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Brane tachyon dynamics

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ABSTRACT

The dynamics of a tachyon attached to a Dvali, Gabadadze and Porrati (DGP) brane is investigated. Exponential potential and inverse power law potential are explored, respectively. The quasi-attractor behavior, for which the universe will eventually go into a phase similar to the slow-roll inflation, is discovered in both cases of exponential potential and inverse power law potential. The equation of state (EOS) of the virtual dark energy for a single scalar can cross the phantom divide in the branch $\theta = -1$ for both potentials, while the EOS of the virtual dark energy for a single scalar cannot cross this divide in the branch $\theta = 1$.

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1. Introduction

$E_8 \times E_8$ heterotic string emerges when one compacts the 11-dim super gravity on an S^1/Z_2 orbifold (Horava–Witten proposal) [1]. The behavior of the string theory in the energy region below the unification scale is not sensitive to the fine structure of the inner Calabi–Yau space, that is, the universe can be effectively described by some 5-dim theory, in which the standard model particles are confined to the 3-brane, while the gravitation can propagate in the whole spacetime. Such string-inspired phenomenological model, called brane world, has been set up and studied extensively [2], especially in the fields of high energy physics and cosmology. According to their behavior in different energy scales, brane world models can be classified into two main categories. One is "high energy theory", that is, its phenomenology becomes different in high energy (ultra-violet) region from general relativity (GR), but recovers to GR in low energy (infrared) region, for example Randall–Sundrum (RS) model [3]. On the contrary, the other type of brane world, "low energy theory", concentrates on the modification in low energy region. Dvali, Gabadadze and Porrati (DGP) model [4] is a leading model in the low-energy-theory models, which is mainly applied to the late time universe (see, however, [5]). Great interest has been aroused in the researches of late time universe since the discovery of cosmic acceleration.

The present cosmic acceleration is one of the most significant cosmological discoveries over the last century [6]. The physical nature of this acceleration remains as a mystery. Various explanations have been proposed, such as a small positive cosmological constant, quintessence, k-essence, phantom, holographic dark energy, etc., see [7] for recent reviews with fairly complete list of references of different models. A cosmological constant is a simple candidate for dark energy. However, following the more accurate data a more dramatic result appears: the recent analysis of the type Ia supernovae data indicates that the time varying dark energy gives a better fit than a cosmological constant, and in particular, the equation of state (EOS) parameter w (defined as the ratio of pressure to energy density) may cross the phantom divide $w = -1$ [8]. Three roads to cross this divide were summarized in a recent review article [9]:

(i) quintom type (two-field) model, for a review see [10],
(ii) interacting model, for example see [11], and
(iii) model in frame of new gravity, especially brane world, for example, see [12].

Inspired by the hopeful unification theory, string/M theory, the models in frame of modified gravity are duly noted since they offer much more extensive possibilities for dark energy. A useful example is that a single scalar cannot cross the phantom divide while it can cross the divide in frame of DGP [12]. Brane world model inherits a key geometric property of 11-dim the Horava–Witten proposal of string/M, which requires standard model particles confined to a brane, while gravity propagates freely throughout the whole manifold. On the other hand an exotic matter with negative pressure, tachyon, coming from string/M theory also has been

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widely applied in cosmology, as inflaton in the early universe [13], and as dark energy in the late time universe [14]. Thus, it is interesting to study the dynamics of a tachyon attached to a brane.

Tachyon is a field at the top of its potential, which has a fairly long history in particle physics. It returns with the studies of string/M recently. It was found that the tachyon modes of open string attached to a Dp-brane described the inherent instability of the Dp-brane [15]. A tachyon field has negative pressure, therefore it may be a proper candidate to drive the universe to accelerate. Generally speaking, a tachyon is always associated to a brane. The behavior of a tachyon in RS type brane world has been investigated in [16], which is concerned with the early universe. In this Letter, we will study the behavior of a tachyon field in DGP. We focus on the late time evolution of the universe in the present Letter. In the standard model (4-dim GR), the equation of state (EOS) of a tachyon is always in the interval $(-1, 0)$. However, we will show that the effective EOS of the dark energy in the tachyon-DGP model can cross the phantom divide, which satisfies the amazing possibility of the crossing behavior of dark energy implied by recent observations.

To find an attractor solution is an important method in cosmology, which is helpful to alleviate the coincidence problem. If there does not exist an attractor in a system, the commonsensible lore tells us that the orbits of the phase portrait never converge: it will look like a turbulent flow. However, we discover the quasi-attractor in the dynamical system without any critical point. The orbits with different initial conditions will converge to a quasi-de Sitter evolution. A useful analogy of this quasi-attractor is the slow-roll inflation. We find that the quasi-attractor behavior is rather robust, which will appear in the cases of different potentials of a tachyon.

