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Analysis of negative parity baryon photoproduction amplitudes in the $1/N_c$ expansion

N.N. Scoccola [a,b,c], J.L. Goity [d,e], N. Matagne [f,*]

[a] Physics Department, Comisión Nacional de Energía Atómica, (1429) Buenos Aires, Argentina
[b] CONICET, Rivadavia 1917, (1033) Buenos Aires, Argentina
[c] Universidad Favaloro, Solís 453, (1078) Buenos Aires, Argentina
[d] Department of Physics, Hampton University, Hampton, VA 23668, USA
[e] Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA
[f] University of Liège, Institute of Physics B5, Sart Tilman, B-4000 Liège 1, Belgium

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ABSTRACT

We study the photoproduction helicity amplitudes of negative parity baryons in the context of the $1/N_c$ expansion of QCD. A complete analysis to next-to-leading order is carried out. The results show sub-leading effects to be within the magnitude expected from the $1/N_c$ power counting. They also show significant deviations from the quark model, in particular the need for 2-body effects.

1. Introduction

In this Letter, the photoproduction helicity amplitudes of the first excited negative parity baryons are analyzed in the framework of the $1/N_c$ expansion of QCD [1]. Those baryons belong to the $[\mathbf{20}', 1^-]$ multiplet of $SU(4) \times O(3)$, where $\mathbf{20}'$ is the mixed symmetric representation of $SU(4)$ (non-strange states in the $SU(6)$ **70**-plet). In terms of masses and widths as well as electromagnetic helicity amplitudes, this is the best known multiplet of excited baryons. In the $1/N_c$ expansion, the masses were analyzed in Refs. [2,3], and the strong transition partial widths were analyzed in Refs. [4,5]. The photoproduction helicity amplitudes have been studied for more than forty years in many works, predominantly using constituent quark models [6], the related single-quark transition model based on $SU(6)_W$ symmetry [7], and dispersion approaches [8]. In the $1/N_c$ expansion, the first analysis of negative parity baryon helicity amplitudes was carried out by Carlson and Carone [9]. Positive parity baryon helicity amplitudes have also been analyzed in recent work [10]. Some model independent relations for helicity amplitudes have been obtained in Ref. [11]. The present work extends the analysis in Ref. [9] by systematically building a complete basis of current operators to sub-leading order in the $1/N_c$ expansion, and by presenting and discussing the results in terms of the multipole contributions to each helicity amplitude. We will compare our analysis with that of Ref. [9] in the discussion of results.

The photoproduction helicity amplitudes are defined by the following matrix elements:

$$A_\lambda = -\sqrt{\frac{2\pi\alpha}{\omega}}\,\eta(B^*)\langle B^*, \lambda|\vec{\epsilon}_{+1}\cdot\vec{J}(\omega\hat{z})|N, \lambda-1\rangle. \tag{1}$$

They correspond to the standard definition as used by the Particle Data Group [12], which includes a sign factor $\eta(B^*)$ that stems from the strong decay amplitude of the excited baryon to a πN state. The amplitudes in Eq. (1) are independent of the phase conventions used to define the excited states. The sign factors are on the other hand convention dependent. Here N and B^* denote respectively the initial nucleon and the final excited baryon, $\lambda = 1/2$ or $3/2$ is the helicity defined along the $\hat{z}$-axis which coincides with the photon momentum, $\vec{\epsilon}_{+1}$ is the photon's polarization vector for helicity $+1$, and $\omega = (M^2_{B^*} - M^2_N)/2M_{B^*}$ is the photon energy in the rest frame of B^*. In the $1/N_c$ expansion, the electromagnetic current $\vec{J}$ is represented as a linear combination of effective multipole current operators with the most general form:

$$\left(k^{[L']}\mathcal{B}^{[LI]}\right)^{[1I]}, \tag{2}$$

where the upper scripts display the angular momentum and isospin, and throughout the neutral component, i.e. $I_3 = 0$, is taken. The $O(3)$ tensor $k^{[L']}$ is expressed in terms of spherical harmonics of the photon momentum, and $\mathcal{B}^{[LI]} = (\xi^{(\ell)}\mathcal{G}^{[\ell' I]})^{[LI]}$ are baryonic operators. $\xi^{(\ell)}$ is the tensor associated with the transition from the $\ell = 0$ $O(3)$ state of the nucleon to the $O(3)$ state of the excited baryon, and is normalized by its reduced matrix element (RME) according to $\langle 0\|\xi^{(\ell)}\|\ell\rangle = \sqrt{2\ell+1}$ ($\ell = 1$ in this work). Finally, $\mathcal{G}^{[\ell' I]}$ is a spin-flavor tensor operator with $I = 0$ or 1. The parity selection rules imply that the helicity amplitudes for photoproduction of the $[\mathbf{20}', 1^-]$ states can only contain $E1$, $M2$ and

* Corresponding author.
E-mail addresses: scoccola@tandar.cnea.gov.ar (N.N. Scoccola), goity@jlab.org (J.L. Goity), nicolas.matagne@theo.physik.uni-giessen.de (N. Matagne).

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$E3$ multipoles. The quantum number L in Eq. (2) determines the multipole: EL for $L=1,3$ and $M2$ for $L=2$. For the EL multipoles the photon orbital angular momentum L' is $L'=L\pm 1$ and for ML multipoles $L'=L$. The multipoles are in addition classified according to their isospin, into isoscalars and isovectors. For general N_c the isovector and isoscalar components of the electric charge can be generalized in different ways [13]. Here we consider them as being both $\mathcal{O}(N_c^0)$, corresponding to the assumption that quark charges are N_c independent.

