Research papers

Estimating the exceedance probability of extreme rainfalls up to the probable maximum precipitation

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A B S T R A C T

If risk-based criteria are used in the design of high hazard structures (such as dam spillways and nuclear power stations), then it is necessary to estimate the annual exceedance probability (AEP) of extreme rainfalls up to and including the Probable Maximum Precipitation (PMP). This paper describes the development and application of two largely independent methods to estimate the frequencies of such extreme rainfalls. One method is based on stochastic storm transposition (SST), which combines the “arrival” and “transposition” probabilities of an extreme storm using the total probability theorem. The second method, based on “stochastic storm regression” (SSR), combines frequency curves of point rainfalls with regression estimates of local and transposed areal rainfalls; rainfall maxima are generated by stochastically sampling the independent variates, where the required exceedance probabilities are obtained using the total probability theorem. The methods are applied to two large catchments (with areas of 3550 km² and 15,280 km²) located in inland southern Australia. Both methods were found to provide similar estimates of the frequency of extreme areal rainfalls for the two study catchments. The best estimates of the AEP of the PMP for the smaller and larger of the catchments were found to be 10⁻⁶ and 10⁻⁸, respectively, but the uncertainty of these estimates spans one to two orders of magnitude. Additionally, the SST method was applied to a range of locations within a meteorologically homogenous region to investigate the nature of the relationship between the AEP of PMP and catchment area.

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1. Introduction

Estimates of Probable Maximum Precipitation (PMP) have been used for many years to derive a “maximum” loading for designs of high hazard infrastructure such as dams and nuclear power plant structures (Myers, 1967; International Commission on Large Dams, 1992; Prasad et al., 2011). The PMP is most commonly used to derive an extreme flood using an appropriate model of the rainfall-runoff process (Newton, 1983; Reed and Field, 1992; Federal Energy Regulatory Commission, 2001), or it can be used to assess the risk of direct flooding on critical infrastructure due to large storms (World Meteorological Organisation, WMO, 1986). Thus, while the theoretical definition infers a physical upper limit with a zero probability of exceedance, in practice such estimates are based on a set of simplifying assumptions that involve extrapolation from the hydrometeorological conditions of observed large events to maximised conditions. It is therefore useful to differentiate between the concept of a theoretical PMP and its “operational estimate”, which represents the “steps followed by hydrometeorologists in arriving at the answers supplied to engineers for hydrological design purposes” (WMO, 1986). While this definition infers a physical upper limit with a zero probability of exceedance, in practice such estimates are based on a set of simplifying assumptions that involve extrapolation from the hydrometeorological conditions of observed large events to maximised conditions. It is therefore useful to differentiate between the concept of a theoretical PMP and its “operational estimate”, which represents the “steps followed by hydrometeorologists in arriving at the answers supplied to engineers for hydrological design purposes” (WMO, 1986). Thus, while the theoretical definition of the PMP implies an event that cannot be exceeded, there is a small, but finite probability that the operational estimate of the PMP may be exceeded.

Estimating the annual exceedance probability (AEP) of the PMP has important practical implications as such estimates have a large influence on the estimated risks of failure. The need to assess the
potential for failure using measures based on risk rather than flood magnitude was given major impetus by the U.S. Nuclear Regulatory Commission (National Research Council, 1988). Since that time, there has been an increasing requirement to incorporate estimates of flood risks in probabilistic risk assessments in both the dams and nuclear industries (Bureau of Reclamation, 1999; Federal Emergency Management Agency, 2001; Australian National Commission of Large Dams, 2003; Nuclear Regulatory Commission, 2013).

A number of studies have been undertaken in the past to estimate the AEP of the PMP. The National Research Council (NRC, 1988) recommended the use of storm transposition techniques to estimate extreme event frequencies. They developed a general approach for estimation of the AEP of extreme rainfall, which included an allowance for storms centred outside the catchment. Subsequently, two studies, Fontaine and Potter (1989) and Wilson and Foufoula-Georgiou (1990), pursued further the NRC approach.

Fontaine and Potter (1989) developed a method of “stochastic storm transposition” (SST) based on the assumption that storms could occur with equal probability anywhere in a transposition region. They calculated the probability that a given areal rainfall \( x \) over a specific catchment would be exceeded by considering the locus of transposed storm centres which just caused the rainfall \( x \) to be exceeded. The probability of this event was computed as the ratio of the area within the locus to the area of the transposition region, and this was evaluated numerically by dividing the transposition into a large number of \((1 \text{ m}^2)\) grid cells. Their study was limited in that they only considered four historical storms, and their depths were not adjusted for changes in magnitude or orientation associated with transposition. They concluded that there was a need to relax the requirement of strict homogeneity so that the area of the transposition region could be maximised, and that this could be achieved by developing procedures that would explicitly account for spatial variations of important characteristics.

Wilson and Foufoula-Georgiou (1990) also applied an SST approach using a framework first developed by Foufoula-Georgiou (1989). They described the occurrence of extreme storms using a joint probability distribution of five storm parameters (magnitude, orientation, shape, and within-storm spatial variability) and two location parameters (for the storm centre). The joint distribution of the five storm parameters was determined by Monte Carlo sampling, and the storm centres were located using a spatial occurrence model that allowed for non-uniform distribution of storm centres in the transposition region. This work was later extended to derive exceedance probabilities of floods using a conceptual rainfall-runoff model, where the initial moisture and temporal pattern characteristics were treated as stochastic variables (Franchini et al., 1996). England et al. (2014) also applied the SST approach to rainfalls and then used a physically-based (deterministic) model to derive a flood frequency curve. They considered fifteen extreme storms with areal extents limited to around 13,000 km\(^2\) within a transposition region that encompassed approximately 10\(^6\) km\(^2\).

Wright et al. (2013) applied the SST approach in combination with an ensemble of storm characteristics developed from high-resolution radar fields (a “storm catalogue”) to derive rainfall frequency estimates as rare as 10\(^{-3}\). They adopted a Poisson-distributed model of storm occurrence and a uniform sampling approach to randomly transpose storms from the storm catalogue over a local region considered to be climatologically homogeneous. One advantage of their approach is that the temporal and spatial characteristics of the rainfall fields are represented in a realistically complex manner, though as currently formulated the use of radar data and the requirement for uniformity of storm occurrence across the transposition region limits the temporal and spatial dimensions of the domain used to trade space for time compared to the methods presented here. Wright et al. (2014) then extended this study and used the synthesised rainfall fields to drive a physically based distributed hydrologic model to derive corresponding flood frequency estimates for a range of catchment areas up to 110 km\(^2\).