The outline of this Letter is as follows. In Section 2 we present our set up of the model. In Section 3, we study the evolution of this system via a dynamical system analysis. In Section 4, we conclude this Letter.

2. The model

Let's start from the action of the DGP model

$$S = S_{\text{bulk}} + S_{\text{brane}}, \tag{1}$$

where

$$S_{\text{bulk}} = \int_{\mathcal{M}} d^5 X \sqrt{-^{(5)}g} \frac{1}{2\kappa_5^2} {}^{(5)}R, \tag{2}$$

and

$$S_{\text{brane}} = \int_M d^4 x \sqrt{-g} \left[\frac{1}{\kappa_5^2} K^{\pm} + L_{\text{brane}}(g_{\alpha\beta}, \psi) \right]. \tag{3}$$

Here κ_5^2 is the 5-dim gravitational constant, $^{(5)}R$ is the 5-dim curvature scalar. x^{μ} ($\mu = 0, 1, 2, 3$) are the induced 4-dim coordinates on the brane, $K^{\pm}$ is the trace of extrinsic curvature on either side of the brane and $L_{\text{brane}}(g_{\alpha\beta}, \psi)$ is the effective 4-dim Lagrangian, which is given by a generic functional of the brane metric $g_{\alpha\beta}$ and matter fields ψ on the brane.

Consider the brane Lagrangian consisting of the following terms

$$L_{\text{brane}} = \frac{\mu^2}{2} R + L_{\text{m}} + L_T, \tag{4}$$

where μ is 4-dimensional reduced Planck mass, R denotes the curvature scalar on the brane, and L_T represents the Lagrangian of a tachyon attached to the brane, L_{m} stands for the Lagrangian of other matters on the brane. Then, assuming a mirror symmetry in the bulk, we have the Friedmann equation on the brane [17],

$$H^2 + \frac{k}{a^2} = \frac{1}{3\mu^2} \left[\rho + \rho_0 + \theta \rho_0 \left(1 + \frac{2\rho}{\rho_0} \right)^{1/2} \right], \tag{5}$$

where $H \triangleq \dot{a}/a$ is the Hubble parameter, a is the scale factor, k is the spatial curvature of the three-dimensional maximally symmetric space in the FRW metric on the brane, and $\theta = \pm 1$ denote the two branches of DGP model, ρ denotes the total energy density, including dust matter and tachyon, on the brane,

$$\rho = \rho_T + \rho_{dm}. \tag{6}$$

The term ρ_0 relates the strength of the 5-dim gravity with respect to the 4-dim gravity,

$$\rho_0 = \frac{6\mu^2}{r_c^2}, \tag{7}$$

where the cross radius is defined as $r_c \triangleq \kappa_5^2 \mu^2$.

A no-go theorem shows that a single field with reasonable conditions in GR cannot cross the phantom divide. We will show that in our model only one field is enough for this crossing behavior via the effect of the 5-dim gravity. In fact, the accelerated expansion of the universe is a joint effect of the tachyon and the competition between 4-dim gravity and the 5-dim gravity.

In the brane world model, the surplus geometric terms relative to the Einstein tensor play the role of the dark energy in GR in part. However, almost all observed properties of dark energy are obtained in frame of GR with a dark energy. To explain the observed evolving EOS of the effective dark energy, we introduce the concept "equivalent dark energy" or "virtual dark energy" in the modified gravity models [9]. We derive the density of virtual dark energy caused by the tachyon and induced gravity term by comparing the modified Friedmann equation in the brane world scenario with the standard Friedmann equation in general relativity. The Friedmann equation in the 4-dimensional GR can be written as

$$H^2 + \frac{k}{a^2} = \frac{1}{3\mu^2} (\rho_{dm} + \rho_{de}), \tag{8}$$

where the first term of RHS in the above equation represents the dust matter and the second term stands for the dark energy. Comparing (8) with (5), one obtains the density of virtual dark energy of DGP,

$$\rho_{de} = \rho_T + \rho_0 + \theta \rho_0 \left(1 + \frac{2\rho}{\rho_0} \right)^{1/2}. \tag{9}$$

Since the dust matter obeys the continuity equation and the Bianchi identity keeps valid, dark energy itself satisfies the continuity equation

$$\frac{d\rho_{de}}{dt} + 3H(\rho_{de} + p_{\text{eff}}) = 0, \tag{10}$$

where p_{eff} denotes the effective pressure of the dark energy. And then we can express the equation of state for the dark energy as

$$w_{de} = \frac{p_{\text{eff}}}{\rho_{de}} = -1 - \frac{1}{3} \frac{d \ln \rho_{de}}{d \ln a}. \tag{11}$$

Observing the above equation, we find that the behavior of w_{de} is determined by the term $\frac{d \ln \rho_{de}}{d \ln a}$. $\frac{d \ln \rho_{de}}{d \ln a} = 0$ (cosmological constant) bounds phantom and quintessence. More intuitively, if ρ_{de} decreases and then increases, or increases and then decreases with the expansion of the universe, we are certain that EOS of dark energy crosses phantom divide. A more important reason why we use the density to describe property of dark energy is that the density is more closely related to observables, hence is more tightly constrained for the same number of redshift bins used [18].

