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## Physics Letters B

[www.elsevier.com/locate/physletb](http://www.elsevier.com/locate/physletb)Realistic calculations of  $\bar{K}NN$ ,  $\bar{K}NNN$ , and  $\bar{K}\bar{K}NN$  quasibound states

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## ABSTRACT

Binding energies and widths of three-body  $\bar{K}NN$ , and of four-body  $\bar{K}NNN$  and  $\bar{K}\bar{K}NN$  nuclear quasibound states are calculated in the hyperspherical basis, using realistic  $NN$  potentials and subthreshold energy dependent chiral  $\bar{K}N$  interactions. Results of previous  $K^-pp$  calculations are reproduced and an upper bound is placed on the binding energy of a  $K^-d$  quasibound state. A self-consistent handling of energy dependence is found to restrain binding, keeping the calculated four-body ground-state binding energies to relatively low values of about 30 MeV. The lightest strangeness  $-2$  particle-stable  $\bar{K}$  nuclear cluster is most probably  $\bar{K}\bar{K}NN$ . The calculated  $\bar{K}N \rightarrow \pi Y$  conversion widths range from approximately 30 MeV for the  $\bar{K}NNN$  ground state to approximately 80 MeV for the  $\bar{K}\bar{K}NN$  ground state.

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## 1. Introduction

Unitarized coupled-channel chiral dynamics in the strangeness  $S = -1$  sector, constrained by fitting to  $K^-p$  low-energy and threshold data, gives rise to a  $(\bar{K}N)_{I=0}$   $s$ -wave quasibound state (QBS), as detailed in recent works [1,2]. The relationship of this QBS to the observed  $\Lambda(1405)$  resonance, which was predicted long ago by Dalitz and Tuan [3] within a phenomenological study of  $\bar{K}N-\pi\Sigma$  coupled channels, has been recently reviewed by Hyodo and Jido [4]. With that strong  $(\bar{K}N)_{I=0}$  interaction,  $\bar{K}$  mesons are expected to bind to nuclear clusters beginning with the  $(\bar{K}NN)_{I=1/2}$   $J^\pi = 0^-$  QBS, loosely termed  $K^-pp$ . While several few-body calculations confirmed that  $K^-pp$  is bound, as reviewed in Ref. [5], we here focus on those calculations using chiral interaction models in which the strong subthreshold energy dependence of the input  $\bar{K}N$  interactions, essential in  $\bar{K}$  nuclear few-body calculations, is under sound theoretical control. Such calculations yield binding energies in the range  $B(K^-pp) \sim 10\text{--}20$  MeV [6,7], in contrast to values of 100 MeV or more obtained upon relegating peaks observed in final-state  $\Delta p$  invariant-mass spectra from FIN-UDA [8] and DISTO [9] to the QBS decay  $K^-pp \rightarrow \Delta p$ . To reinforce this discrepancy we note that none of the other published  $K^-pp$  calculations based on  $\bar{K}N$  phenomenology [10–13] managed to get as large  $K^-pp$  binding energy as 100 MeV.

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Given this unsettled state of affairs for  $K^-pp$ , it is desirable to provide chiral model predictions for heavier  $\bar{K}$  nuclear clusters starting with four-body systems and, in particular, to study the onset of binding for  $S = -2$  clusters.<sup>1</sup> A good candidate is  $\bar{K}\bar{K}NN$  which of all four-body  $\bar{K}$  nuclear clusters has the largest number of  $\bar{K}N$  bonds (four out of six). Furthermore, for the  $I = 0$ ,  $J^\pi = 0^+$  lowest energy QBS, and limiting the nuclear isospin to  $I_N = 1$  corresponding to the dominant  $s$ -wave  $NN$  configuration, this QBS has the most advantageous  $I_{\bar{K}N} = 0, 1$  composition of  $V_{\bar{K}N}^{(I)}$ ,  $3 \div 1$  in favor of the strong  $V_{\bar{K}N}^{(0)}$ , same as in  $K^-pp$ .

In this Letter we present fully four-body nonrelativistic calculations of the  $\bar{K}$  nuclear clusters  $\bar{K}NNN$  and  $\bar{K}\bar{K}NN$  in the hyperspherical basis. Realistic  $NN$  interactions and effective subthreshold  $\bar{K}N$  interactions derived within a chiral model [15] are used. The energy dependence of the subthreshold  $\bar{K}N$  interactions is treated self consistently, extending a procedure suggested and practised in Refs. [16–18]. This provides a robust mechanism to restrain the calculated binding energies of  $\bar{K}$  nuclear clusters. Our calculations in the three-body sector reproduce the  $K^-pp$  calculations of Doté et al. [6] and provide an upper bound on the binding energy of a  $K^-d$   $J^\pi = 1^-$  QBS. In the four-body sector we find binding energies close to 30 MeV, in strong disagreement with predictions of over 100 MeV made in phenomenological, non-chiral models for  $\bar{K}NNN$  [19] and  $\bar{K}\bar{K}NN$  [20,21].