One of the key factors influencing the complexity of the adopted stochastic storm transposition approach is the need to accommodate the systematic variation in rainfall depth associated with site factors (eg elevation, aspect, and the distance from the moisture source). Foufoula-Georgiou (1989) defines the transposition region as the area within which all the occurred storms can be transposed anywhere with an adjustment to their probability of occurrence (or alternatively, could be transposed anywhere with the same occurrence probability but with an adjustment to their depth). Agho et al. (2000) avoided much of the complexity involved in the SST approach by transforming the storm rainfalls into non-dimensional depths, such that the exceedance probability of a particular non-dimensional depth is the same anywhere in the transposition region. They standardised the rainfalls using a non-dimensional approach introduced by Schaefer (1994) in which extreme storm depths were expressed as a fraction of the PMP derived for that location. This approach assumes that the PMP estimate explicitly allows for the effects of convergence and topography, and standardisation thus removes the systematic spatial variability of extreme rainfall from consideration. Schaefer (1994) originally fitted an exponential distribution to the non-dimensional storm rainfall depths, and then used this to estimate the arrival probabilities of extreme storms for a given transposition region; these probabilities were then combined with Alexander’s (1963) simplistic calculation of transposition probability to provide estimates of the AEP of the PMP in the western United States. Nathan et al. (1999) used a similar approach to estimate the AEP of PMPs for inland and coastal areas of south-eastern Australia based on the storm catalogue used to develop generalised PMP estimates for that region (Meighen and Kennedy, 1995).

Another approach used to estimate the AEP of the PMP was developed by Klemes (1993), who developed a combinatorial method that considered the joint distributions of the independent components (precipitable water, and orographic and storm efficiencies) that combined to produce the PMP. Klemes applied his approach to a site in Canada (Coquitlam Lake) using only at-site data, and this was then later extended by Neudorf (1994) to estimate the AEP of the Probable Maximum Flood (PMF). Pearse and Laurenson (1997) also adopted a joint probability approach to estimate the AEP of the PMP, and they combined a regional distribution of storm efficiency with a distribution of convergence rainfall developed for a specific site in eastern Australia. The reliance of these estimates on at-site data precludes consideration of extreme storm types that meteorologically might be expected to occur over the catchment but which have not yet appeared in the historic record. While the use of combinatorial approaches is attractive in that they provide estimates of extremes based on feasible combinations of their formative components, without transposition from a wider region the combinations considered will always be limited by events that have occurred in the local historic record.

In an alternative approach, Schaefer (2005) derived an estimate of areal rainfall exceedance probabilities by stochastically combining a frequency curve of point rainfalls for a location within a catchment (the “index station”) with a regression relationship that provided estimates of areal catchment rainfalls as a function of point rainfalls at the index station. The areal rainfalls were obtained from a Thiessen analysis of the largest storm depths recorded over the catchment, and the point rainfall frequency...
The Dartmouth Dam catchment is 3550 km² and it lies within the Murray which is the largest river in Australia (Fig. 1). The area of the Snowy Mountain region and has a total area of 15,280 km². The Hume Dam catchment rises from an elevation of 154 m above sea level to 2228 m at Mount Kosciuszko. The upper region of the catchment is dominated by the elevated plateau of the Snowy Mountains, from where the terrain falls steeply down to the alluvial flood plains in the north western part of the catchment. While the Hume Dam catchment might be considered “small” compared to the area of the whole Murray Darling Basin, it is “large” in the design context relevant to this paper. Dams represent the most common form of infrastructure requiring estimates of risks beyond 10⁻³, and most dams are built in upland areas to minimise the size of the barrier relative to the volume of water impounded; the catchment areas upstream of dams thus tend to be smaller than catchments relevant to floodplain management.

The Hume Dam catchment receives most of its precipitation during the winter and early spring as both rainfall and snow. During the months April to November, rainfall in the catchment is generally caused by the movement of cold fronts. From December to March, the influence of tropical systems to the north can produce significant rainfall. There is substantial variation in annual average precipitation, with rainfall ranging from 500 mm in the areas of plains to the west to over 2000 mm in the higher peaks. Snow occurs at higher elevations in winter months, with the area above the normal August winter snowline accounting for about 7% of the catchment. Average annual evaporation varies from 620 mm to 750 mm.

The transposition region relevant to this study is the inland zone of south-eastern Australia (Fig. 1). This area represents a meteorologically homogenous region in terms of the types of storms that have produced the largest rainfall depths on record. The precise extent of the region was defined by Minty et al. (1996) for development of the Generalised South-east Australia Method (GSAM), which is a method used to estimate PMP in those regions of Australia where tropical storms are not the source of the greatest depths of rainfall, and where topographic influences vary markedly. The Hume catchment represents about 1% of the total area covered by the inland GSAM zone. Fig. 1 also shows the location of the storms used in development of the GSAM method of PMP estimation, where the notional maximum areal extent of each storm is represented by a red-bordered parallelogram. A small number of these events lie outside the boundary of the GSAM region, but these storms were assessed by Minty et al. (1996) to be suitable for consideration in development of the GSAM method.

Three primary datasets were used in development and application of the methods and these are summarised in Table 1. Both the SSR and SST methods rely heavily on the Australian Water Availability Project (AWAP) dataset (Jones et al., 2009), which provides a gridded analysis of historic daily gauged rainfalls collected over the past 113 years. Additionally, the SSR method relies on the catalogue of significant rainfall events used to develop the GSAM method (Meighen and Kennedy, 1995), and on gauged records of daily rainfalls within the study catchments and surrounding region.

It is worth noting the analysis of these data sets is based on the assumption that the historic climate is stationary. This approach is consistent with the findings of Bates et al. (2015) in which they found that in eastern Australia rainfall frequency estimates derived assuming stationarity lie within the uncertainty bounds derived using non-stationary assumptions. Also, Green et al. (2015) found a lack of spatial consistency in stationarity tests of historic rainfall maxima, and this was used to justify a stationary climate assumption for the current estimates of rainfall frequencies for design purposes in Australia. However, other authors (Verdon-Kidd and Kiem, 2015) have concluded that regime shifts in annual maxima do need to be considered. Overall it is considered that the issue of non-stationarity requires further investigation, but it is not unreasonable to exclude this consideration from the present analyses.