3. Dynamics of tachyon-DGP

In this section, we will analyze the dynamics of a tachyon in the late time universe on a DGP brane with two different potentials, respectively. We show that the quasi-attractor (which we will explain in detail later) appears in both of the two cases.

For a tachyon field in a curved background, the action in L_T of (4) takes a Dirac–Born–Infeld (DBI) form,

$$L_T = -V(T)\sqrt{1+X},\tag{12}$$

where

$$X = g^{\mu\nu}\partial_\mu T \partial_\nu T.\tag{13}$$

One sees that a tachyon has a dimension of [length] rather than [mass], which is different from an ordinary scalar. The equation of motion for tachyon reads,

$$\frac{1}{V(T)}\frac{dV(T)}{dT} - \left(\frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}\right)g^{\mu\nu}\partial_\nu T$$
$$+ \frac{1}{2(1+X)}g^{\mu\nu}\partial_\nu T\left(\partial_\mu g^{\alpha\beta}\partial_\alpha T\partial_\beta T + 2g^{\alpha\beta}\partial_\alpha\partial_\mu T\partial_\beta T\right)$$
$$- \partial_\mu g^{\mu\nu}\partial_\nu T - g^{\mu\nu}\partial_\mu\partial_\nu T = 0,\tag{14}$$

which degenerates to

$$\frac{\ddot{T}}{1-\dot{T}^2} + 3H\dot{T} + \frac{1}{V(T)}\frac{dV(T)}{dT} = 0,\tag{15}$$

in an FRW universe, where a dot denotes the derivative with respect to time. Note that our result (14) is different from the result in [19], which cannot degenerate to (15) in an FRW universe.

Varying the action with respect to the metric tensor we obtain the energy momentum of the tachyon field,

$$T^{\mu\nu} = -V\left[g^{\mu\nu}(1+X)^{1/2} - (1+X)^{-1/2}\partial^\mu T\partial^\nu T\right],\tag{16}$$

which reduces to

$$\rho = \frac{V(T)}{\sqrt{1-\dot{T}^2}},\tag{17}$$

$$p = -V(T)\sqrt{1-\dot{T}^2},\tag{18}$$

in an FRW universe. Thus the (local) equation of state of tachyon reads,

$$w = \dot{T}^2 - 1.\tag{19}$$

The reality conditions for ρ and $\dot{T}$ require $0 \leqslant \dot{T}^2 \leqslant 1$, which yields,

$$-1 \leqslant w \leqslant 0.\tag{20}$$

For a more detailed research of the evolution of the variables in this model we write them in a dynamical system, which can be derived from the Friedmann equation (5) and continuity equation (10). We first define some new dimensionless variables,

$$x \triangleq \dot{T},\tag{21}$$

$$y \triangleq \frac{\sqrt{V}}{\sqrt{3}\mu H},\tag{22}$$

$$l \triangleq \frac{\sqrt{\rho_{dm}}}{\sqrt{3}\mu H},\tag{23}$$

$$b \triangleq \frac{\sqrt{\rho_0}}{\sqrt{3}\mu H}.\tag{24}$$

The physical meanings of these new variables are clear: x denotes kinetic energy of the tachyon, y marks the relative strength of potential energy to the Hubble parameter, l represents the relative strength of the dust density to the Hubble parameter, and b stands for the Hubble parameter. The exact form of the potential of a tachyon is still under research. In the following two subsections, we will discuss two examples of potentials, say, exponential potential and inverse power law potential.