<sup>1</sup> We disregard the  $\bar{K}\bar{K}N$  QBS which was calculated within a chiral interaction model to practically coincide with the  $\bar{K} + (\bar{K}N)_{I=0}$  threshold [14].

## 2. Input and methodology

In this section we (i) briefly review the hyperspherical basis in which  $\bar{K}$ -nuclear cluster wavefunctions are expanded and in which calculations of ground-state energies are done, (ii) specify the two-body  $NN$ ,  $\bar{K}N$ ,  $\bar{K}\bar{K}$  input interactions, and (iii) discuss the choice of  $\bar{K}N$  subthreshold energy to be used self consistently in the binding energy calculations.

### 2.1. Hyperspherical basis

The hyperspherical-harmonics (HH) formalism is used here similarly to its application in light nuclei [22] and recently in four-quark clusters [23]. In the present case, the  $N$ -body wavefunction ( $N = 3, 4$ ) consists of a sum over products of isospin, spin and spatial components, antisymmetrized with respect to nucleons and symmetrized with respect to  $\bar{K}$  mesons. Focusing on the spatial components, translationally invariant basis functions are constructed in terms of one hyper-radial coordinate  $\rho$  and a set of  $3N - 4$  angular coordinates  $[\Omega_N]$ , substituting for  $N - 1$  Jacobi vectors. The spatial basis functions are of the form

$$\Phi_{n,[K]}(\rho, [\Omega_N]) = R_n^{[N]}(\rho) \mathcal{Y}_{[K]}^{[N]}([\Omega_N]), \quad (1)$$

where  $R_n^{[N]}(\rho)$  are hyper-radial basis functions expressible in terms of Laguerre polynomials and  $\mathcal{Y}_{[K]}^{[N]}([\Omega_N])$  are the HH functions in the angular coordinates  $\Omega_N$  expressible in terms of spherical harmonics and Jacobi polynomials. Here, the symbol  $[K]$  stands for a set of angular-momentum quantum numbers, including those of  $\hat{L}^2$ ,  $\hat{L}_z$  and  $\hat{K}^2$ , where  $\hat{K}$  is the total grand angular momentum which reduces to the total orbital angular momentum for  $N = 2$ . The HH functions  $\mathcal{Y}_{[K]}^{[N]}$  are eigenfunctions of  $\hat{K}^2$  with eigenvalues  $K(K + 3N - 5)$ , and  $\rho^K \mathcal{Y}_{[K]}^{[N]}$  are harmonic polynomials of degree  $K$ .

### 2.2. Interactions

For the  $NN$  interaction we used the Argonne AV4' potential [24] derived from the full AV18 potential by suppressing the spin-orbit and tensor interactions and readjusting the central spin and isospin dependent interactions. The AV4' potential provides an excellent approximation in  $s$ -shell nuclei to AV18. Its accuracy in  $\bar{K}$  nuclear cluster calculations has been confirmed here by comparing our results for  $K^-pp$  using AV4' with those of Ref. [6] using AV18.

For  $\bar{K}h$  interactions, where the hadron  $h$  is a nucleon or  $\bar{K}$  meson, following Refs. [14,15] we have used a generic finite-range potential

$$V_{\bar{K}h}^{(I)}(r; \sqrt{s}) = V_{\bar{K}h}^{(I)}(\sqrt{s}) \exp(-r^2/b^2) \quad (2)$$

with  $b = 0.47$  fm, where the superscript  $I$  denotes the isospin of the  $\bar{K}h$  pair and  $\sqrt{s}$  is the Mandelstam variable reducing to the total energy in the two-body c.m. system. For  $\bar{K}\bar{K}$ , owing to Bose–Einstein statistics for  $\bar{K}$  mesons, it is safe to assume that  $V_{\bar{K}\bar{K}}^{(I=0)} = 0$  at low energies where  $s$  waves dominate. For  $I_{\bar{K}\bar{K}} = 1$ ,  $V_{\bar{K}\bar{K}}^{(I=1)} = 313$  MeV was obtained in Ref. [14] by fitting to the chiral leading-order Tomozawa–Weinberg  $s$ -wave scattering length. In the absence of nearby thresholds of coupled channels, no significant energy dependence is anticipated for this weakly repulsive  $\bar{K}\bar{K}$  interaction.