3. Methodology

3.1. Stochastic storm regression

The SSR approach as devised by Schaefer combines the analysis of historic storms with stochastic simulation of a frequency-based regression function. The approach is based on the key premise that it is easier to derive a frequency relationship for point rainfalls than it is for areal rainfalls; its application involves the derivation of frequency relationships for a small number of point locations within the catchment of interest, and a frequency curve of areal rainfalls is obtained by the stochastic simulation of terms in a regression relationship that predicts areal rainfalls as a function of these point rainfalls.
In the development presented here use is made of certain key terms, and these are italicised when first introduced to clarify their intended meaning. The SSR approach involves the following five key elements: (i) the preparation of a data set of storm areal rainfall depths based on large historic events that have occurred within the catchment (local storms) and within a wider transposition region (regional storms), (ii) the transformation of these local and regional storms into non-dimensional rainfall depths (standardised storms), (iii) the development of a regression relationship to estimate average areal storm depths as a function of point rainfalls at a small number of sites (key sites) located within the catchment (the storm regression relationship), (iv) the derivation of frequency curves of point rainfalls for the key sites, and (v) the derivation of a frequency curve of areal rainfalls based on the stochastic simulation of point rainfalls in the storm regression relationship in a manner that accounts for the natural variability (and uncertainty) in the governing process. Schaefer progressively developed this approach over a number of years in unpublished studies for dam-owning agencies wishing to take a risk-based approach to their decision-making. A brief description of each of the above elements is provided below, and this is followed by a description of the simulation scheme developed here for its implementation.

3.1.1. Use of local and regional storm data to derive standardised storms

The identification of historic local and regional storms required to develop the storm regression relationship is most easily undertaken using gridded rainfall data sets as these facilitate the identification of areal rainfalls. Gridded daily rainfalls are available at a resolution of 0.05° × 0.05° (approximately 5 km by 5 km) for the whole of Australia (Jones et al., 2009), and similar data sets are available in other countries (eg Rajeevan et al., 2006; Klok and Klein Tank, 2009; Herrera et al., 2012). As the process of gridding involves interpolation from a relatively sparse network of rainfall stations, it is necessary to check and correct for the presence of bias in gridded data sets when estimating rainfall extremes (Tozer et al., 2012; King et al., 2013).

The derivation of standardised storms is a particularly important step as it allows regional storms that are rarer than have been observed in the local catchment to be included in the development of the storm regression relationship. The objective of the standardisation is to transform the observed rainfall maxima such that it can be assumed that the exceedance probability of a particular non-dimensional depth is the same anywhere in the transposition region. Perhaps the simplest approach to this is to divide each rainfall depth by a suitable probability quantile (index variable) that is relevant to the rare rainfalls of interest. For areal rainfalls this is most readily achieved by deriving rainfall quantiles for all observed data in the transposition region, and then using spatial interpolation techniques in conjunction with covariates based on elevation (and other factors) to derive a regular field for the areas of interest (Hutchinson 1995; Daly et al., 2008). However, the implicit assumption of using a single rainfall quantile as the index variable is that the scale and shape of the governing probability distributions over the range of observed maxima are the same for any location within the region. Accordingly, it is best to select a rainfall quantile that balances the need to be representative of the observed extremes of interest whilst minimising the uncertainty involved in its estimation. More sophisticated standardisation schemes that take account of variation in both the location and scale attributes of the distribution (Majone and Tomirotti, 2004) could also be considered.

3.1.2. Development of storm regression relationship

Once the standardised storms have been derived they can be transposed to the catchment of interest by the inverse of the transformation, that is, by multiplying the non-dimensional rainfall depths by the index variable relevant to the catchment of interest. The data set comprised of both local and transposed regional storms is then used to develop a regression relationship between point rainfall depths at k selected key sites (Rk) and areal rainfall depths (D):

\[ D = \beta_0 + \sum_{i=1}^{k} \beta_i R_k + \epsilon \]  

(1)

where \( \beta_i \) are the regression coefficients for the key site rainfalls (\( R_k \)) and \( \epsilon \) is the error term. The site which explains the majority of the...
variance in the storm regression relationship is referred to here as the prime key site ($R_1$), and the other locations are referred to as the secondary key sites ($R_{2...k}$). This has the same format as a Thiessen polygon approach for computation of areal rainfall where the regression coefficients reflect the relative contribution from each of the key sites.

3.1.3. Frequency curves of point rainfalls at key sites

Frequency curves of point rainfalls at the key sites are best obtained using techniques that “substitute space for time”, that is, using methods in which rainfall maxima at different locations are used to compensate for short records at the sites of interest (National Research Council, 1988). The techniques for doing this are well established and are particularly suited to the estimation of rainfall extremes as in general their frequency characteristics are more well-behaved over large areas than floods (Stedinger et al., 1993). Approaches based on regional L-Moments (Hosking and Wallis, 2005) are typically used for this, and parameters can be averaged across homogeneous regions or varied in accordance with site characteristics (Schafer, 1990; Wallis et al., 2007). Other approaches adopt a modified station-year method in which data from several sites are pooled to create a longer record, whilst appropriate regard is given to the influence of inter-site correlation (Nandakumar et al., 1997; Reed et al., 1999). It should be noted that the focus of these techniques is on the estimation of point rainfalls, and the subsequent steps as outlined below have been devised to account for the reduction in average rainfall depths associated with areal storms.

3.1.4. Stochastic simulation of areal frequency curve

The stochastic simulation of the areal frequency curve developed here is best described by reference to the flow chart provided in Fig. 2, where a distinction is made in the simulation scheme between epistemic and aleatory uncertainty (Beven, 2016). The steps involved in the simulation of hydrologic variability (aleatory uncertainty) are shown in the left side of the figure, and those representing parameter uncertainty (epistemic uncertainty) are shown on the right. The consideration of aleatory uncertainty yields a single frequency curve of areal rainfalls for a given set of model parameters (steps A–D), and this is repeated many times to reflect the epistemic uncertainty associated with parameter identification (steps E–G). The steps involved in the former may be briefly described as follows: (A) point rainfalls at the prime key site are stochastically generated from an appropriate probability distribution using a stratified sampling scheme in which the probability domain is divided into $n$ intervals; (B) correlated rainfalls are generated for the other key sites from the marginal distribution of extreme point rainfalls relevant to each location; the desired degree of correlation is achieved by first generating independent random variates then applying a rotational transformation of the coordinate system (Saucier, 2000); (C) the key site point rainfalls obtained in Steps A and B are used to derive areal rainfalls using the storm regression relationship (Eq. (1)); and (D) once Steps A–C have been computed over the full stratified probability domain, a frequency curve of areal rainfalls is derived using the Total Probability Theorem, in which the probability that a particular areal rainfall depth $D$ equals or exceeds $d$ is conditioned upon the probability of the point rainfall at the prime key site ($R_1$) using:

$$P(D > d) = \int_{R_1} P(D > d|R_1)p(R_1) \, dR_1$$  \hspace{1cm} (2)

In practice this integral is easily solved for the discrete case based on the $n$ intervals used to stratify the probability domain, where the exceedance probabilities are computed by a summation over the selected class intervals of $R_1$:

$$P(D > d) = \sum_{j=1}^{n} P(D > d|R_{1j}) \cdot P(R_{1j})$$  \hspace{1cm} (3)