3.1. Exponential potential

The exponential potential is an important example which can be solved exactly in the standard model for a scalar. We first study the dynamics of a tachyon with an exponential potential,

$$V = V_0 e^{-\lambda T},\tag{25}$$

where V_0 and λ are two constants. With the evolution of the universe, the tachyon rolls down, which can be described by,

$$x' = 3(1-x^2)(-x+jb),\tag{26}$$

$$y' = \frac{3}{2}\alpha y - \frac{3}{2}jbxy,\tag{27}$$

$$l' = \frac{3}{2}\alpha l - \frac{3}{2}l,\tag{28}$$

$$b' = \frac{3}{2}\alpha b,\tag{29}$$

where

$$j = \frac{\mu\lambda}{\sqrt{3}\rho_0},\tag{30}$$

and

$$\alpha \triangleq \left[l^2 + x^2 y^2 (1-x^2)^{-1/2}\right]$$
$$\times \left[1 + \theta\left(1 + 2\frac{y^2(1-x^2)^{-1/2}+l^2}{b^2}\right)^{-1/2}\right],\tag{31}$$

and a prime stands for derivation with respect to $s \triangleq \ln a$. α has significant physical sense in a dynamical universe. In fact it is just the slow-roll parameter in the language of inflation,

$$\alpha = -\frac{2\dot{H}}{3H^2}.\tag{32}$$

$\alpha \ll 1$ implies the universe enters a quasi-de Sitter phase.

In the above system we have set $k = 0$, which is implied either by theoretical side (inflation in the early universe), or observational side (CMB fluctuations [20]). One can check this system degenerates to a tachyon with dust matter in standard GR. Note that the 4 equations (26), (27), (28), (29) of this system are not independent. By using the Friedmann constraint, which can be derived from the Friedmann equation,

$$y^2(1-x^2)^{-1/2} + l^2 + b^2$$
$$+ \theta b^2\left(1 + 2\frac{y^2(1-x^2)^{-1/2}+l^2}{b^2}\right)^{1/2} = 1,\tag{33}$$

the number of the independent equations can be reduced to 3. There are two critical points of this system satisfying $x' = y' = l' = b' = 0$ appearing at

$$x = y = l = b = 0;\tag{34}$$

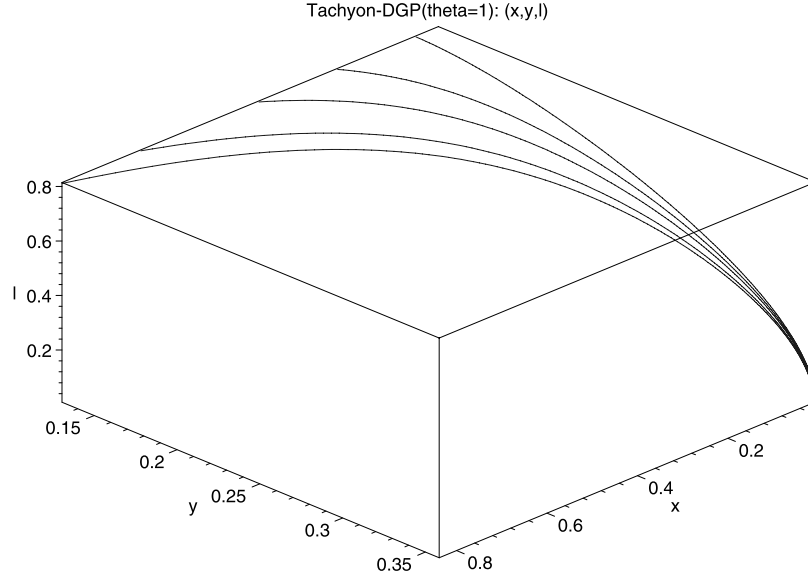$$x = 1, \qquad y = l = b = 0.\tag{35}$$

Fig. 1. The evolution of a tachyon on a DGP in the branch $\theta = 1$. The different initial conditions for the curves are $x(s=0) = 0.1$, $x(s=0) = 0.08$, $x(s=0) = 0.05$, $x(s=0) = 0.03$, $x(s=0) = 0.01$ from the left to the right, respectively.

It is easy the check that neither of them satisfies the Friedmann constraint (33). So there is no de Sitter type attractor in this system.

However, through a detailed numerical investigation, we find a future "quasi-attractor" in this system. The physical picture is that for a fairly large space of the initial conditions, the tachyon on a DGP will enter such a quasi-de Sitter space, where the tachyon rolls down the potential very slowly such that its kinetic energy is effectively zero and the background space is in fact a de Sitter one. One can make an analogy to the situation of slow-roll inflation, where we omit the kinetic energy of the inflaton and we treat the spacetime as a de Sitter.

It is difficult to obtain the analytical solution of the quasi-attractor. We show the typical obits of this system in the two branches, respectively. Since l is an explicit function of b,

$$l \sim ba^{-3/2} = be^{-3s/2}, \tag{36}$$

we just plot 3-dim phase portraits in the subspace x–y–l in Fig. 1. To show it more clearly, the projections on x–y, x–l and y–l planes are also plotted in Fig. 2.