The  $\bar{K}N$  interaction is an effective interaction based on chiral SU(3) meson–baryon coupled-channel dynamics with low-energy constants fitted to near-threshold  $K^-p$  scattering and reaction data plus threshold branching ratios [15]. Its HNJH version [25] used here reproduces, a-posteriori, within error bars the  $K^-p$  scattering length determined from the recent SIDDHARTA measurement of

the  $1s$  level shift and width of kaonic hydrogen [26]. The energy-dependent complex potential strengths  $V_{\bar{K}N}^{(I)}(\sqrt{s})$  were fitted in Ref. [15] by third-order polynomials in  $\sqrt{s}$  in the range  $1300 \leq \sqrt{s} \leq 1450$  MeV, covering the  $\pi\Sigma$  threshold at  $\sqrt{s} \approx 1330$  MeV, as well as the  $\bar{K}N$  threshold with isospin-averaged value  $\sqrt{s_{\text{th}}} = 1434.6$  MeV. The attractive real parts  $\text{Re} V_{\bar{K}N}^{(I)}(\sqrt{s})$  become gradually weaker for subthreshold arguments  $\sqrt{s} \lesssim 1420$  MeV, a property shown below to be crucial in restraining the calculated binding energies of  $\bar{K}$  nuclear clusters. The absorptive imaginary parts  $\text{Im} V_{\bar{K}N}^{(I)}(\sqrt{s})$  that originate from  $\bar{K}N \rightarrow \pi Y$  conversion also become weaker, but much faster, practically vanishing at the  $\pi\Sigma$  threshold.

### 2.3. Energy dependence

The issue of energy dependence in near-threshold  $\bar{K}N$  interactions deserves discussion. For a single  $\bar{K}$  meson bound together with  $A$  nucleons we define an average  $\bar{K}N$  Mandelstam variable  $\sqrt{s_{\text{av}}}$  by

$$A\sqrt{s_{\text{av}}} = \sum_{i=1}^A \sqrt{(E_K + E_i)^2 - (\vec{p}_K + \vec{p}_i)^2}, \quad (3)$$

approximating it near threshold,  $\sqrt{s_{\text{th}}} \equiv m_N + m_K = 1434.6$  MeV, by

$$A\sqrt{s_{\text{av}}} \approx A\sqrt{s_{\text{th}}} - B - (A-1)B_K - \sum_{i=1}^A (\vec{p}_K + \vec{p}_i)^2 / 2E_{\text{th}}, \quad (4)$$

where  $B$  is the total binding energy of the system and  $B_K = -E_K$ . Note that all the terms on the r.h.s. following  $A\sqrt{s_{\text{th}}}$  are negative definite, so that  $\sqrt{s_{\text{av}}} \approx \sqrt{s_{\text{th}}} + \delta\sqrt{s}$  with  $\delta\sqrt{s} < 0$ . Hence, the relevant two-body energy argument of  $V_{\bar{K}N}$  resides in the subthreshold region, forming a continuous distribution. The state of the art in non-Faddeev  $\bar{K}$  nuclear calculations is to replace this distribution by an expectation value taken in the calculated QBS [6,16–18]. Transforming squares of momenta in (4) to kinetic energies, the following expression is derived:

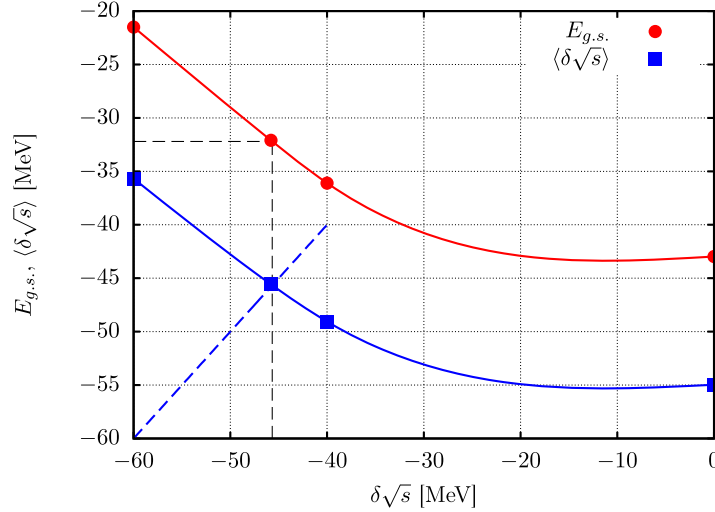
$$\langle \delta\sqrt{s} \rangle = -\frac{B}{A} - \frac{A-1}{A} B_K - \xi_N \frac{A-1}{A} \langle T_{N:N} \rangle - \xi_K \left( \frac{A-1}{A} \right)^2 \langle T_K \rangle, \quad (5)$$

where  $\xi_{N(K)} \equiv m_{N(K)} / (m_N + m_K)$ ,  $T_K$  is the kaon kinetic energy operator in the total c.m. frame and  $T_{N:N}$  is the pairwise  $NN$  kinetic energy operator in the  $NN$  pair c.m. system. Eq. (5) refines the prescription  $\langle \delta\sqrt{s} \rangle = -\eta B_K$ , with  $\eta = 1, 1/2$ , used in the two types of  $K^-pp$  variational calculations in Ref. [6]. In the limit  $A \gg 1$ , it agrees with the nuclear-matter expression given in Ref. [16] for use in kaonic atoms and  $\bar{K}$  nuclear quasibound states.