The parameters of each of the above relationships (Eqs. (1)–(3)) are subject to the epistemic uncertainty arising from the finite nature of the available data. This uncertainty can be characterised using a parametric bootstrap method. To this end, the relationships used to represent hydrologic variability are themselves stochastically sampled to reflect the uncertainty involved in their parameterisation. This is done for each of the relationships used in steps A–C, where (with reference to Fig. 2): uncertainty in the marginal distributions of point rainfalls at the key sites is estimated by bootstrapping correlated samples of rainfall maxima with the same length of record as the historic series (step E); uncertainty in the correlation between rainfall maxima at the key sites is based on the assumption that Fisher’s transformation of the sample correlation coefficient is normally distributed with a standard error (Yevjevich, 1984) that is a function of sample size (step F); and uncertainty in the slope and intercept parameters of the storm regression relationship is characterised by bootstrapping samples of areal rainfall maxima using the same number of events that

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**Table 1**

<table>
<thead>
<tr>
<th>Data source</th>
<th>Stochastic storm regression</th>
<th>Stochastic storm transposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSAM storms: catalogue of significant rainfall events used to develop estimates of Probable Maximum Precipitation for south-east Australia (Meighen and Kennedy, 1995)</td>
<td>Identification of largest storms to have occurred over the transposition region</td>
<td>Not used</td>
</tr>
<tr>
<td>AWAP: re-analysis of historical daily rainfalls (1900–2013) at a grid resolution of 0.05° x 0.05° (approximately 5 km by 5 km) for the whole of Australia (Jones et al., 2009)</td>
<td>(a) Identification of largest storms to have occurred over the study catchments (b) Transposition of regional storms to the catchments of interest (using non-dimensional rainfall depths)</td>
<td>(a) Estimation of the arrival probability of (non-dimensional) rainfall events of a given area and duration occurring anywhere in the transposition region (b) Derivation of relationship between rainfall depth, duration and area.</td>
</tr>
<tr>
<td>Daily rainfall observations at 60 sites located within and around the study catchments</td>
<td>(a) derivation of rainfall frequency curves using at-site/ regional methods for selected locations within the study catchments (b) Development of regression relationship between point rainfalls and local/transposed areal storm rainfalls</td>
<td>Not used</td>
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</tbody>
</table>

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are contained in the catalogue of local and regional storms (step G). Expected probability quantiles of catchment average rainfalls are derived using the Total Probability Theorem for each set of simulations, and confidence limits representing the epistemic uncertainty are obtained from the relevant percentiles of the results.

A number of different approaches could be used to implement each of the above steps, and the application of the adopted approach to two large catchments located in inland southern Australia is described in Section 5.

3.2. Dimensionless stochastic storm transposition

The basis of the SST approach is to compute the probabilities of extreme rainfalls by the separate consideration of the “arrival” and “transposition” probabilities. A flowchart of the main steps involved is presented in Fig. 3. The arrival probability (step A, Fig. 3) represents the likelihood that the centre of an extreme storm occurs somewhere in the transposition region, and the transposition probability (step C) describes the likelihood that, once the storm has arrived in the transposition region, it exceeds a given average depth of rainfall over the catchment of interest. The transposition probability is computed by taking into account the inverse relationship between rainfall depth and storm area (step B). The Total Probability Theorem (step D) is used to combine the joint probabilities from both these components to derive a frequency curve of areal rainfalls relevant to a particular catchment.

The form of the arrival and transposition probability models is discussed below, but it is first useful to clarify the meaning of transposition region.

The definition of the transposition region is based on a non-dimensional interpretation of the concept discussed by National Research Council (1988): the region is considered to be meteorologically homogenous such that a major storm occurring somewhere in the region could occur anywhere else in the region (ie the storm-producing mechanisms are similar everywhere); it is also considered to be statistically homogenous whereby it is possible to transform storm rainfall depths into non-dimensional depths which have the same probability of exceedance everywhere in the transposition region.

3.2.1. Non-dimensional arrival distribution

A range of approaches could be used to transform the rainfall depths to render the transposition region statistically homogeneous. Schaefer (1994) transformed a data base of observed storms into non-dimensional depths by dividing storm depths by the generalised PMP for the same location, duration and area. On the assumption that this transformation provided a statistically homogeneous data set, he then fitted an empirical distribution to the data to estimate the arrival probability of a rainfall event anywhere in the transposition region. Nathan et al. (1999) adopted a similar approach using a two-parameter exponential distribution fitted to a sample of observed storms which also had been standardised by...
the generalised PMP for the same location, duration and area. While the simplicity of this approach is attractive, it suffers from two problems. Firstly, the available data sets of observed storms have been generally prepared for the analysis of extremes and for enveloping depth-area-duration characteristics for the establishment of PMP procedures (eg World Meteorological Organisation, 2009; Meighen and Kennedy, 1995; Cerveny et al., 2007). Such data sets represent an incomplete partial series which is not well suited to formal statistical inference, and there is potential for the selection of storms to be spatially biased on the basis of their absolute magnitude not their relative rarity. Secondly, standardisation by the PMP depth assumes that all PMP estimates are equally likely to occur. While this may be the case for storms of the same area and duration within a region covered by the one generalised procedure, the enveloping of storms used to develop depth-area-duration relationships is based on a mixture of storm severities. Since maximisation is undertaken without explicit regard to the exceedance probability of the contributing events, it is possible that the degree of severity of the PMP varies across storm area, duration, and the procedure used.

To avoid these problems, in this study the arrival distribution is derived from the analysis of gridded rainfall data that has been standardised based on the frequency of occurrence. The details of the analyses involved are provided in Sections 4.1 and 4.3, but in concept the objective of this standardisation is to ensure that a given non-dimensional depth has the same exceedance probability everywhere in the transposition region. Accordingly, the same index variable used to transpose storms for the SSR approach can be used, whereby rainfall depths are transformed using the location (and perhaps scale) attributes of the distribution. If the index variable used to standardise the rainfall depths is gridded, then it is straightforward to identify the storms of a given duration and area with the largest non-dimensional depth anywhere in the transposition region in each year of available record. This series represents a complete annual series that is not biased by the systematic variation of factors that control storm depth (such as orography and distance from moisture sources). An appropriate probability distribution can be identified and fitted to this sample of maxima using L-Moments (Hosking and Wallis, 2005) and then used to estimate the arrival probability (step A, Fig. 3).

The defensibility of this approach rests on the efficacy of the adopted standardisation to render the transposition region statistically homogeneous. Various approaches are available to assess this, and an example of the test used in this investigation is provided in Section 4.1.