Fig. 1 illustrates the evolution of a tachyon attached to a DGP brane in the phase space x–y–l. One clearly see that the orbits with different initial conditions converge to one orbit, which is helpful to explain the present amplitude of the cosmological constant. In this converging flow of orbits, different initial conditions yield almost the same universe. But we have proved that it does not exist a strict attractor in this system. Thus, it is only a quasi-attractor. Different orbits are associated with different initial conditions. Fig. 1 describes the evolution of the universe from $s = -1$ to $s = 3$. The slow-roll parameter $\alpha \approx 8.6 \times 10^{-6} \ll 1$ at the quasi-attractor. Therefore, slow-roll is a perfect approximation and the universe is effectively a de Sitter one. The detailed parameters for this quasi-attractor are listed as follows: $j = 0.01$, $\Omega_{dm0} = 0.3$, $\Omega_{r_c} = 0.2$. Ω_{dm0} and Ω_{r_c} are present partitions of the dust matter and geometric term, which are defined as

$$\Omega_{dm0} \triangleq \frac{\rho_{dm0}}{\rho_c}, \tag{37}$$

and

$$\Omega_{r_c} \triangleq \frac{\rho_0}{\rho_c}, \tag{38}$$

where ρ_{dm0} labels the present density of dust matter and ρ_c denotes the present critical density.

For the branch $\theta = -1$, we have a similar conclusion. We show the phase portraits of x–y–l in Fig. 3 and its projections in Fig. 4. Fig. 3 describes the evolution of the universe from $s = -1$ to $s = 3$. For comparison, the parameters in Figs. 3 and 4 are adopted as the same of the branch $\theta = 1$. From the panel l–y in Fig. 2, one sees that the different curves almost coincide, which implies that the phase space is almost reduced to a lower dim subspace.

From Fig. 3, one sees that the quasi-attractor appears again in the branch $\theta = -1$. $\alpha \approx 3.2 \times 10^{-5} \ll 1$ at the quasi-attractor, which marks slow-roll of the tachyon. The corresponding density and pressure read,

$$\frac{\rho_T}{\rho_c} = 1.72106, \tag{39}$$

and

$$\frac{p_T}{\rho_c} = -1.72101 \approx -\frac{\rho_T}{\rho_c}. \tag{40}$$

Though it looks the tachyon is a perfect approximation of vacuum, we stress that the density ρ_{de} and p_{eff} in (10) are different from ρ_T and p_T. The evolution of the universe around the quasi-attractor is determined by the joint effect of the tachyon and geometric contribution, for the dust matter has been completely diluted away.

So, we further study the behavior of the virtual dark energy, which carries the combining effect of the tachyon and geometric term. We find that on the way to the quasi-attractor the crossing -1 behavior of the EOS of the virtual dark energy will appear in the branch $\theta = -1$. We show a concrete numerical example of this crossing behaviors in Fig. 5, in which we take the parameter set as $j = 0.01$, $\Omega_{dm0} = 0.3$, $\Omega_{r_c} = 0.2$.