The multipole components of the helicity amplitudes are expressed in terms of the matrix elements of the effective operators as follows:

$$A_\lambda^{ML} = \sqrt{\frac{3\alpha N_c}{4\omega}}(-1)^{L+1}\eta(B^*)\sum_{n,I} g_{n,L}^{[L,I]}(\omega) \times \langle J^*,\lambda; I^*, I_3; S^*|(\mathcal{B}_n)_{[1,0]}^{[L,I]}|1/2,\lambda-1;1/2,I_3\rangle, \tag{3}$$

$$A_\lambda^{EL} = \sqrt{\frac{3\alpha N_c}{4\omega}}(-1)^{L}\eta(B^*) \times \sum_{n,I}\left[\sqrt{\frac{L+1}{2L+1}}\, g_{n,L-1}^{[L,I]}(\omega) + \sqrt{\frac{L}{2L+1}}\, g_{n,L+1}^{[L,I]}(\omega)\right] \times \langle J^*,\lambda; I^*, I_3; S^*|(\mathcal{B}_n)_{[1,0]}^{[L,I]}|1/2,\lambda-1;1/2,I_3\rangle, \tag{4}$$

where J^*, I^* and S^* denote the spin, isospin and quark-spin of the excited baryon and the sum over n is over all operators with given $[L,I]$ quantum numbers. The factor $\sqrt{N_c}$ appears as usual for transition matrix elements between excited and ground state baryons [14]. In the electric multipoles we have a combination of the coefficients $g_{n,L-1}^{[L,I]}$ and $g_{n,L+1}^{[L,I]}$, and because the operators appearing in these multipoles do not appear in the magnetic multipoles, we may as well replace that combination of coefficients by a single term without any loss of generality. Thus, in what follows we will only keep $g_{n,L-1}^{[L,I]}$. These and the coefficients $g_{n,L}^{[L,I]}(\omega)$ are going to be determined by fits to the empirical helicity amplitudes.

It is convenient to express these matrix elements in terms of reduced matrix elements (RMEs) via the Wigner–Eckart theorem:

$$\langle J^*,\lambda; I^*, I_3; S^*|(\mathcal{B}_n)_{[1,0]}^{[L,I]}|1/2,\lambda-1;1/2,I_3\rangle = \frac{(-1)^{L+I+J^*+I^*-1}}{\sqrt{(2I^*+1)(2J^*+1)}}\langle L,1;1/2,\lambda-1|J^*,\lambda\rangle \times \langle I,0;1/2,I_3|I^*,I_3\rangle\langle J^*;I^*;S^*\|\mathcal{B}_n^{[L,I]}\|1/2,1/2\rangle. \tag{5}$$

If one wishes, one can further express the RMEs of the baryonic operators in terms of RMEs involving only the spin-flavor pieces of those operators [10].

For the purpose of carrying out the group theoretical calculations, and without any loss of generality, one can consider that the $[\mathbf{20}',1^-]$ baryon states are made of a ground state core composed of N_c-1 quarks coupled to an excited quark. The states can then be expressed as follows [2,15]:

$$|J,J_3;I,I_3;S\rangle = \sum_{m,s_3,i_3,\eta}\langle \ell,m;S,J_3-m\mid J,J_3\rangle c_{\mathrm{MS}}(I,S,\eta) \times \langle S_c,S_3-s_3;1/2,s_3\mid S,S_3\rangle\langle I_c,I_3-i_3;1/2,i_3\mid I,I_3\rangle \times |S_c,S_3-s_3;I_c=S_c,I_3-i_3\rangle|1/2,s_3;1/2,i_3\rangle|1,m\rangle, \tag{6}$$

where $\ell=1$, $\eta=\pm 1/2$, $S_c=I_c=S+\eta$ are the spin and the isospin of the core, and $c_{\mathrm{MS}}(I,S,\eta)$ are isoscalar factors of the permutation group of N_c particles [16], which for the mixed symmetric representation $[N_c-1,1]$ can be found in Ref. [2]. In the following, the generators of $SU(4)$ which act on the core will carry a subscript c, while operators acting on the excited quark will be denoted in lower case. For $N_c=3$, the states contained in the $[\mathbf{20}',1^-]$ are as follows: two N states with $J^*=1/2$, two with $J^*=3/2$ and one with $J^*=5/2$, and one Δ with $J^*=1/2$ and one with $J^*=3/2$. There are two mixing angles, θ_1 for the pair of excited N states with $J^*=1/2$, and θ_3 for the N pair with $J^*=3/2$. The mixing angles are defined in the standard fashion [2], and have been determined in different ways. In the $1/N_c$ expansion in particular, they can be obtained from an analysis of the masses [2], and more precisely from analyzing strong transitions [5]. We use the latter in this work.

