi_0^\alpha; \phi_{0\alpha}^*; t_0; t_0^*\}$, and integrate out the first-level Lagrange multipliers $\{\lambda^{\widetilde{\alpha}}\} = \{\lambda^\alpha; \lambda_0\}$, such that the resulting zero-level total action A is a lower truncated Laurent series in the $t \equiv e^{t_0}$ variable

$$A = \widetilde{A}\left(\phi_0; \phi_0^* = \frac{\partial\psi}{\partial\phi_0}; t_0; t_0^* = \frac{\partial\psi}{\partial t_0}; \lambda = 0; \lambda_0 = 0; \hbar; \kappa\right) = \sum_{k=-M}^{\infty} A_{(k)} e^{kt_0}, \qquad A_{(k)} = A_{(k)}(\phi_0; \hbar; \kappa). \tag{11.2}$$

For a theory that is perturbative in the original z-variables, (minus) the lower limit is $M \leqslant 2$. If we furthermore integrate out the Schwinger proper time variable t_0, then the $\widetilde{\mathcal{Z}}$ partition function (9.4) becomes

$$\widetilde{\mathcal{Z}} = \int_{-\infty}^{0} dt_0 \int [d\phi_0]\, e^{\frac{i}{\hbar}A}$$
$$= \begin{cases} \frac{1}{M}\sum_{m=0}^{\infty}\frac{1}{m!}\sum_{k_1,\ldots,k_m \geqslant 1-M}\int[d\phi_0]\big(-\frac{i}{\hbar}A_{(-M)}\big)^{\frac{\Sigma k}{M}}\Gamma\big(-\frac{\Sigma k}{M}; -\frac{i}{\hbar}A_{(-M)}\big)\prod_{i=1}^{m}\frac{i}{\hbar}A_{(k_i)} & \text{for } M > 0, \\ \sum_{m=0}^{\infty}\frac{1}{m!}\sum_{k_1,\ldots,k_m \geqslant 1}\frac{1}{\Sigma k}\int[d\phi_0]\, e^{\frac{i}{\hbar}A_{(0)}}\prod_{i=1}^{m}\frac{i}{\hbar}A_{(k_i)} & \text{for } M = 0, \\ \sum_{m=0}^{\infty}\frac{1}{m!}\sum_{k_1,\ldots,k_m \geqslant -M}\frac{1}{\Sigma k}\int[d\phi_0]\prod_{i=1}^{m}\frac{i}{\hbar}A_{(k_i)} & \text{for } M < 0, \end{cases} \tag{11.3}$$

where $\Sigma k := \sum_{i=1}^{m} k_i$; where $\Gamma(s;\varepsilon) := \int_\varepsilon^\infty \frac{du}{u} u^s e^{-u}$ is the incomplete Gamma function; and in the case $M > 0$, it has been assumed that $\mathrm{Im}(A_{(-M)}) > 0$. The case $M < 0$ can be viewed as the case $M = 0$ with $A_{(0)} = 0$. The formula (11.3) is an expansion in Planck's constant $\hbar$ if all the subleading terms $A_{(k>-M)} = \mathcal{O}(\hbar)$ are quantum corrections. We stress that the world-line path integral $\widetilde{\mathcal{Z}}$ does *not* reproduce the standard field–antifield path integral [1] in the undeformed limit $\kappa \to 0$, as only the former contains a Schwinger proper time integration.[8]