3.2.2. Transposition probability

Once the probability that a storm has arrived in the transposition region has been determined, it is then necessary to evaluate the probability that the storm partially or completely overlaps the catchment of interest and yields an average rainfall depth that equals or exceeds a given threshold value. Based on the stochastic transposition concepts developed by National Research Council (1988), Fontaine and Potter (1989), Wilson and Foufoula-Georgiou (1990), and Agho et al. (2000), the exceedance probability that the average catchment rainfall $D$ exceeds a given depth $d$ can be estimated using the total probability integral

$$
P(D > d) = \int f(D > d|f) \int_{A_{\text{thresh}}} p(f|A_{i}) p(A_{i}) dA_{i} df$$

where $f$ represents the non-dimensional areal rainfall depth of the storm with area $A_{i}$ (step D, Fig. 3). The term $P(D > d|f)$ is the conditional probability that the average catchment rainfall $D$ exceeds a depth $d$ given the non-dimensional depth $f$, the term $p(f|A_{i})$ is the probability density function of a storm of area $A_{i}$ occurring in the transposition zone with a non-dimensional depth $f$, and $p(A_{i})$ is the probability density function of sampling storm areas (assumed to be uniform). The inner integral (involving the term $p(f|A_{i})$) provides the expected density of non-dimensional depth $f$ by averaging over all storm areas.

The conditional probability term $P(D > d|f)$ is determined using the concept of a threshold area, as introduced by Fontaine and Potter (1989). The threshold area $A_{\text{thresh}}$ is defined as the area within which a storm centre must lie in order to produce an average rainfall depth that is greater than the threshold value $d$. This is illustrated in Fig. 4, where if the storm centre lies inside the locus of points defined by $A_{\text{thresh}}$ the average rainfall depth exceeds $d$, and if it lies on the boundary of $A_{\text{thresh}}$ the average storm depth exactly equals $d$. If the storm centre lies outside $A_{\text{thresh}}$ then the average storm depth is less than $d$. For a statistically homogeneous transposition region, the probability that an extreme storm with non-

![Fig. 3. Flowchart of the steps involved in the Stochastic Storm Transposition method.](image)

![Fig. 4. Illustration of storm isohyets over a threshold area required to exceed a given catchment average rainfall depth used in the Stochastic Storm Transposition method (adapted from Agho, 2001).](image)
where \( A_{\text{trans}} \) is the area of the transposition region. The threshold area for each given storm depth (step C, Fig. 3) may be estimated numerically by the following steps: the full arc around the centroid of the catchment is divided into a large number of equal-angle wedges; the length of the ray \( W \) (Fig. 4) is iteratively solved along the side of each wedge by finding the radius from the catchment centroid at which the storm centre produces the target depth of rainfall \( d \) over the catchment; the area of each wedge is calculated for the radii \( W \) derived for both sides of the wedge; and the threshold area is then the sum of the areas of all the computed wedges.

The average catchment rainfall \( D \) depth over the catchment produced by a given storm with centre located on ray \( W \) (in Fig. 4) is obtained from

\[
D = \frac{1}{A_c} \int_{A_c} r(x, y) \, dx \, dy
\]

where \( r(x, y) \) is the depth of rain at coordinate \((x, y)\) and \( A_c \) is the area of the catchment. The integral can be numerically evaluated by placing a regularly spaced grid over the catchment. For a given storm centroid position, the rainfall depth is computed as the mean of the rainfall depths at both of the grid points. It is assumed that the storm shapes are elliptical with fixed eccentricity and orientation.

To compute the inner integral of Eq. (4) it is necessary to define a relationship between rainfall depth and storm area (step B, Fig. 3). Agho et al. (2000) adopted a three parameter equation of the form

\[
D = c_1 + c_2 (\log_{10} A_c)^{c_3}
\]

where \( c_1, c_2, \) and \( c_3 \) are parameters fitted to depth-area relationships relevant to the region. These relationships can be derived from arrival distributions developed for different storm areas, as discussed in Section 3.1, or else they are typically available from studies undertaken to develop PMP estimates (WMO, 2009).

It should be noted that for the present work a somewhat pragmatic approach was adopted to characterise the uncertainty of the SST estimates. The majority of the epistemic uncertainty involved in the approach is contained in the uncertainty in the arrival distribution, and thus an approximation of the confidence intervals surrounding the SST results was obtained by obtaining results for the upper and lower bounds of the input arrival distributions.

4. Application of the methods

As indicated in Section 2, the AWAP gridded rainfall data was a prime input to both methods, and the manner in which this data was prepared for analysis is described below. This is followed by sections that describe selected aspects of both applications, and the results from both methods are then compared.

4.1. Standardisation and bias correction of gridded rainfalls

The Australian Water Availability Project (AWAP) dataset provides estimates of daily rainfalls for a 113 year period commencing January 1900, at a resolution of 0.05° × 0.05° (approximately 5 km by 5 km) for the whole of Australia (Jones et al., 2009). The AWAP gridded daily rainfall data is used for two main purposes (Table 1): in the SST method the AWAP data provides the means to estimate the arrival probability of a rainfall event occurring anywhere in the transposition region; in the SSR method, it is used to transpose regional storms to the catchments of interest in order to develop the storm regression relationship. For both these applications it is necessary to have confidence that the gridded rainfall depths provide an unbiased estimate of the areal rainfall depths, and also that the method of standardisation satisfies the assumption that the exceedance probability of a particular non-dimensional depth is the same anywhere in the transposition region.

There are several limitations to the use of gridded data products when considering the analysis of extreme rainfall events (Tozer et al., 2012; King et al., 2013), and accordingly a bias-correction equation was developed that rectified the errors of underestimation at high rainfalls in a manner that progressively reduced with increasing catchment area. The bias-correction equation was developed by comparing the AWAP rainfalls depths to independent estimates of extreme storm depths prepared by specialist hydrologists. For this comparison, rainfall depths relevant to 21 separate historic events were extracted from AWAP gridded rainfalls. Data was extracted for selected combinations of storm areas and durations which corresponded to published storm analyses (Meighen and Kennedy, 1995); this yielded 116 comparisons of rainfall depths for storm areas ranging between 100 km² and 60,000 km² and durations between 1 and 7 days. A bias correction equation was developed as a function of rainfall depth and storm area, and the nature of the required adjustment is evident in the difference in the scatter plots of raw and bias-corrected rainfalls in Fig. 5.

The AWAP rainfalls were also transformed into non-dimensional depths by dividing each gridded rainfall estimate by an index variable relevant to the rare rainfalls of interest. The index variable selected is the rainfall quantile associated with an exceedance probability of 0.02. This quantile was selected to balance the competing needs involved in choosing an index variable that is relevant to the observed extremes of interest whilst being able to be robustly estimated from the 113 years of available AWAP record. The GEV distribution was fitted by L-Moments (Hosking and Wallis, 2005) to the annual maxima obtained for each of the (approximately) 40,000 individual grid cells that comprise the transposition region. The GEV distribution was selected as it has been shown to be a suitable choice of model in previous regionalisation studies (Nandakumar et al., 1997, 2012). No attempt was made to use regional information to fit the distribution parameters at each individual grid cell, as the maxima used to fit the distributions were already spatially smoothed in the gridding process (Jones et al., 2009); also, information on the standardised storm areas of interest involved the averaging of index variables for between 40 and 600 grid cells, and consideration of this number of grid cells would minimise the influence of any locally anomalous estimates of the index variable.