Table 1

Baryon operator basis. The upper labels $^{[L,I]}$ denote angular momentum and isospin and how these are coupled. The NLO operators $E1_5^{(1)}$, $E1_6^{(1)}$, and $M2_4^{(1)}$ involve linear combinations with LO operators in order to eliminate the LO component

Operator	Order	Type
$E1_1^{(0)} = (\xi^{[1,0]}s)^{[1,0]}$	1	1B
$E1_2^{(0)} = \frac{1}{N_c}(\xi^{[1,0]}(s\ S_c)^{[0,0]})^{[1,0]}$	$\frac{1}{N_c}$	2B
$E1_3^{(0)} = \frac{1}{N_c}(\xi^{[1,0]}(s\ S_c)^{[1,0]})^{[1,0]}$	$\frac{1}{N_c}$	2B
$E1_4^{(0)} = \frac{1}{N_c}(\xi^{[1,0]}(s\ S_c)^{[2,0]})^{[1,0]}$	$\frac{1}{N_c}$	2B
$E1_1^{(1)} = (\xi^{[1,0]}t)^{[1,1]}$	1	1B
$E1_2^{(1)} = (\xi^{[1,0]}g)^{[1,1]}$	1	1B
$E1_3^{(1)} = \frac{1}{N_c}(\xi^{[1,0]}(s\ G_c)^{[2,1]})^{[1,1]}$	1	2B
$E1_4^{(1)} = \frac{1}{N_c}(\xi^{[1,0]}(s\ T_c)^{[1,1]})^{[1,1]}$	$\frac{1}{N_c}$	2B
$E1_5^{(1)} = \frac{1}{N_c}(\xi^{[1,0]}(s\ G_c)^{[0,1]})^{[1,1]} + \frac{1}{4\sqrt{3}}\ E1_1^{(1)}$	$\frac{1}{N_c}$	2B
$E1_6^{(1)} = \frac{1}{N_c}(\xi^{[1,0]}(s\ G_c)^{[1,1]})^{[1,1]} + \frac{1}{2\sqrt{2}}\ E1_2^{(1)}$	$\frac{1}{N_c}$	2B
$M2_1^{(0)} = (\xi^{[1,0]}s)^{[2,0]}$	1	1B
$M2_2^{(0)} = \frac{1}{N_c}(\xi^{[1,0]}(s\ S_c)^{[1,0]})^{[2,0]}$	$\frac{1}{N_c}$	2B
$M2_3^{(0)} = \frac{1}{N_c}(\xi^{[1,0]}(s\ S_c)^{[2,0]})^{[2,0]}$	$\frac{1}{N_c}$	2B
$M2_1^{(1)} = (\xi^{[1,0]}g)^{[2,1]}$	1	1B
$M2_2^{(1)} = \frac{1}{N_c}(\xi^{[1,0]}(s\ G_c)^{[2,1]})^{[2,1]}$	1	2B
$M2_3^{(1)} = \frac{1}{N_c}(\xi^{[1,0]}(s\ T_c)^{[1,1]})^{[2,1]}$	$\frac{1}{N_c}$	2B
$M2_4^{(1)} = \frac{1}{N_c}(\xi^{[1,0]}(s\ G_c)^{[1,1]})^{[2,1]} + \frac{1}{2\sqrt{2}}\ M2_1^{(1)}$	$\frac{1}{N_c}$	2B
$E3_1^{(0)} = \frac{1}{N_c}(\xi^{[1,0]}(s\ S_c)^{[2,0]})^{[3,0]}$	$\frac{1}{N_c}$	2B
$E3_1^{(1)} = \frac{1}{N_c}(\xi^{[1,0]}(s\ G_c)^{[2,1]})^{[3,1]}$	1	2B

The basis of baryon operators $\mathcal{B}$ can be built using leading and sub-leading spin-flavor operators by following a procedure similar to that described in Ref. [5] for the case of the strong decays. The basis used in this work is depicted in Table 1, which indicates the multipole to which the operator contributes and the order in $1/N_c$. More specifically, a baryonic operator $\mathcal{B}$ is given by the corresponding operator in the basis of Table 1 multiplied by a scaling factor α, depicted in the last column of Table 2, which is introduced in order for the operator to have matrix elements of natural size. This factor α is chosen in such a way that the largest RME of the operator $\mathcal{B}$ is equal to 1 (1/3) if the operator is $\mathcal{O}(N_c^0)$ ($\mathcal{O}(1/N_c)$). This allows one to easily see the importance of the different operators by just looking at the magnitude of their coefficients. At leading order (LO) in $1/N_c$ there are a total of eight operators, one $E1$ and one $M2$ isoscalars, and three $E1$, two $M2$ and one $E3$ isovectors. It is important to emphasize that this distribution in the different multipoles is basis independent. At subleading order (NLO), there are eleven new operators. This exhausts the basis because the number of helicity amplitudes for the photoproduction of the $[\mathbf{20}',1^-]$ baryons is equal to nineteen. The analysis shows, therefore, that neither sub-sub-leading operators nor three-body operators are needed for a full description of the helicity amplitudes. This in particular means that there is no way of sorting out such contributions.