A similar procedure is applied to the  $\bar{K}\bar{K}NN$  system by summing up the four pairwise  $\bar{K}N$   $\sqrt{s}$  contributions and expanding about  $\sqrt{s_{\text{th}}}$ :

$$\langle \delta\sqrt{s} \rangle = -\frac{1}{2} (B + \xi_N \langle T_{N:N} \rangle + \xi_K \langle T_{K:K} \rangle), \quad (6)$$

where  $T_{K:K}$  is the pairwise  $\bar{K}\bar{K}$  kinetic energy operator in the  $\bar{K}\bar{K}$  pair c.m. system. Eqs. (5) and (6) provide a self-consistency cycle in  $\bar{K}$  nuclear cluster calculations by requiring that the expectation value  $\langle \delta\sqrt{s} \rangle$  derived from the solution of the Schrodinger equation agrees with the input value  $\delta\sqrt{s}$  used for  $V_{\bar{K}N}(\sqrt{s})$ . This is demonstrated in Fig. 1 for the lowest  $\bar{K}\bar{K}NN$  configuration, with  $l = 0$ ,  $J^\pi = 0^+$ . Its ground-state (g.s.) energy  $E_{g.s.}$ , calculated upon



**Fig. 1.** Self-consistency construction in  $(\bar{K}\bar{K}NN)_{I=0, J^\pi=0^+}$  binding-energy calculations. The upper (red) and lower (blue) curves show calculated values of  $E_{g.s.}$  and  $\langle\delta\sqrt{s}\rangle$  from Eq. (6), respectively, vs. input  $\delta\sqrt{s}$  values. The points connected by a vertical dashed line are the self-consistent values of  $E_{g.s.}$  and  $\langle\delta\sqrt{s}\rangle$ , the latter is obtained by the intersection of the dashed diagonal in the left-low corner with the lower (blue) curve. (For interpretation of the references to color, the reader is referred to the web version of this Letter.)

suppressing  $\text{Im } V_{\bar{K}N}$ , is shown by the upper (red) curve as a function of the input  $\delta\sqrt{s}$ . As one goes further down beginning approximately 15 MeV below threshold, the  $\bar{K}N$  effective interaction from Ref. [15] becomes gradually weaker, resulting in less binding energy. In this subthreshold energy range the expectation values  $\langle-\delta\sqrt{s}\rangle$ , calculated from Eq. (6) by solving the Schroedinger equation, also decrease upon increasing the input  $-\delta\sqrt{s}$  values as shown by the lower (blue) curve. This allows to locate a self-consistent value  $\langle\delta\sqrt{s}\rangle$  by drawing a diagonal to the lower curve and connecting it by a vertical line to the upper curve to identify the associated self-consistent value of  $E_{g.s.}$ . For  $(\bar{K}\bar{K}NN)_{I=0, J^\pi=0^+}$ , this construction yields a self-consistent value  $\langle\delta\sqrt{s}\rangle = -46$  MeV and a corresponding value  $E_{g.s.}(\langle\delta\sqrt{s}\rangle) = -32.1$  MeV. If the energy dependence of  $V_{\bar{K}N}(\sqrt{s})$  were neglected, and  $V_{\bar{K}N}(\sqrt{s}_{\text{th}})$  corresponding to  $\delta\sqrt{s} = 0$  were used instead, a considerably stronger binding would have emerged:  $E_{g.s.}(\delta\sqrt{s} = 0) = -43.0$  MeV.

### 3. Results and discussion

We now present the results of self-consistent three-body and four-body calculations of  $\bar{K}$  and  $\bar{K}\bar{K}$  nuclear clusters. The three-body calculations have been tested by comparing with similar calculations for  $K^-pp$  [6].

For a  $\bar{K}$  nuclear cluster with global quantum numbers  $I, L, S, J^\pi$ , the potential and kinetic energy matrix elements were evaluated in the HH basis. The interactions specified in Section 2.2 conserve  $L$  and  $S$ , the latter is given by the nuclear spin  $S_N$ . Since no  $L \neq 0$  QBS are likely to become particle stable upon switching off  $\text{Im } V_{\bar{K}N}$ , we limit our considerations to  $L = 0$ , resulting in  $J = S = S_N$  with parity  $\pm$  for even/odd number of  $\bar{K}$  mesons, respectively. Although the total isospin  $I$  is conserved by these charge-independent interactions, the isospin dependence of  $V_{\bar{K}N}$  induces  $\Delta I_N = 1$  nuclear charge-exchange transitions, so that the nuclear isospin  $I_N$  need not generally be conserved. Suppressing  $\text{Im } V_{\bar{K}N}$ , the g.s. energy  $E_{g.s.}$  was calculated in a model space spanned by HH basis functions with eigenvalues  $K \leq K_{\text{max}}$ . Self-consistent calculations were done for  $\sqrt{s}$  from the  $\bar{K}N$  threshold down to 80 MeV below, at which value the error incurred by the near-threshold approximation (4) is only 2.4 MeV. Self consistency in  $\delta\sqrt{s}$  was reached after typically five cycles. The convergence of binding energy calculations for particle-stable g.s. configurations