The objective of the standardisation is to transform the observed rainfall maxima such that it can be assumed that the exceedance probability of a particular non-dimensional depth is the same anywhere in the transposition region. To test the homogeneity of the standardised rainfalls, the transposition region was divided into two contiguous sub-regions of roughly equal area on the basis of mean annual rainfall (MAR). The “dry” sub-region (with MAR < 300 mm) covers the north-western half of the transposition region, and the “wet” sub-region (with MAR ≥ 300 mm) extended over the south-east. The adopted standardisation can be considered fit for purpose if the frequency curves of arrival distributions for the two sub-regions are statistically similar. If not, then the region must still be considered statistically heterogeneous.

To derive the sub-region arrival distributions, areal annual maxima corresponding to storms of 5000 km² and 20,000 km² area were extracted from the standardised rainfalls. The maximum standardised rainfall of a given storm area was found by searching
all the grid cells within each sub-region; the largest average depth of standardised rainfall occurring anywhere in each sub-region in each year of record was identified, where the storms were assumed to be circular in shape. This process yielded a set of 113 annual area maxima for each sub-region, and a GEV distribution was fitted to each set of maxima to derive the standardised arrival distributions. The resulting frequency curves for the 5000 km² and 20,000 km² areal storms of two-day’s duration for the “wet” and “dry” sub-regions are shown in Fig. 6. It is seen that the difference between the best estimates of each frequency distribution are small compared to the width of the corresponding confidence limits, and hence it is concluded that the selected index variable has rendered the region statistically homogeneous for the purposes of this investigation.

4.2. Application of the SSR method

The application of the SSR method follows the five steps outlined in Section 2. The AWAP gridded rainfalls were used to identify the ten largest one- and two-day local storms to have occurred over the Hume and Dartmouth catchments, and these were found to be associated with 21 unique events. The largest regional storms to have occurred over the transposition region were identified from the published storm catalogue (Meighen and Kennedy, 1995). The regional storms were identified from the standard storm areas provided in the catalogue that encompassed the extent of the Dartmouth and Hume catchments, and a total of 15 unique events were selected from the (approximately) 100 years of records available. The regional storms were transformed to non-dimensional depths by dividing each gridded rainfall value by its corresponding index variable (i.e., the rainfall quantile associated with an exceedance probability of 0.02). The non-dimensional storms were then centred over the Hume and Dartmouth catchments and the depths were then transformed back into absolute depths by multiplying by the index variable for the new locations. The storms were transposed without rotation as testing (at increments of 90° rotations) indicated that differences in catchment rainfall depths for rotated storms were generally less than 5% of the mean value.

Storm regression relationships were developed to predict 1-day and 2-day catchment areal rainfall depths as a function of point rainfalls at a number of key sites. The regression relationships were fitted using ordinary least squares. The key sites were selected by trial and error from a candidate set of predictors comprised of 12 daily gauges with more than 80 years of record located within the boundaries of the catchment. A total of four key sites within the boundary of Hume catchment and two key sites for the Dartmouth catchment were selected to minimise bias and variance in the model fit. It is expected that the number of key sites required to adequately capture the systematic variation in catchment rainfalls will increase with catchment size, though in concept the scheme developed to derive areal rainfalls (steps B and C, Fig. 2) is scalable to any number of sites. A scatter plot of the regression estimate versus derived storm data for 2-day areal rainfalls over
the Hume catchment is shown in Fig. 7(a). This figure illustrates the relative magnitudes of the transposed regional and local storms, where it is seen that inclusion of regional storms significantly increases the range of events included in the analysis.

Fig. 7(b) shows an example of the frequency curve of point rainfalls derived for the key sites. The point rainfall frequency curves were derived using the CRC-FORGE method (Nandakumar et al., 2012) where confidence limits were derived using parametric bootstrapping techniques. The CRC-FORGE method was selected as this is the method adopted across Australia for design purposes. The method uses at-site data to fit a GEV distribution to more frequent events, and then trades space for time by progressively including rainfall maxima from an increasing number of surrounding gauges. For comparison, a best estimate was also obtained using a regional L-moment approach (Hosking and Wallis, 2005; Schaefer, 1990) based on the analysis of 50 rainfall gauges with an average record length of (almost exactly) 100 years. The resulting point rainfall curve is also shown in Fig. 7(b), where is seen that there is no statistical difference between the two best estimates.

Fig. 8 illustrates selected simulations involved in applying the SSR method. A stratified sampling scheme is used to generate point rainfalls from the marginal distributions of each key site, whilst preserving the degree of correlation between them (steps A and B, Fig. 2). An example of one such simulation is shown in the scatter plots of Fig. 8(a), which also shows the historic correlations determined from analysis of the local and regional storms. The correlated point rainfall maxima and are then used as input to the storm regression relationship (Eq. (1); Step C Fig. 2) to derive estimates of catchment rainfalls, and these are then analysed using the total probability theorem to derive expected probability quantiles (Step D, Fig. 2). The method used to construct the stratified sample and derive the expected probability quantiles is described by Nathan et al. (2003). This process is repeated 1000 times using bootstrap techniques to generate correlated marginal distributions of point rainfalls at the key sites, where parameters defining cross-correlations and storm regression relationships are also generated to reflect the uncertainty arising from the limited historical data used in their estimation (steps E–G, Fig. 2). An example of stochastic storm regression relationships is shown in Fig. 8(a), and the resulting confidence limits surrounding the expected probability quantiles of areal rainfall depth are shown in Fig. 8(b).

4.3. Application of the SST method

Application of the SST method involves the two key elements of first estimating the arrival distribution, then computing the transposition probabilities for the given catchment area and transposition region. The areal maxima used to fit the arrival distributions were obtained by searching for the largest average depth of standardised rainfall occurring anywhere in the transposition region in each year of record. Arrival distributions were derived for four storm durations (1–4 days) and nine storm areas (ranging between 100 and 60,000 km²), and the derived maxima were corrected for bias using the function illustrated in Fig. 5. For simplicity areal rainfalls were defined by circles of the required area, as testing indicated that using ellipses of various eccentricity (varying between 0.0 and 0.6) and orientation (between 90° and 170°) yielded differences in extreme rainfall quantities ranging between only 1% and 2% of the average of all combinations trialled. The density of rainfall gauges in the western 20% of the transposition region is markedly lower than elsewhere, thus the areal rainfall maxima were extracted only from the portion of the transposition region with adequate gauging density. The arrival probabilities were adjusted to account for the differences in sampling area using a correction factor based on the assumption that the number of storms arriving each year follows a Poisson process (Agho, 2001). The centroids of the annual areal maxima extracted from the 113 years of record were found to be evenly distributed across the sampling region, and no spatial trends were evident in respect to their year of occurrence.