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[8] However, we mention an alternative procedure in the special situation where $\kappa\Delta_* S = 0$, which includes both (i) the undeformed case $\kappa = 0$ with action $\widetilde{W} = \frac{W}{t^2}$, and (ii) the truncated case $\widetilde{W} = \frac{W}{t^2} = \lfloor S \rfloor$ with $\Delta_* S = 0$. In these two cases, shift the $\widetilde{W}$ action with a one-loop contribution $\widetilde{W} = \frac{W}{t^2} \to \widetilde{W} = \frac{W}{t^2} + i\hbar\ln(1-t^2) = \frac{W}{t^2} + \frac{\hbar}{i}\sum_{k=1}^{\infty}\frac{t^{2k}}{k}$. One may check that the shifted $\widetilde{W}$ action also satisfies the quantum master equation (8.2). Now choose the t integration contour as a small circle around $t = 1$. The one-loop correction $\oint_1 \frac{dt}{t} e^{\frac{i}{\hbar}\cdot i\hbar\ln(1-t^2)} = -\oint_1 \frac{dt}{t}\frac{1}{t+1}\frac{1}{t-1}$ creates a simple pole at $t = 1$, and thereby one implements the condition $t = 1$. Therefore the $\widetilde{\mathcal{Z}}$ path integral (9.4) reduces (up to a constant multiplicative factor) to the standard W–X-form $\widetilde{\mathcal{Z}} = \int[dz][d\lambda]\, e^{\frac{i}{\hbar}(W+X)} = \mathcal{Z}$. In the undeformed case $\kappa = 0$, the W action (8.7) at $t = 1$ becomes the standard loop expansion, which satisfies the standard quantum master equation $\Delta e^{\frac{i}{\hbar}W} = 0$.

Appendix A. Proof of Eq. (7.1)

$$T^{-1}\Delta_\tau T = \Delta_\tau T = \Delta_\tau(1+\kappa\theta\Delta_{\tau*}) = \Delta_\tau + \kappa[\Delta_\tau,\theta]\Delta_{\tau*} = \Delta_\tau + K_\tau\Delta_\tau\frac{1}{1-K_\tau} = \Delta_\tau\frac{1}{1-K_\tau} = \Delta_{\tau*}. \tag{A.1}$$

Appendix B. Proof of Eq. (7.3)

$$\begin{aligned} T^{-1}T &:= (1-\kappa\theta\Delta_\tau)(1+\kappa\theta\Delta_{\tau*}) = 1-\kappa\theta\Delta_\tau+\kappa\theta\Delta_{\tau*}-\kappa^2\theta[\Delta_\tau,\theta]\Delta_{\tau*} \\ &= 1-\kappa\theta\Delta_\tau+\kappa\theta\Delta_\tau\frac{1}{1-K_\tau}-\kappa\theta K_\tau\Delta_\tau\frac{1}{1-K_\tau} = 1, \end{aligned} \tag{B.1}$$

$$\begin{aligned} TT^{-1} &:= (1+\kappa\theta\Delta_{\tau*})(1-\kappa\theta\Delta_\tau) = 1-\kappa\theta\Delta_\tau+\kappa\theta\Delta_{\tau*}-\kappa^2\theta\left[\Delta_\tau\frac{1}{1-K_\tau},\theta\right]\Delta_\tau \\ &= 1-\kappa\theta\Delta_\tau+\kappa\theta\Delta_\tau\frac{1}{1-K_\tau}-\kappa\theta K_\tau\frac{1}{1-K_\tau}\Delta_\tau = 1. \end{aligned} \tag{B.2}$$

Appendix C. Proof of Eq. (7.7)

$$(B,B)_{\tau*} := T^{-1}(TB,TB)_\tau = (1-\kappa\theta\Delta_\tau)(TB,TB)_\tau = I - II = (B,B)_\tau + 2(\Delta_{\tau*}B)\cdot(K_\tau B), \quad B\in\mathcal{A}_\tau, \qquad \varepsilon(B)=0, \tag{C.1}$$

where

$$\begin{aligned} I &:= (TB,TB)_\tau = (B+\kappa\theta\Delta_{\tau*}B, B+\kappa\theta\Delta_{\tau*}B)_\tau \\ &= (B,B)_\tau - 2\kappa(\Delta_{\tau*}B)\cdot(\theta,B)_\tau + 2\kappa\theta(\Delta_{\tau*}B,B)_\tau + 2\kappa^2\theta(\Delta_{\tau*}B,\theta)_\tau\cdot\Delta_{\tau*}B, \end{aligned} \tag{C.2}$$