is shown in Fig. 2 as a function of  $K_{\text{max}}$ . With the exception of the  $(\bar{K}NNN)_{I=1}$  cluster, good convergence was reached for values of  $K_{\text{max}} \approx 30\text{--}40$ . The poorer convergence for  $(\bar{K}NNN)_{I=1}$  is apparently due to its proximity to the  $(\bar{K}NN)_{I=1/2} + N$  threshold. Asymptotic values of  $E_{g.s.}$  were found by fitting the constants  $C$  and  $\gamma$  of the parametrization

$$E(K_{\text{max}}) = E_{g.s.} + \frac{C}{K_{\text{max}}^\gamma} \quad (7)$$

to values of  $E(K_{\text{max}})$  calculated for sufficiently high values of  $K_{\text{max}}$ . The accuracy reached is better than 0.1 MeV in the three-body calculations and about 0.2 MeV in the four-body calculations.

The conversion width  $\Gamma$  was then evaluated through the expression

$$\Gamma = -2\langle\Psi_{g.s.}|\text{Im } \mathcal{V}_{\bar{K}N}|\Psi_{g.s.}\rangle, \quad (8)$$

where  $\mathcal{V}_{\bar{K}N}$  sums over all pairwise  $\bar{K}N$  interactions. Since  $|\text{Im } \mathcal{V}_{\bar{K}N}| \ll |\text{Re } \mathcal{V}_{\bar{K}N}|$ , this is a reasonable approximation for the width. The dependence of the calculated width  $\Gamma$  of  $\bar{K}$  nuclear clusters on the input  $\delta\sqrt{s}$  value used for the subthreshold  $\bar{K}N$  energy is demonstrated in Fig. 3 for the same  $\bar{K}$  nuclear clusters depicted in Fig. 2. The width is seen almost invariably to decrease upon increasing  $-\delta\sqrt{s}$ , i.e. upon going deeper below threshold. This is similar to the dependence of  $E_{g.s.}$  on the input  $\delta\sqrt{s}$ , as displayed for  $(\bar{K}\bar{K}NN)_{I=0}$  in Fig. 1. It is worth noting that the calculated widths of the single- $\bar{K}$  nuclear systems are clustered roughly in a range of 30–40 MeV. Given a calculated width  $\Gamma_{\bar{K}N} = 43.6$  MeV for the underlying  $(\bar{K}N)_{I=0}$  QBS, a scale of  $\Gamma$  (single  $\bar{K}$ ) approximately 40 MeV appears quite natural. In contrast, the width calculated for the double- $\bar{K}$  system  $(\bar{K}\bar{K}NN)_{I=0}$  is about twice larger, approximately 80 MeV.

In Table 1 we compare results of the present work for  $(\bar{K}N)_{I=0}$  and  $(\bar{K}NN)_{I=1/2}$  QBS with those by Doté et al. [6]. Our  $(\bar{K}N)_{I=0}$  calculation reproduces that of Ref. [14] and agrees with that in Ref. [6] to within 0.1 MeV out of binding energy  $B \approx 11.5$  MeV and 0.2 MeV out of width  $\Gamma \approx 43.7$  MeV, a precision of better than 1%. We note that this  $\Lambda(1405)$ -like QBS is bound considerably weaker than a QBS required by construction to reproduce  $\Lambda(1405)$  nominally, with  $B_{\Lambda(1405)} \approx 27$  MeV [19]. For a more complete discussion of this point we refer to [15].

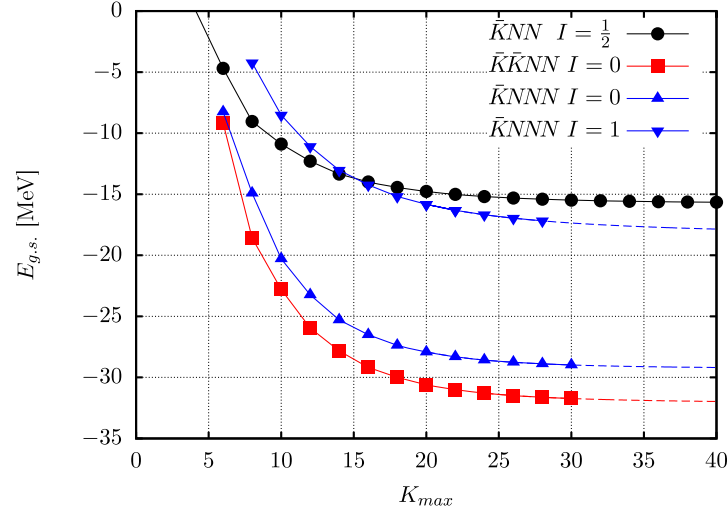


Fig. 2. Ground-state energies of  $\bar{K}$  nuclear clusters, calculated self consistently, as a function of  $K_{max}$ . The dashed lines show extrapolation according to Eq. (7).