One important check on the basis we have constructed is the counting of the number of operators for each multipole and isospin

Table 2
Reduced matrix elements of basis operators depicted in Table 1. The notation ${}^{2S^*}N^*_{J^*}$ is used for the nucleon states. The columns must be multiplied by the corresponding overall factor shown in the last row, where $A \equiv ((1-\frac{1}{N_c})(1+\frac{3}{N_c}))^{1/2}$ and $B \equiv (1-\frac{1}{N_c})^{1/2}$. The scaling factor α explained in the text is depicted in the last column

	${}^2N^*_{1/2}$	${}^2N^*_{3/2}$	${}^4N^*_{1/2}$	${}^4N^*_{3/2}$	${}^4N^*_{5/2}$	$\Delta^*_{1/2}$	$\Delta^*_{3/2}$	α
$E1^{(0)}_1$	$-\sqrt{\frac{2}{3}}$	$\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{2}{3}}$	$\sqrt{\frac{10}{3}}$	0	0	0	$\frac{-3}{\sqrt{10}}$
$E1^{(0)}_2$	$\frac{-1}{2N_c}$	$\frac{-1}{N_c}$	0	0	0	0	0	$\sqrt{\frac{3}{2}}$
$E1^{(0)}_3$	0	0	$\frac{-\sqrt{3}}{2N_c}$	$\frac{\sqrt{15}}{2N_c}$	0	0	0	$\frac{-2}{\sqrt{5}}$
$E1^{(0)}_4$	0	0	$\frac{\sqrt{5}}{2N_c}$	$\frac{1}{2N_c}$	0	0	0	$\frac{-\sqrt{12}}{\sqrt{5}}$
$E1^{(1)}_1$	1	2	0	0	0	$-\sqrt{2}$	2	$\frac{-\sqrt{3}}{2\sqrt{2}}$
$E1^{(1)}_2$	$\frac{-\sqrt{2}}{3}$	$\frac{\sqrt{2}}{3}$	$\frac{1}{3\sqrt{2}}$	$\frac{-\sqrt{5}}{3\sqrt{2}}$	0	$-\frac{1}{3}$	$\frac{-1}{3\sqrt{2}}$	$\frac{-3\sqrt{3}}{2\sqrt{2}}$
$E1^{(1)}_3$	0	0	$\sqrt{\frac{5}{3}}\,\frac{N_c+2}{4N_c}$	$\frac{N_c+2}{4\sqrt{3}N_c}$	0	0	0	$\frac{-36}{5\sqrt{5}}$
$E1^{(1)}_4$	$\frac{-1}{3\sqrt{2}N_c}$	$\frac{1}{3\sqrt{2}N_c}$	$\frac{-2\sqrt{2}}{3N_c}$	$\frac{2\sqrt{10}}{3N_c}$	0	$\frac{1}{3N_c}$	$\frac{1}{3\sqrt{2}N_c}$	$\frac{-\sqrt{27}}{\sqrt{40}}$
$E1^{(1)}_5$	$\frac{1}{4\sqrt{3}N_c}$	$\frac{1}{2\sqrt{3}N_c}$	0	0	0	$\frac{1}{\sqrt{6}N_c}$	$\frac{-1}{\sqrt{3}N_c}$	$\frac{3}{\sqrt{2}}$
$E1^{(1)}_6$	0	0	0	0	0	$\frac{1}{2\sqrt{2}N_c}$	$\frac{1}{4N_c}$	$-\sqrt{12}$
$M2^{(0)}_1$	0	$\sqrt{\frac{10}{3}}$	0	$-\sqrt{\frac{2}{3}}$	$\frac{3}{\sqrt{2}}$	0	0	$\frac{-3}{2\sqrt{5}}$
$M2^{(0)}_2$	0	0	0	$\frac{-\sqrt{3}}{2N_c}$	$\frac{9}{4N_c}$	0	0	$\frac{-2}{3}$
$M2^{(0)}_3$	0	0	0	$\frac{3}{2N_c}$	$\frac{\sqrt{3}}{4N_c}$	0	0	$\frac{-2}{\sqrt{3}}$
$M2^{(1)}_1$	0	$\frac{\sqrt{10}}{3}$	0	$\frac{1}{3\sqrt{2}}$	$\frac{-\sqrt{3}}{2\sqrt{2}}$	0	$\frac{-\sqrt{5}}{3\sqrt{2}}$	$\frac{-\sqrt{27}}{\sqrt{20}}$
$M2^{(1)}_2$	0	0	0	$\frac{\sqrt{3}(N_c+2)}{4N_c}$	$\frac{N_c+2}{8N_c}$	0	0	$\frac{-12}{5}$
$M2^{(1)}_3$	0	$\frac{\sqrt{5}}{3\sqrt{2}N_c}$	0	$\frac{-2\sqrt{2}}{3N_c}$	$\frac{\sqrt{6}}{N_c}$	0	$\frac{\sqrt{5}}{3\sqrt{2}N_c}$	$\frac{-\sqrt{3}}{2\sqrt{2}}$
$M2^{(1)}_4$	0	0	0	0	0	0	$\frac{\sqrt{5}}{4N_c}$	$\frac{-2\sqrt{6}}{\sqrt{5}}$
$E3^{(0)}_1$	0	0	0	0	$\frac{\sqrt{21}}{2\sqrt{2}N_c}$	0	0	$-\sqrt{\frac{6}{7}}$
$E3^{(1)}_1$	0	0	0	0	$\frac{\sqrt{7}}{4\sqrt{2}}\,\frac{N_c+2}{N_c}$	0	0	$\frac{-18\sqrt{2}}{5\sqrt{7}}$
Factor	A	$\frac{-A}{\sqrt{2}}$	$\frac{-B}{\sqrt{2}}$	$\frac{-B}{\sqrt{2}}$	$-\sqrt{\frac{2}{3}}B$	$-B$	$-B$	