Preliminary frequency analysis of these maxima using a range of probability distributions indicated that the upper tails of the fitted distributions varied in an inconsistent manner with storm duration and area. While such inconsistencies are expected to arise when extrapolating frequency curves beyond the period of available record, it became apparent that such inconsistencies were largely resolved if the maxima were censored on the basis of assumed storm type. Storms in this region are derived from two dominant mechanisms, namely cold fronts and monsoon troughs. Cold fronts are associated with low pressure systems originating from the south of the continent and commonly occur throughout the year. Conversely, monsoon troughs bring moist air from the northern oceans, and only occur in the warmer months; such events are far less common than cold fronts and do not occur every year. Analysis of the seasonality of the storm maxima indicated that all but one of the largest 20% of storms occurred in the summer months, whereas the lower 80% of events are more uniformly distributed across the different seasons. This distribution of storm type is consistent with the relative frequency of frontal and monsoon synoptic drivers (Pook et al., 2006; Gallant and Karoly, 2012), and it was assumed that fitting the arrival distributions to the top 20% of events was a reasonable surrogate for explicit storm typing. Censoring on the basis of assumed storm type largely resolved the inconsistencies in the upper tails of distributions with storm duration and area.

The selection of the most appropriate distribution to fit was obtained from examination of the plots of sample L-skew and L-Kurtosis derived using both censored and uncensored series. It
was found that the spread of the sample L-Moments lay within the range covered by the Kappa distribution, i.e. by the Generalised Logistic, Generalised Extreme Value, and the Generalised Pareto distributions which represent special cases of the Kappa distribution with second shape parameters of $-1$, 0 and $+1$, respectively. The sample L-Moments values were found to be centred on the Kappa distribution with a shape parameter set to 0.25, and it was decided to adopt this as the single “parent” distribution for all arrival distributions, with fixed values of skewness adopted for all 9 storm areas but varying with duration. This single distribution was considered to best represent the characteristics of all 36 samples fitted (9 storm areas by 4 durations). While it might be argued that adoption of a second shape parameter corresponding to one of the generalised three parameter distributions would be a more conventional choice, the adopted distribution also represents a 3-parameter “special case” of the Kappa distribution. An example fit of the Kappa distribution to one of the censored series is shown in Fig. 9(a); also shown in this figure is a fit obtained using the GEV distribution, demonstrating that the difference is shown in Fig. 9(a); also shown in this figure is a fit obtained using the GEV distribution, demonstrating that the difference is modest compared to the width of the associated uncertainty limits.

Logistic, Generalised Extreme Value, and the Generalised Pareto distributions were also derived from the AWAP data. Distributions were fitted using Eq. (7) to selected quantiles obtained from the 28 arrival probability distributions derived for areas larger than 1000 km$^2$ (ie 4 storm durations for each of 7 storm areas ranging between 1000 and 60,000 km$^2$). The parameters of Eq. (7) were fitted by least squares subject to the constraint that the average rainfall depth reproduced the expected PMP depths for storms with the same area as the catchments of interest.

Results of the SST analyses are presented and discussed in the following section, but it is first worth noting one important attribute of the approach. Fig. 9(b) illustrates the contribution of the uncertainty in the arrival distribution to the overall estimate of the AEP for three extreme rainfall events. The blue curve shows that for a design rainfall depth of 30% of the PMP, 90% of the contribution to the estimated AEP of this event is generated by rainfall events with an arrival AEP more frequent than $10^{-2}$. Such events are easily estimated using the 113 years of available record, and hence the estimate of the AEP of a 30% PMP event is largely insensitive to assumptions about the shape of the arrival probability distribution. It is also seen from this figure that 90% of the AEP of events that are 65% and 100% of the PMP depth are generated by rainfalls with arrival AEPs more common than $2 \times 10^{-4}$ and $2 \times 10^{-6}$, respectively. As seen in the following section, these AEPs are around an order of magnitude more frequent than the AEP of the extreme event of interest, where the balance of contribution is due to the transposition probability.

4.4. Comparison of methods

Results were derived for the Hume and Dartmouth catchments using both methods and the derived frequency curves for the Hume catchment are shown in Fig. 10. It was found that there is good agreement between the SSR and SST estimates for the Hume 1-day and Dartmouth 2-day events for AEPs of $10^{-4}$ and rarer. For the Hume 2-day and Dartmouth 1-day events there is an appreciable level of difference between the SSR and SST estimates, though the average of these estimates corresponds to the results obtained for the other durations.

Also shown in Fig. 10 are independent estimates of rainfall quantiles obtained using current design techniques (Nandakumar et al., 2012; Green et al., 2015; Podger et al., 2015). This “design rainfall” information is used by engineers for general design purposes, and is available for exceedance probabilities as rare as $5 \times 10^{-4}$. It is seen that the design rainfalls for the 2-day event lie midway between the SST and SSR results, and those for the 1-day event lie below. The narrow confidence intervals for the SST results reflect the accuracy of the arrival probabilities for the frequent events that contribute to this AEP range; as discussed above in relation to Fig. 9(b), the arrival probabilities of most relevance are more common than $10^{-2}$, where the balance of the uncertainty contribution is due to the transposition component.

A final set of frequency curves was adopted giving equal weight to the estimates from the two methods. For the Hume catchment both methods yielded the same estimate for the AEP of the 1-day PMP event, namely $10^{-6}$; since the average of the two estimates for the 2-day event is also $10^{-6}$, this value was used to anchor...
the top end of the frequency curve. A similar situation exists for Dartmouth catchment, though here the two methods both indicate that the AEP of the 2-day PMP is $10^{-7}$, which is similar to the average of the two estimates derived for the 1-day event.

The uncertainty of these AEP estimates is considerable and spans at least one to two orders of magnitude about the best estimate. Analyses were undertaken to assess the sensitivity to various assumptions, including the selected probability distribution adopted to characterise the arrival distributions of areal and point rainfalls used in the SSR and SST methods, storm shape and orientation used in the SST methods, and different numbers of key sites and correlation dependency characteristics in the SSR method. While these factors vary in their degree of influence on the final estimates, the uncertainty associated with these assumptions is less than half the width of the confidence intervals shown in Fig. 10. That is, the major uncertainty in these estimates is due to the limited period (113 years) of available record and the finite size of the transposition region. The AEP of PMP estimates obtained from Fig. 10 are about one order of magnitude less than the current design recommendations suggested by Laurenson and Kuczera (1999), but the magnitude of the associated uncertainties are similar. The regional implications of these results are further explored in the following section.