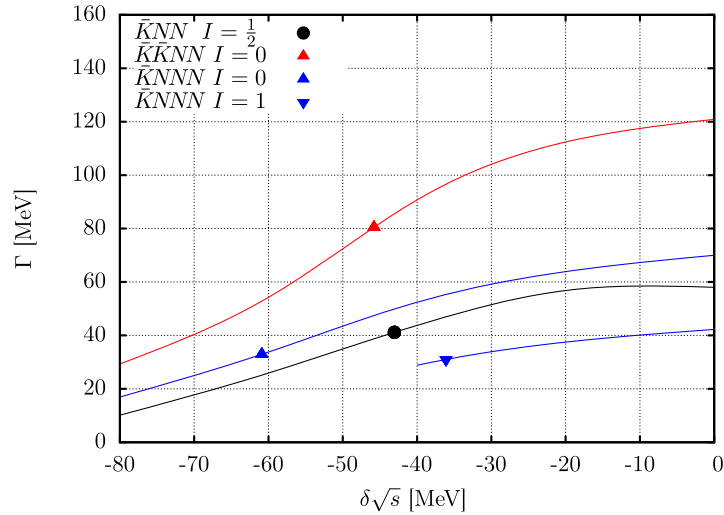


Fig. 3. Conversion widths  $\Gamma$  of  $\bar{K}$  nuclear clusters calculated from Eq. (8) as a function of  $\delta\sqrt{s}$ . The widths appropriate to the self-consistent values  $\langle\delta\sqrt{s}\rangle$  are denoted on each one of the curves.

**Table 1**  
Comparison of  $\bar{K}N$  and  $\bar{K}NN$  QBS calculations, BGL (present) vs. DHW [6].

QBS	$I, J^\pi$	Ref.	$\langle\delta\sqrt{s}\rangle$ [MeV]	$B$ [MeV]	$\Gamma$ [MeV]	$B_K$ [MeV]	$r_{NN}$ [fm]	$r_{KN}$ [fm]
$\bar{K}N$	$0, \frac{1}{2}^-$	BGL	-11.4	11.4	43.6	11.4	-	1.87
		DHW	-11.5	11.5	43.8 <sup>†</sup>	11.5	-	1.86
$\bar{K}NN$	$\frac{1}{2}, 0^-$	BGL	-43	15.7	41.2	35.5	2.41	2.15
		DHW	-39	16.9	47.0	38.9	2.21	1.97
& $I_N = 1$		BGL	-35	11.0	38.8	27.9	2.33	2.21
		DHW	-31	12.0	44.8	31.0	2.13	2.01

<sup>†</sup> We thank Dr. A. Doté for communicating to us this width value.

For  $\bar{K}NN$  with  $I = 1/2$  and  $J^\pi = 0^-$ , loosely termed  $K^-pp$ , we compare the present calculation with the type-I HNJH-versed DHW variational calculation [6] for which the implied effective  $\langle\delta\sqrt{s}\rangle$  value is close to our self-consistent  $\langle\delta\sqrt{s}\rangle$  value. From their type-I,II calculations one concludes that  $\delta B/\langle\delta\sqrt{s}\rangle \approx 0.24$ , so that our binding energy value  $B$  should come out smaller by approximately 1 MeV than their listed type-I  $B$ . The remainder 0.2 MeV of the 1.2 MeV difference between rows 3 and 4 in the table is attributed to using slightly different  $NN$  interactions: AV4' here, AV18 in Ref. [6]. Rows 5 and 6 of the table demonstrate the effect of limiting the model space to  $I_N = 1$ ,

compatible with the dominant  $s$ -wave  $NN$  configuration. This results in a decrease of the calculated binding energy by  $4.8 \pm 0.1$  MeV. The 1 MeV difference between rows 5 and 6 is consistent with the estimate made above for  $\delta B/\langle\delta\sqrt{s}\rangle$ , with no room within  $NN$   $s$  waves for any marked difference arising from the difference between using AV4' (BGL) and AV18 (DHW). Finally, the differences of order 10–15% between the two width calculations, and between the two r.m.s. distance calculations, reflect the sensitivity of these entities to details of the three-body wavefunction, particularly through the effective  $\langle\delta\sqrt{s}\rangle$  value used.