Fig. 5 explicitly illuminates that the EOS crosses -1 at $s \sim -0.2$. Also, we plot the evolution of the deceleration parameter q. It is one of the most significant parameters from the viewpoint of observations, which carries the total effects of cosmic fluids. q is defined as,
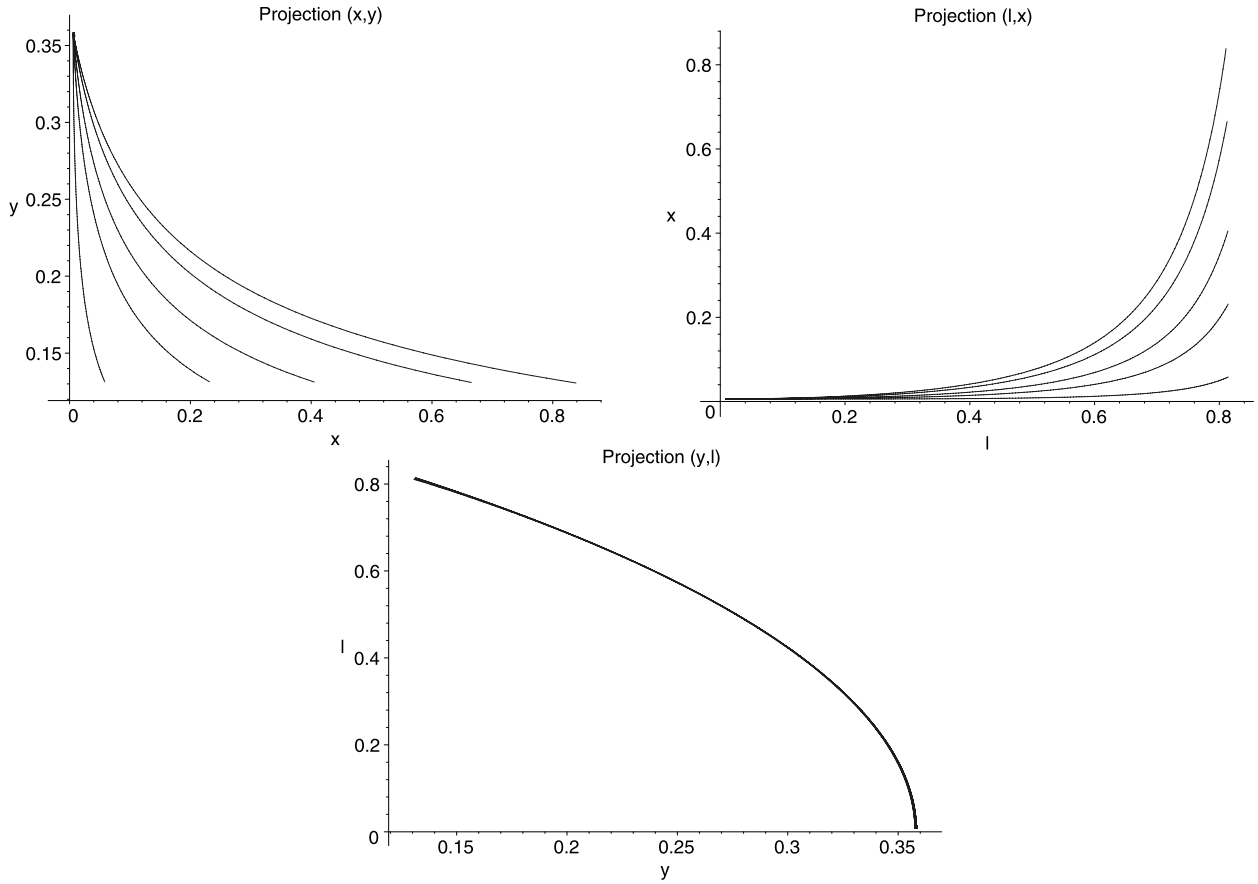
Fig. 2. The projections of Fig. 1 on x–y, x–l, l–y planes, respectively.
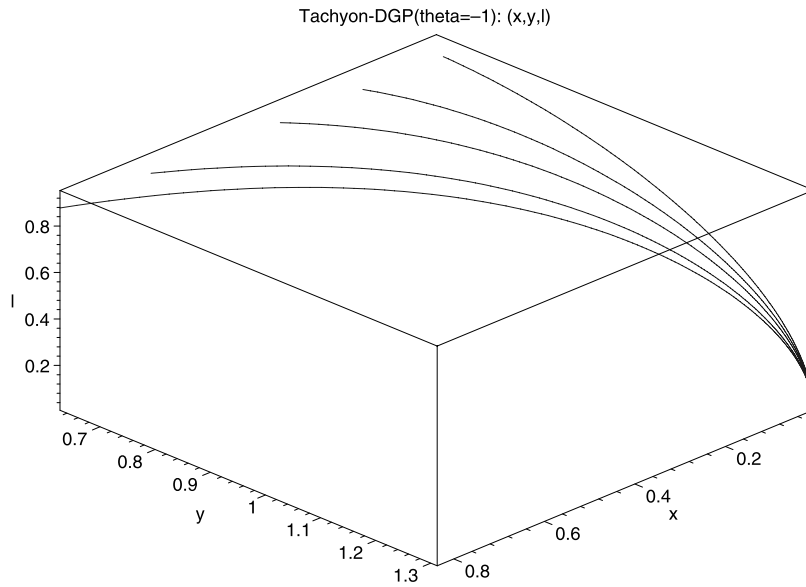

Fig. 3. The evolution of a tachyon on a DGP in the branch $\theta = -1$. The different initial conditions for the curves are $x(s = 0) = 0.1$, $x(s = 0) = 0.08$, $x(s = 0) = 0.05$, $x(s = 0) = 0.03$, $x(s = 0) = 0.01$ from the left to the right, respectively.

$$q \triangleq -\frac{\ddot{a}a}{\dot{a}^2} = -1 + \frac{3}{2}\alpha. \tag{41}$$

Fig. 6 illuminates the evolution of the deceleration parameter for a tachyon on a DGP in the branch $\theta = -1$ with the same parameters in the above figure.