type. In general, we are interested in transitions of the form $N\gamma \to B^*$ where $N = p, n$ and $B^* = N(1535), N(1520), N(1650), N(1700), N(1675), \Delta(1620), \Delta(1700)$. It is clear that the following two requirements have to be fulfilled: (i) isoscalar operators can only contribute to $N\gamma \to N^*$, while isovector operators can contribute to both $N\gamma \to N^*$ and $N\gamma \to \Delta^*$, and (ii) for each multipole and transition (independently of the spin/isospin projections) there should be one and only one independent element in the operator basis. Using these, one can proceed to count. For example, for $N\gamma \to N(1535)$ one finds that there is one independent $E1^{(0)}$ element and one independent $E1^{(1)}$ element. Similarly, for $N\gamma \to \Delta(1700)$ we have one independent $E1^{(1)}$ element and one independent $M2^{(1)}$ element. Carrying out this procedure to the whole $[\mathbf{20}', 1^-]$ multiplet, one obtains that the maximum number of independent operators in the different multipoles are as follows: $E1^{(0)}$ (4), $E1^{(1)}$ (6), $M2^{(0)}$ (3), $M2^{(1)}$ (4), $E3^{(0)}$ (1), and $E3^{(1)}$ (1). Table 1 shows that the basis we constructed is consistent with this count. The RMEs $\langle J^*, I^*; S^* \| \mathcal{B}^{[L,I]}_n \| 1/2, 1/2\rangle$ of the operators in the basis are shown in Table 2. They have been obtained using standard angular momentum techniques.

One more important input needed from the strong transitions is the sign $\eta(B^*)$ that appears in Eqs. (3)–(4). That sign is obtained from the strong amplitude for $B^* \to \pi N$, and is given in terms of the corresponding RME defined in Ref. [5] by

$$\eta(B^*) = (-1)^{J^*-\frac{1}{2}}\,\mathrm{sign}\big(\langle \ell_\pi N \| H_{\mathrm{QCD}} \| J^* I^*\rangle\big), \tag{7}$$

where ℓ_π corresponds to the pion partial wave. Note that the sign η can be determined up to an overall sign for each pion partial wave, which cannot be fixed by strong transitions alone. Since the partial waves involved in our case are S and D waves, we have one extra relative sign, which we will call ξ as customary [17,18]. In addition, the analysis of the strong transitions gives two consistent but different results for the mixing angle θ_3. The values (in radians) $\theta_3 = 2.82$ and $\theta_3 = 2.38$ cannot be distinguished from the strong fits. One finds that some of the η signs are different for these two values. We take into account this with an extra sign factor κ, which is equal to $+1(-1)$ for $\theta_3 = 2.82(2.38)$.

Table 3 displays the empirically known helicity amplitudes taken from [12] along with the strong sign η, and the amplitudes resulting from the fits to be discussed in the next section.

2. Analysis and results

In this section we present and analyze the different fits to the helicity amplitudes. The coefficients to be fitted $g^{[L,I]}_{n,L'}(\omega)$ are expressed by including the barrier penetration factor: $g^{[L,I]}_{n,L'} \times (\omega/\Lambda)^{L'}$, where $L' = 0$ for $E1$ operators and $L' = 2$ for $M2$ and $E3$ operators. Throughout we will choose the scale $\Lambda = m_\rho$. We performed several LO and NLO fits. A first analysis concerns the choices left by the values of the mixing angle θ_3, and the signs ξ and κ. Using all the LO operators, the choices are made by considering the χ^2 for all possibilities. The sign $\xi = -1$ is strongly favored. This is in agreement with an old determination based on the single-quark-transition model [17,18]. The second choice that is favored, although less markedly than the one for ξ, is $\theta_3 = 2.82$. Finally, for κ there is no indication of a preference from the fits; for the sake a definiteness we will take $\kappa = +1$ in our fits. This latter sign basically depends on strong amplitudes which are small and have large relative errors, which imply that its determination is subject to a degree of uncertainty. The helicity amplitudes show here their importance by allowing to determine the relative sign ξ between the strong S- and D-wave amplitudes, and by selecting between the two possible values of θ_3 consistent with the

Table 3
Helicity amplitudes (in units of $10^{-3}\text{Gev}^{-1/2}$) for the fits in Table 4. The sign η is indicated in the last column. In the fits we have set $\xi = -1$, and $\kappa = +1$. Numbers in parenthesis indicate the individual contribution to the total χ^2