**Table 2**  
Results of  $\bar{K}NNN$  and  $\bar{K}\bar{K}NN$  QBS calculations.

QBS	$I, J^\pi$	$\langle \delta\sqrt{s} \rangle$ [MeV]	$B$ [MeV]	$\Gamma$ [MeV]	$B_K$ [MeV]	$r_{NN}$ [fm]	$r_{NK}$ [fm]	$r_{KK}$ [fm]
$\bar{K}NNN$	$0, \frac{1}{2}^+$	-61	29.3	32.9	36.6	2.07	2.05	-
	$1, \frac{1}{2}^+$	-36	18.5	31.0	21.0	2.33	2.55	-
$\bar{K}\bar{K}NN$	$0, 0^+$	-46	32.1	80.5	33.6	1.84	1.88	2.31
	& $V_{\bar{K}\bar{K}} = 0$	-52	36.1	83.2	37.9	1.71	1.70	2.01

We have also searched for a  $\bar{K}NN$  QBS with  $I = 1/2$  and  $J^\pi = 1^-$ , loosely termed  $K^-d$ . The possibility of a QBS with these quantum numbers has hardly been discussed in the literature, apparently since it was realized from the very beginning [27] that  $K^-d$  is less exposed than  $K^-pp$ , by a ratio close to  $1 \div 3$ , to the strongly attractive  $V_{\bar{K}N}^{(0)}$  interaction. We are not aware of any genuine three-body calculation for  $K^-d$ .<sup>2</sup> Our calculations did not produce any  $I = 1/2, J^\pi = 1^-$  QBS below the  $(\bar{K}N)_{I=0} + N$  threshold, i.e. with total binding energy exceeding 11 MeV. Whether or not such a QBS exists above the  $(\bar{K}N)_{I=0} + N$  threshold is an open question which cannot be resolved within the present HH calculations that normally converge at the lowest energy state for given quantum numbers.

In Table 2 we present new results for  $\bar{K}NNN$  and  $\bar{K}\bar{K}NN$  QBS. The first two rows concern the  $\bar{K}NNN$  system essentially based on the  $I_N = 1/2$  mirror nuclei  ${}^3\text{H}$  and  ${}^3\text{He}$  which are bound by 8.99 MeV in this calculation. The  $\bar{K}$  nuclear interaction splits the two resultant  $I = 0, 1$   $\bar{K}NNN$  QBS such that the  $I = 0$  QBS is the lower of the two. The 11 MeV isospin splitting is small compared to the approximately 30 MeV conversion width of each of these states. We note that the  $I = 0$  QBS is bound weakly compared to the tight binding over 100 MeV predicted for it by Akaishi and Yamazaki [19]. Its spatial dimensions, with interparticle distances all exceeding 2 fm, also do not indicate a very tight structure. The imposition of self-consistency in the binding energy calculation is responsible for the relatively low value  $B(\bar{K}NNN)_{I=0} = 29.3$  MeV, compared to a considerably higher value  $B(\bar{K}NNN)_{I=0}^{\delta\sqrt{s}=0} = 42.1$  MeV upon using the threshold  $\bar{K}N$  interaction. Higher values  $B(\bar{K}NNN)_{I=0,1}$  would also have been obtained had we used the self-consistency requirement  $\langle \delta\sqrt{s} \rangle = -B_K$  [6] which for  $K^-pp$  gave a value of  $B$  close to the one found by using (5), see Table 1.

The last two rows of Table 2 report on the  $S = -2$  ( $\bar{K}\bar{K}NN$ ) $_{I=0}$  QBS which has been highlighted as a possible gateway to kaon condensation in self-bound systems, given its large binding energy over 100 MeV predicted by Yamazaki et al. [20]. Our calculated value  $B = 32.1$  MeV is comparable with that for the  $S = -1$  ( $\bar{K}NNN$ ) $_{I=0}$  QBS, and is a factor of two larger than for the lowest  $\bar{K}NN$  QBS with  $I = 1/2$  and  $J^\pi = 0^-$ . Note, however, that ( $\bar{K}\bar{K}NN$ ) $_{I=0}$  is bound by less than 10 MeV with respect to the threshold for decay to a pair of ( $\bar{K}N$ ) $_{I=0}$   $\Lambda(1405)$ -like QBS. This apparent relatively weak binding of ( $\bar{K}\bar{K}NN$ ) $_{I=0}$  is owing to the restraining effect of handling self consistently the energy dependent  $\bar{K}N$  interaction. Finally, the last row of the table shows what happens when the repulsive  $V_{\bar{K}\bar{K}}$  is switched off. The effect is mild, increasing  $B$  by only 4 MeV. Nevertheless, inspection of the r.m.s. distances in ( $\bar{K}\bar{K}NN$ ) $_{I=0}$  reveals a more compact structure than ( $\bar{K}NNN$ ) $_{I=0}$ , which is also reflected by the large value of  $\Gamma(\bar{K}\bar{K}NN)_{I=0}$ .

