URL: http://www.elsevier.nl/locate/entcs/volume2.html 2 pages

Logic and Graphs

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Abstract

The lecture is based on a chapter of a forthcoming handbook of graph grammars, edited by G. Rozenberg. This chapter is entitled: "The expression of graph properties and graph transformations in monadic second-order logic".

By considering graphs as logical structures, one can express formally their properties by logical formulas. One can thus describe classes of graphs by formulas of appropriate logical languages expressing characteristic properties.

There are two main motivations for doing this : the first one, originating from the work by Fagin, consists in giving logical characterizations of complexity classes ; the second one consists in using logical formulas as finite devices, comparable to grammars or automata, to specify classes of graphs and to establish properties of such classes from their logical descriptions. We shall only consider here the second of these motivations. The ideal language is in this respect monadic second-order logic, as we shall demonstrate. It is crucial for establishing "easily" results like this one :

the set of planar graphs belonging to a HR set of graphs (i.e., a set of graphs generated by a Hyperedge Replacement graph grammar) is HR,

and this one :

the set of Hamiltonian graphs belonging to a HR set of graphs is HR,

by essentially the same proofs, using the facts that planarity and Hamiltonicity can both be described by MS (monadic second-order) formulas, and the theorem saying that the intersection of a HR set and an MS-definable set is a HR set. These two results do not concern logic, but their proofs use logic as a tool.

The deep reason why MS logic is so crucial in the theory of context-free graph grammars is that it replaces for graphs the notion of a finite automaton which is very important in the theory of formal languages. It "replaces" because no convenient notion of finite automaton is known for graphs.

The notion of a transformation from words or trees to words or trees is also essential in language theory. These transformations are usually defined in terms of finite automata, that produce an output while traversing the given word or tree. Since we have no notion of finite graph automaton, we cannot define graph transformations in terms of automata. However, we can define such transformations in terms of MS formulas. We call them *definable transductions* : "definable" refers to logic and "transduction" to the way transformations of words and trees are usually named.

The lecture will survey the following notions: representations of graphs by logical structures, expressive power of monadic second-order logic from the point of view of graph theory, representation of context-free graph grammars by systems of recursive equations (so that the generated sets of graphs are the least solutions of the corresponding systems), the fundamental theorem on the intersection of context-free and MS-definable sets of graphs, definable transductions and characterizations of context-free sets of graphs as images of the set of finite binary trees under definable transductions.

These results have been obtained by M. Bauderon, B. Courcelle, J. Engelfriet and G. Rozenberg.

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