Amplitude	Empirical	LO	NLO1	NLO2	NLO3	η
$A^p_{1/2}[N(1535)]$	$+90 \pm 30$	76(0.2)	90	111(0.5)	86(0.0)	$-\xi$
$A^n_{1/2}[N(1535)]$	-46 ± 27	−54(0.1)	−46	−78(1.4)	−72(0.9)	$-\xi$
$A^p_{1/2}[N(1520)]$	-24 ± 9	−25(0.0)	−24	−20(0.2)	−16(0.8)	−1
$A^n_{1/2}[N(1520)]$	-59 ± 9	−6(8.8)	−59	−46(1.9)	−43(3.1)	−1
$A^p_{3/2}[N(1520)]$	$+166 \pm 5$	66(4.0)	166	163(0.4)	162(0.7)	−1
$A^n_{3/2}[N(1520)]$	-139 ± 11	−55(4.0)	−139	−143(0.1)	−135(0.1)	−1
$A^p_{1/2}[N(1650)]$	$+53 \pm 16$	45(0.3)	53	52(0.0)	39(0.8)	ξ
$A^n_{1/2}[N(1650)]$	-15 ± 21	−12(0.0)	−15	−20(0.1)	−25(0.2)	ξ
$A^p_{1/2}[N(1700)]$	-18 ± 13	−18(0.0)	−18	−20(0.0)	26(11.7)	κ
$A^n_{1/2}[N(1700)]$	0 ± 50	41(0.7)	0	47(0.9)	−13(0.1)	κ
$A^p_{3/2}[N(1700)]$	-2 ± 24	1(0.0)	−2	−10(0.1)	−46(3.3)	κ
$A^n_{3/2}[N(1700)]$	-3 ± 44	47(1.3)	−3	47(1.3)	61(2.1)	κ
$A^p_{1/2}[N(1675)]$	$+19 \pm 8$	15(0.3)	19	8(2.0)	2(4.4)	−1
$A^n_{1/2}[N(1675)]$	-43 ± 12	−45(0.0)	−43	−50(0.4)	−43(0.0)	−1
$A^p_{3/2}[N(1675)]$	$+15 \pm 9$	10(0.3)	15	11(0.2)	3(1.7)	−1
$A^n_{3/2}[N(1675)]$	-58 ± 13	−53(0.1)	−58	−71(1.0)	−61(0.0)	−1
$A^N_{1/2}[\Delta(1620)]$	$+27 \pm 11$	53(5.7)	27	32(0.2)	81(24.5)	$-\xi$
$A^N_{1/2}[\Delta(1700)]$	$+104 \pm 15$	80(0.6)	104	108(0.1)	90(0.9)	+1
$A^N_{3/2}[\Delta(1700)]$	$+85 \pm 22$	70(0.3)	85	112(1.5)	67(0.6)	+1

strong transitions. Note that $\theta_3 = 2.82$ corresponds to "small" mixing, while 2.32 corresponds to "large" mixing. A simultaneous fit of strong transitions and photoproduction amplitudes is the best way of extracting the mixing angles. This will be carried out in a future project [20].

As already mentioned, the helicity amplitudes resulting from the fits we have carried out are given in Table 3; the corresponding fit coefficients are displayed in Table 4. In the fits we expand the operator matrix elements in powers of $1/N_c$ to the order corresponding to the fit. In the LO fits, we have set the errors in the input helicity amplitudes to be equal to 0.3 of the value of the helicity amplitude or the experimental value if this is larger. The point of this is to test whether or not the LO analysis is consistent in the sense that it gives a χ^2 per degree of freedom (χ^2_{dof}) close to unity. For the NLO fits, we of course use the empirical errors.

We now proceed to discuss the results.

– The LO fit shows a χ^2_{dof} of 2.42. This indicates that there are NLO effects to be taken into account for a satisfactory fit. The main deficiencies are in fitting of the $N(1520)$ and the $\Delta(1620)$ amplitudes as one can readily ascertain from their individual contributions to the total χ^2 (numbers in parenthesis in Table 3). If one keeps only the LO operators with the largest coefficients (say coefficients bigger than 2), the χ^2_{dof} does not change much from the one obtained with all LO operators. Notice that one 2-body LO operator seems to be significant, namely $M2^{(1)}_2$. We have checked that a fit taking $\kappa = -1$ leads to similar results except that the coefficient of $M2^{(1)}_2$ turns out to be only 40% of the case $\kappa = +1$. If indeed 2-body operators should give small effects, then this would be a way to discriminate about the sign κ. In fact, a LO fit using only 1-body operators gives respectively $\chi^2_{\text{dof}} = 2.48$ and 2.12 for $\kappa = +1$ and -1.

– One can perform a LO fit motivated by the single-quark-transition model [17,18], which is also commonly used in quark model calculations. In that model, the photon only couples to the excited quark with a fixed ratio for the isoscalar versus the isovector coupling as given by the bare quark charges. Here this is achieved by locking 1-body operators as follows: $(\frac{1}{6}E1^{(0)}_1 + E1^{(1)}_2)$, $(\frac{1}{6}M2^{(0)}_1 + M2^{(1)}_1)$, and $E1^{(1)}_2$ whose isoscalar counterpart does not appear in the operator basis because it is spin-flavor singlet. The fit has $\chi^2_{\text{dof}} \sim 2.5$ at LO, which is similar to the result with unlocked operators, thus indicating that at LO one cannot draw a clear conclusion.

– As it is well known, in the single-quark-transition model the so-called Moorhouse selection rule [7] holds. That rule states that the amplitudes for photoexcitation of protons to $^4N^*$ states

Table 4
Results for the dimensionless coefficients $g^{[L,I]}_{n,L'}$ from different fits. Two partial NLO fits are given. Fit NLO2 keeps the minimum number of dominant operators needed for $\chi^2_{\text{dof}} \leqslant 1$, and fit NLO3 only keeps 1-body operators