From Figs. 5 and 6, clearly, the EOS of effective dark energy crosses -1. At the same time the deceleration parameter is consistent with observations. It is well known that the EOS of a single scalar in standard GR never crosses the phantom divide. Therefore, the induced term, through the "energy density" of r_c, ρ_0, plays a critical role in this crossing.
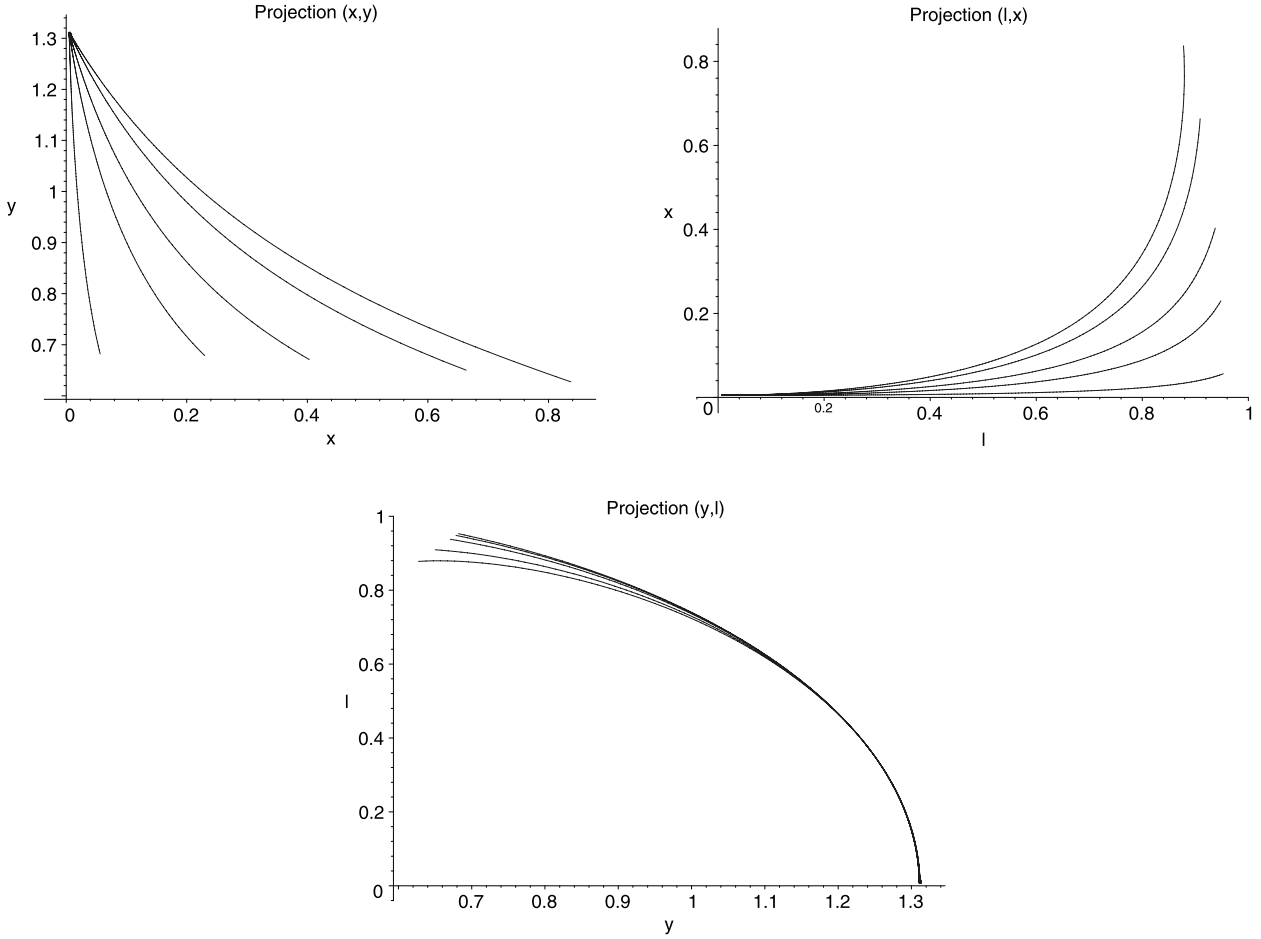
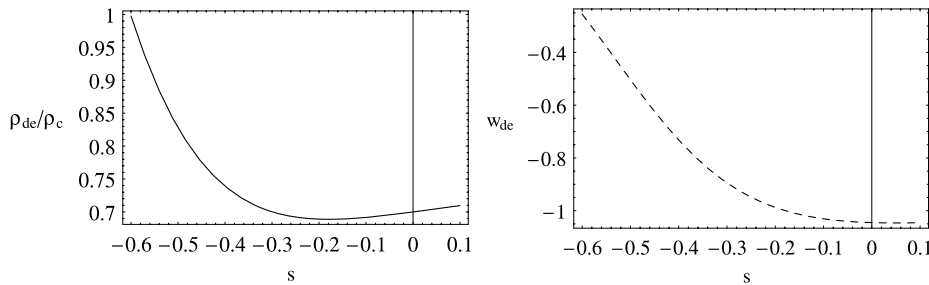**Fig. 4.** The projections of Fig. 3 on x–y, x–l, l–y planes, respectively.