<sup>2</sup> However, very recently Oset et al. [28] made an estimate within the Fixed Center Approximation for a  $K^-d$  QBS with total binding energy  $B = 9$  MeV and conversion width  $\Gamma \approx 30$  MeV. Alternatively, extrapolating below threshold the best educated guess for the scattering length  $a_{K^-d}$  [29] yields an estimate of  $B = 6.6$  MeV and  $\Gamma \approx 29$  MeV.

## 4. Conclusion

In conclusion, we have performed calculations of three-body  $\bar{K}NN$  and four-body  $\bar{K}NNN$  and  $\bar{K}\bar{K}NN$  QBS systems. Using practically identical interactions to those used in the  $K^-pp$  chiral model calculations by Doté et al. [6], we were able to test our calculations for this QBS against theirs. Given the low binding energy  $B(K^-pp) \approx 16$  MeV and sizable conversion width  $\Gamma_{\text{conv}}(K^-pp) \approx 40$  MeV, it might be difficult to identify such a near-threshold QBS unambiguously in ongoing experimental searches. This situation gets further complicated by two additional factors: (i) the possible presence of a near-threshold  $K^-d$  QBS in the same charge state as the one in which  $K^-pp$  is searched on, and (ii) additional two-nucleon absorption widths  $\Delta\Gamma_{\text{abs}}$  accounting for the poorly understood non-pionic processes  $\bar{K}NN \rightarrow YN$ . For  $K^-pp$  we note the estimate  $\Delta\Gamma_{\text{abs}}(K^-pp) \lesssim 10$  MeV [6]. Appreciable  $p$ -wave contributions to the  $K^-pp$  width were also suggested in Ref. [6], but doubts have been recently expressed on the effectiveness of a  $p$ -wave  $\bar{K}N$  interaction by testing its role in kaonic atoms [16]. Altogether, the widths of  $\bar{K}NN$  QBS are likely to be dominated by their conversion widths.

For the four-body QBS systems  $\bar{K}NNN$  and  $\bar{K}\bar{K}NN$  we found relatively modest binding, of order 30 MeV in both, with conversion widths ranging from about 30 MeV for each of the  $\bar{K}NNN$  QBS to about 80 MeV for the lowest  $\bar{K}\bar{K}NN$  QBS. These systems, although somewhat more compact than  $K^-pp$ , are not as compact as suggested by Yamazaki et al. [19–21]. Their  $\bar{K}N$  r.m.s. distances do not fall below that of the  $\Lambda(1405)$ -like  $\bar{K}N$  QBS, and their  $NN$  r.m.s. distances exceed that of nuclear matter ( $\approx 1.7$  fm). For a conservative estimate of the absorption widths  $\Delta\Gamma_{\text{abs}}$  in these systems, we count the number of nucleons  $n$  available to join a given  $\bar{K}N$  correlated pair, one pair per each  $\bar{K}$ . This gives twice as large  $n$  for each of the four-body systems ( $n = 2$ ) with respect to  $K^-pp$  ( $n = 1$ ). Hence, neglecting three-nucleon absorption,  $\Delta\Gamma_{\text{abs}}(\bar{K}NNN, \bar{K}\bar{K}NN) \sim 20$  MeV.

The energy dependence of the subthreshold  $\bar{K}N$  effective interaction, constructed in Ref. [15] within a coupled channel chiral model, was found to be instrumental in restraining the binding of the four-body  $\bar{K}$  nuclear clusters through the self-consistency requirement derived here for these light systems. A strong  $\bar{K}N$  interaction operates to form tightly bound compact structures, necessarily accompanied by large kinetic energies. This leads by Eqs. (5) and (6) to substantial values of the energy shift  $\langle \delta\sqrt{s} \rangle$  which give rise to weaker input  $\bar{K}N$  interactions, resulting in less binding as demonstrated in Fig. 1 for  $\bar{K}\bar{K}NN$ . However, dispersive contributions to the binding energy of QBS cannot be excluded. Recent fits to kaonic atoms [16,17] suggest that  $\Delta B_{\text{disp}} \sim \Delta\Gamma_{\text{abs}}$ , so that these binding energies could reach values  $B(K^-pp) \sim 25$  MeV and  $B(\bar{K}NNN, \bar{K}\bar{K}NN) \sim 50$  MeV. For heavier  $\bar{K}$ -nuclear clusters where the nuclear density is closer to nuclear-matter density, a restraining mechanism similar to the one discussed here has been shown to be operative [18]. Other restraining, or saturation mechanisms are likely to be operative such as the increased  $\bar{K}\bar{K}$  repulsion upon adding  $\bar{K}$  mesons [30]. It is therefore quite unlikely that strange dense matter is realized through  $\bar{K}$  mesons as argued repeatedly by Yamazaki et al. [20,21].

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