Operator	LO	NLO1	NLO2	NLO3
$E1^{(0)}_1$	-0.36 ± 0.19	-0.34 ± 0.22	-0.34 ± 0.15	-0.15 ± 0.14
$E1^{(0)}_2$		0.52 ± 0.62		
$E1^{(0)}_3$		1.02 ± 0.85		
$E1^{(0)}_4$		0.50 ± 0.63		
$E1^{(1)}_1$	2.34 ± 0.31	3.03 ± 0.20	3.54 ± 0.13	3.26 ± 0.22
$E1^{(1)}_2$	-0.68 ± 0.36	0.40 ± 0.27		0.21 ± 0.25
$E1^{(1)}_3$	0.41 ± 0.53	-0.21 ± 0.41		
$E1^{(1)}_4$		-1.95 ± 1.42		
$E1^{(1)}_5$		-0.18 ± 0.90		
$E1^{(1)}_6$		4.17 ± 0.89	3.92 ± 0.77	
$M2^{(0)}_1$	0.76 ± 0.21	1.52 ± 0.32	1.27 ± 0.17	1.21 ± 0.17
$M2^{(0)}_2$		-1.22 ± 1.34		
$M2^{(0)}_3$		-1.18 ± 1.75		
$M2^{(1)}_1$	3.02 ± 0.62	3.81 ± 0.56	3.95 ± 0.40	4.69 ± 0.37
$M2^{(1)}_2$	-3.11 ± 1.00	-2.33 ± 1.12	-2.73 ± 0.62	
$M2^{(1)}_3$		-0.15 ± 1.13		
$M2^{(1)}_4$		-1.49 ± 2.38		
$E3^{(0)}_1$		0.34 ± 0.83		
$E3^{(1)}_1$	0.75 ± 0.89	0.35 ± 0.53		
dof	11	0	13	14
χ^2_{dof}	2.42	–	0.94	4.00

vanish. In the present analysis, the rule is violated by the unlocking of the 1-body operators, and by 2-body operators. At the level of physical states, the rule tends to suppress the amplitudes $p\gamma \to N(1650), N(1700)$, and $N(1675)$. In the first two cases, the mixing angles θ_1 and θ_3 work against that suppression as they give to these states a component $^2N^*$. In the case of $N(1675)$, the rule turns out to be mostly violated by 2-body effects, at least for $\kappa = +1$.

– The NLO order fit NLO1, involves all operators in the basis. It gives values for the coefficients of the LO operators which are, within the expected deviations from $1/N_c$ counting, consistent with the values obtained in the LO fits. Moreover, none of the coefficients of the NLO operators has a magnitude larger than that of the largest LO coefficients. This is a strong indication of the consistency of the $1/N_c$ expansion. We find that this consistency is more clearly manifested here than in the case of the positive parity baryons analyzed in [10]. From the magnitude of the coefficients, it is obvious that only a few NLO operators are needed for a consistent fit. In fact, as shown by the fit NLO2 in Table 4, a consistent fit is obtained with only five LO and one NLO operators. Of these dominant operators four are one-body and LO, and two are two-body with one of them LO and the other NLO. Note also that none of the 2-body $E3$ operators is required. It is remarkable that out of eleven NLO operators only one is essential for obtaining consistent fits. At this point it is important to mention that many of the empirical amplitudes have errors that are larger than what is needed for an accurate NLO analysis. It is for this reason that one cannot draw a more precise NLO picture which could unveil the role of other operators.

– To test for deviations from the single-quark-transition model at NLO, we have performed a NLO fit including all operators with locked the 1-body operators. The result is a $\chi^2_{\mathrm{dof}} \sim 2.5$, which gives a good indication that there are deviations from that model.

– The fit NLO3 depicted in Table 4 including only 1-body operators gives rather large χ^2_{dof}, with a similar result for a 1-body fit with locked operators. One can conclude that, although the gross features of the set of helicity amplitudes are described by 1-body operators, the deviations can be pinpointed quite clearly, in particular the need for 2-body effects.

– The dominant operator in terms of the magnitude of its contributions is $E1_1^{(1)}$, as it can be seen from Table 5, which depicts the partial contribution to each amplitude by the operators included in the fit NLO2. This operator is expected to dominate in a non-relativistic quark model as it corresponds to the usual orbital electric dipole transition. The contributions of the other relevant $E1$ and $M2$ operators, needed for a consistent fit, turn out to be rather similar in magnitude.

Table 5
Partial contributions to the helicity amplitudes by the different operators. This table corresponds to the NLO2 fit

Amplitude	$E1_1^{(0)}$	$E1_1^{(1)}$	$E1_6^{(1)}$	$M2_1^{(0)}$	$M2_1^{(1)}$	$M2_2^{(1)}$	Total
$A_{1/2}^p[N(1535)]$	17	95	0	0	0	0	111
$A_{1/2}^n[N(1535)]$	17	−95	0	0	0	0	−78
$A_{1/2}^p[N(1520)]$	−4	70	0	−29	−46	−11	−20
$A_{1/2}^n[N(1520)]$	−4	−70	0	−29	46	11	−46
$A_{3/2}^p[N(1520)]$	−7	120	0	17	26	6	163
$A_{3/2}^n[N(1520)]$	−7	−120	0	17	−26	−6	−143
$A_{1/2}^p[N(1650)]$	16	36	0	0	0	0	52
$A_{1/2}^n[N(1650)]$	16	−36	0	0	0	0	−20
$A_{1/2}^p[N(1700)]$	11	−21	0	2	32	−44	−20
$A_{1/2}^n[N(1700)]$	11	21	0	2	−32	44	47
$A_{3/2}^p[N(1700)]$	20	−36	0	−1	−18	26	−10
$A_{3/2}^n[N(1700)]$	20	36	0	−1	18	−26	47
$A_{1/2}^p[N(1675)]$	0	0	0	−21	19	10	8
$A_{1/2}^n[N(1675)]$	0	0	0	−21	−19	−10	−50
$A_{3/2}^p[N(1675)]$	0	0	0	−30	27	14	11
$A_{3/2}^n[N(1675)]$	0	0	0	−30	−27	−14	−71
$A_{1/2}^N[\Delta(1620)]$	0	85	−53	0	0	0	32
$A_{1/2}^N[\Delta(1700)]$	0	57	18	0	32	0	108
$A_{3/2}^N[\Delta(1700)]$	0	99	31	0	−19	0	112

It is instructive to briefly discuss the individual helicity amplitudes, as they differ very significantly in the type of contributions involved. For this discussion, we take fit NLO2, which contains the most significant contributions. As already mentioned, the individual contributions by the various operators to the helicity amplitudes are shown in Table 5.