$$\begin{aligned} II &:= \kappa\theta\Delta_\tau(TB,TB)_\tau = 2\kappa\theta(\Delta_\tau TB,TB)_\tau = 2\kappa\theta\big(\Delta_\tau B+\kappa[\Delta_\tau,\theta]\Delta_{\tau*}B,TB\big)_\tau \\ &= 2\kappa\theta\left(\Delta_\tau B+K_\tau\Delta_\tau\frac{1}{1-K_\tau}B,TB\right)_\tau = 2\kappa\theta(\Delta_{\tau*}B,\ B+\kappa\theta\Delta_{\tau*}B)_\tau \\ &= 2\kappa\theta(\Delta_{\tau*}B,B)_\tau + 2\kappa^2\theta(\Delta_{\tau*}B,\theta)_\tau\cdot\Delta_{\tau*}B. \end{aligned} \tag{C.3}$$

Now use polarization of Eq. (C.1) to prove Eq. (7.7), cf. e.g., Ref. [23].

Appendix D. Proof of Eq. (7.13)

$$\begin{aligned} B*B &= T^{-1}(TB)^2 = T^{-1}\big(B+\kappa\theta(\Delta_{\tau*}B)\big)^2 = (1-\kappa\theta\Delta_\tau)\big(B^2+2\kappa\theta B\Delta_{\tau*}B\big) \\ &= I - II - III = B^2 - \kappa\theta(B,B)_\tau - 2\kappa\theta(K_\tau B)\cdot\Delta_{\tau*}B \\ &= B^2 - \kappa\theta(B,B)_{\tau*}, \quad B\in\mathcal{A}_\tau, \qquad \varepsilon(B)=0, \end{aligned} \tag{D.1}$$

where

$$I := B^2 + 2\kappa\theta B\Delta_{\tau*}B = B^2 + 2\kappa\theta B\Delta_\tau\frac{1}{1-K_\tau}B, \tag{D.2}$$

$$II := \kappa\theta\Delta_\tau\big(B^2\big) = 2\kappa\theta B\Delta_\tau B + \kappa\theta(B,B)_\tau, \tag{D.3}$$

$$III := 2\kappa^2\theta\Delta_\tau\theta B\Delta_{\tau*}B = 2\kappa^2\theta[\Delta_\tau,\theta]B\Delta_{\tau*}B = 2\kappa\theta K_\tau B\Delta_{\tau*}B = 2\kappa\theta(K_\tau B)\cdot\Delta_{\tau*}B + 2\kappa\theta BK_\tau\Delta_\tau\frac{1}{1-K_\tau}B. \tag{D.4}$$

Now use polarization of Eq. (D.1) to prove Eq. (7.13).

Appendix E. Proof of Eq. (7.18)

$$\begin{aligned} e_*^B &= T^{-1}e^{(TB)} = T^{-1}e^{B+\kappa\theta(\Delta_{\tau*}B)} = (1-\kappa\theta\Delta_\tau)e^B(1+\kappa\theta\Delta_{\tau*}B) \\ &= I - II - III = e^B\left(1-\frac{1}{2}\kappa\theta(B,B)_\tau - \kappa\theta(K_\tau B)\cdot\Delta_{\tau*}B\right) \\ &= e^B\left(1-\frac{1}{2}\kappa\theta(B,B)_{\tau*}\right), \quad B\in\mathcal{A}_\tau, \qquad \varepsilon(B)=0, \end{aligned} \tag{E.1}$$

where

$$I := e^B(1+\kappa\theta\Delta_{\tau*}B) = e^B\left(1+\kappa\theta\Delta_\tau\frac{1}{1-K_\tau}B\right), \tag{E.2}$$

$$II := \kappa\theta\big(\Delta_\tau e^B\big) = \kappa\theta e^B\left(\Delta_\tau B + \frac{1}{2}(B,B)_\tau\right), \tag{E.3}$$

$$III := \kappa^2\theta\Delta_\tau\theta e^B\Delta_{\tau*}B = \kappa^2\theta[\Delta_\tau,\theta]e^B\Delta_{\tau*}B = \kappa\theta K_\tau e^B\Delta_{\tau*}B = \kappa\theta e^B(K_\tau B)\cdot\Delta_{\tau*}B + \kappa\theta e^B K_\tau\Delta_\tau\frac{1}{1-K_\tau}B. \tag{E.4}$$

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