The Stochastic Storm Regression method is attractive in that it makes use of the detailed hydrometeorological analysis of a number of large events that have occurred in the transposition region; its weakness, however, is that it relies on a large degree of extrapolation of point rainfall frequency curves and its implementation requires the undertaking of somewhat complex statistical analysis to correctly account for the correlations between point and areal rainfalls.

The Stochastic Storm Transposition approach is attractive in that it requires few inputs, is based on a rigorous statistical framework that takes good advantage of regionalised inputs, and requires an extrapolation of the arrival distribution to an AEP that is only one to two orders of magnitude less than the computed frequency of rainfall exceedances over the target catchment; its weakness, however, is that it relies heavily on the defensibility of the arrival distribution of areal rainfalls, which is difficult to identify robustly from the available data. The lengths of rainfall records available are very short compared to the extremes of interest, and perhaps the most promising means of decreasing the uncertainties in the arrival distribution is to explore the utility trading space for time with continental and global data sets (eg PRISM, Di Luzio et al., 2008; GPCP, Huffman et al., 2001; TRMM, Huffman et al., 2007).

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The Stochastic Storm Transposition (SST) analyses for a 2-day event of 20,000 km$^2$ area showing (a) an example arrival distribution and (b) the contribution to the annual exceedance probability of different catchment rainfalls (expressed as fractions of the Probable Maximum Precipitation, PMP) by events of differing regional arrival exceedance probabilities.

Fig. 9. Illustration of Stochastic Storm Transposition (SST) analyses for a 2-day event of 20,000 km$^2$ area showing (a) an example arrival distribution and (b) the contribution to the annual exceedance probability of different catchment rainfalls (expressed as fractions of the Probable Maximum Precipitation, PMP) by events of differing regional arrival exceedance probabilities.
4.5. Preliminary assessment of regional design estimates

The recommendations for estimating the AEP of the PMP for design purposes in Australia were developed by Laurenson and Kuczera (1999) and were originally presented as “interim” advice due to the limited evidence on which they were based. The nature of the results obtained for the Hume and Dartmouth catchments does prompt the need to consider whether such differences might be site-specific or consistent across the transposition region. Accordingly, AEP of PMP estimates were derived for the storm locations shown in Fig. 1 using the SST procedure. Given the large degree of extrapolation involved it would be preferable to apply both the SSR and SST methods and reconcile the derived estimates, but for this preliminary assessment just the SST method was adopted as the only additional information required for its application anywhere in the transposition region is the value of the location-specific index variable used to standardise the arrival distribution (as discussed in Section 4.2). PMP estimates for seven storm areas (ranging between 1000 and 60,000 km²) and four durations (1–4 days) for the locations shown in Fig. 1 were obtained from estimates prepared by Nathan et al. (1999) using the generalised methodology developed by Minty et al. (1996).

The AEP of PMP estimates for the different storm areas and durations at the selected locations are summarised in Fig. 11. Also shown in this figure are the values recommended for design, along with their notional 75% and 100% confidence limits, as developed by Laurenson and Kuczera (1999). It is seen that the AEP of PMP estimates derived using the SST method for 2–4 day duration storms (filled symbols, Fig. 11) largely fall within the suggested 75% confidence limits, but the majority of the 1-day estimates (hollow symbols, Fig. 11) fall outside these limits. The difference in AEP estimates for these durations is largely attributed to the relatively shallow arrival distributions for one day storms which lead to a much larger change in arrival probability for a given change in PMP fraction (compared with the other durations). The variation in AEP of the PMP for the different combinations of storm duration, area, and location is speculated to be due to the site-specific nature of several factors involving: (a) the index variable used to scale the non-dimensional arrival distribution, (b) the enhancement factor used to estimate the topographic component of the PMP estimate, and (c) the moisture adjustment factor used in PMP estimation that accounts for variation in extreme precipitable water with different locations; additionally, there is variation in these estimates across storm area for a given duration associated with (d) differences in the severity of the storms used to construct the standardised convergence component of the depth-area curves. The extent to which the variation in these estimates is due to assumptions in the PMP methodology or the parameterisation of the SST method requires further research, but the current Australian flood guidelines (Nathan and Weinmann, 2016) recommends that the Laurenson and Kuczera relationship be retained for routine design applications, but that consideration should be given to designs with high potential consequences of failure.

The recommendations developed by Laurenson and Kuczera were presented as “interim” advice due to the limited evidence on which they were based. The large degree of uncertainty involved in these estimates is unfortunate given the practical need to make risk-informed investment decisions about high hazard structures. Tempting as it might be to regard such uncertain analyses as “mathematistry” (Klemes, 1987) – whereby effort is expended on analyses that are divorced from practical reality – the uncomfortable fact remains that hydrologists are required to provide “best estimates” to satisfy the extreme life safety criteria associated with high hazard structures (ANCOLD, 2003; FERC, 2016). While it is to be expected that further research may yield different estimates of such extremes, it is less clear to what extent the uncertainties can be materially reduced given the limitations inherent in regional data sets comprised of approximately 100 years of correlated maxima.

Fig. 11. Estimates of the annual exceedance probability (AEP) of Probable Maximum Precipitation (PMP) as recommended by Laurenson and Kuczera (1999), and as obtained from application of the Stochastic Storm Transposition (SST) method to selected locations within the transposition region.
5. Conclusions

The estimation of exceedance probabilities of extreme events that lie beyond the observed record is a vexing area of hydrology as it necessarily involves making extrapolations that have a high degree of uncertainty. Nevertheless, owners of high hazard infrastructure have an ongoing responsibility to manage their assets in a risk-informed manner, and they require estimates of extreme hydrologic risks to assist them with their decision making.

The Stochastic Storm Regression (SSR) and Stochastic Storm Transposition (SST) methods were found to provide similar estimates of the frequency of extreme areal rainfall events. The catchment-specific estimates of the annual exceedance probability of Probable Maximum Precipitation (PMP) events for the Hume and Dartmouth catchments are estimated to be $10^{-6}$ and $10^{-7}$, respectively. The difference in estimates largely reflects the fourfold difference in catchment areas, which is reflected in the transposition component of the SSR method and in the parameterisation of the storm regression relationship in the SSR method. However, it is important to note that the uncertainty in these AEP estimates is considerable and spans at least one to two orders of magnitude about the best estimate. The AEP of PMP estimates derived using the SST method for a range of storm areas and durations across the transposition region exhibit considerable variation with location, and this variation lies within the notional confidence limits suggested by Laurenson and Kuczera (1999) for this region.

The study raises a number of issues that would benefit from further research, and these are primarily associated with factors involved in the estimation of the arrival distribution, the scaling variables used for transposition, and reconciliation of more frequent estimates with other available information. However, any estimates of such extremes are subject to the limitations imposed by the finite sample of observations available, and this limitation is largely irreducible in the short term.

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