– $N(1535)$: The amplitudes are not very well established, with various analyses giving significantly different results [12,19]. One can however establish that $E1_1^{(1)}$ plays an important role, in particular because its coefficient is primarily determined by other better known amplitudes. For this reason, we find that it is very difficult to reconcile the values obtained for the amplitudes on p and n which result from the analysis carried out in Ref. [19].

– $N(1520)$: This, as well as $N(1700)$, receive several contributions $E1$ and $M2$, which involve some important cancellations. One manifestation of such cancellations is in the $\lambda = 1/2$ amplitudes in which the isoscalar component turns out to be larger than the isovector one (only case where this occurs). In the quark model such a cancellation seems unproblematic to be explained [6], and thus it can be understood in simple terms. On the other hand, the $\lambda = 3/2$ amplitudes are dominated by the operator $E1_1^{(1)}$, with small contributions from other operators, and a particularly small total isosinglet component.

– $N(1650)$: The $p\gamma$ amplitude would vanish in the limit in which the Moorhouse rule is valid. The dominant effect driving this amplitude is the mixing by the angle θ_1. One can check that the effect of unlocking operators gives small contributions, and in particular tends to reduce the Moorhouse allowed $n\gamma$ amplitude (for the latter there are however some discrepancies between different analyses [12,19]).

– $N(1700)$: These amplitudes are poorly known empirically, as they seem to be small (some of them on the grounds of the Moorhouse rule). In addition several operators contribute, which according to our analysis will tend to have large cancellations. Thus, one expects that a clear understanding of the physics contained in this case will not be easy.

– $N(1675)$: These are the only amplitudes admitting $E3$ contributions, and show through the fit that they are irrelevant. Note that the $E3$ operators are 2-body. In this case the $p\gamma$ amplitudes only proceed because of violations to the Moorhouse rule due to the unlocking of 1-body operators and due to 2-body operators. We find that the main contribution is due to the 2-body LO operator $M2_2^{(1)}$. On the other hand the unsuppressed $n\gamma$ amplitudes are dominated by the $M2$ 1-body contributions.

– $\Delta(1620)$: Various analyses are inconsistent with each other, but all of them strongly indicate that this helicity amplitude is small. It is an interesting amplitude, because it receives a large $E1$ contribution from $E1_1^{(1)}$, and the only way to have a small amplitude is to have a large cancellation. In our analysis that cancellation is shown to come from the 2-body operator $E1_6^{(1)}$; in fact the need for this cancellation largely determines in the fit the importance of that operator. Taken at face value, this is a strong indication for 2-body effects. In the single-quark-transition model, as well as in quark models [6] one finds that the calculated amplitude is much larger than the empirical one. This is due to the absence of the 2-body effects in those models.

– $\Delta(1700)$: These are among the most clearly established and understood amplitudes. $E1_1^{(1)}$ plays the dominant role, with the other two operators $M2_1^{(1)}$ and $E1_6^{(1)}$ giving contributions of similar magnitude. Since the 1-body LO operators already give a good description, it is not surprising that these amplitudes are well described in the quark model [6].

At this point we can compare our analysis with that of Carlson and Carone [9]. We have checked that their set of operators, eleven in total, corresponds to a subset of our operator basis, which can be obtained by locking several pairs of operators using the isoscalar to isovector ratio of the electric charge operator as we explained earlier. In this case, 1- as well as 2-body operators are locked. A fit with that set of locked operators gives a $\chi^2_{\rm dof} \sim 3.2$. This result clearly indicates the necessity for the more general basis we use in this work. However, one should emphasize that the main features of most helicity amplitudes are obtained in the analysis of Ref. [9]. Another point where we differ with Ref. [9] is in the mixing angles: in our analysis we take the mixing angles from the strong decays, while in Ref. [9] some of the fits include fitting the mixing angles. Their mixing angles are somewhat different from ours, leaving an open issue which should be sorted out. We plan to carry out simultaneous fits of strong decays and helicity amplitudes [20], from where we expect to extract more reliable values for the mixing angles.

3. Summary

The aim of this work was to extend the $1/N_c$ expansion analysis of baryon photoproduction helicity amplitudes to the negative parity baryons, improving on the approach used in earlier work [9]. The most important outcome of the analysis is that the expected hierarchies implied by the $1/N_c$ power counting are respected. Another important aspect is that only a reduced number of the operators in the basis turn out to be relevant. Several of those operators can be easily identified with those in quark models, but there are also 2-body operators not included in quark models which are necessary for an accurate description of the empirical helicity amplitudes. With this analysis one can select between the two possible values of the mixing angle θ_3 which are consistent with strong decays, as well as the relative sign ξ between the S and D-wave strong amplitudes. A comprehensive analysis that includes strong and helicity amplitudes will further refine the results of this work, and will be presented elsewhere.

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