Fig. 5. Density of virtual dark energy and its EOS in the branch $\theta = -1$ for an exponential potential. Left panel: The evolution of the density of virtual dark energy as a function of $s = \ln a$. Right panel: The evolution of the EOS of the virtual dark energy as a function of s. The present epoch is denoted by $s = 0$.
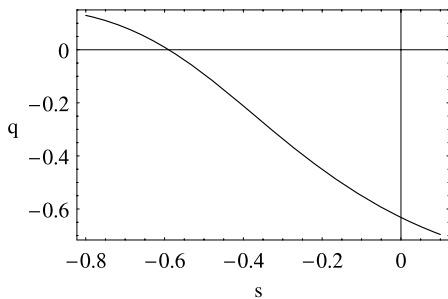

Fig. 6. The deceleration parameter as a function of s corresponding to Fig. 5.

For the branch $\theta = 1$, this crossing behavior is impossible. We demonstrate this point by using (11) and the explanation following it. In the branch $\theta = 1$, the virtual dark energy density ρ_{de} in (9) becomes,

$$\rho_{de} = \frac{V(T)}{\sqrt{1 - \dot{T}^2}} + \rho_0 + \rho_0 \left(1 + \frac{2\left(\frac{V(T)}{\sqrt{1 - \dot{T}^2}} + \rho_{dm}\right)}{\rho_0} \right)^{1/2}. \quad (42)$$

Clearly, every term in RHS of the above equation is decreasing in an expanding universe (ρ_T will decrease since its $w > -1$, and ρ_{dm} decreases with the scale factor). This conclusion is unchanged for the $\theta = 1$ branch of DGP with an essence whose $w > -1$ and dust matter confined to it. It is also independent of the concrete form of the potential (for a positive potential).
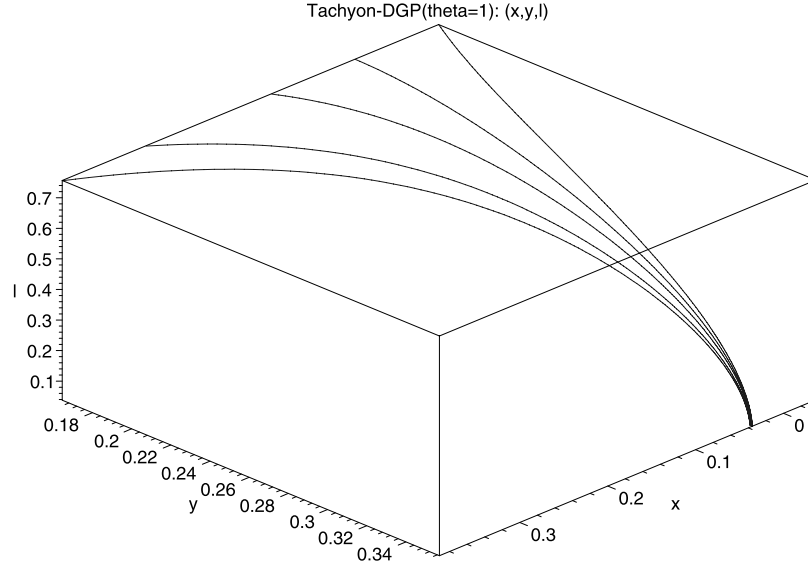
Fig. 7. The evolution of a tachyon on a DGP in the branch $\theta = 1$. The parameters used for this figure are $\mu/A = 0.1$, $\Omega_{dm0} = 0.3$, $\Omega_{r_c} = 0.2$. The different initial conditions for the curves are $x(s=0) = 0.1$, $x(s=0) = 0.08$, $x(s=0) = 0.05$, $x(s=0) = 0.03$, $x(s=0) = 0.01$ from the left to the right, respectively.

3.2. Inverse power law potential

The discussions of this subsection is parallel to the last subsection.

Exponential potential is an important case in the standard model. Some researches imply that an analogy of exponential potential in the context of tachyon dynamics is inverse square potential. Here we set

$$V = \frac{4A^2}{3T^2}, \tag{43}$$

where A is a constant. The dynamics of the universe can be described by the following dynamical system with the dimensionless variables x, y, l, b,

$$x' = 3(1 - x^2)\left(-x + \frac{\mu}{A}y\right), \tag{44}$$

$$y' = \frac{3}{2}\alpha y - \frac{3\mu}{2A}xy^2, \tag{45}$$

$$l' = \frac{3}{2}\alpha l - \frac{3}{2}l, \tag{46}$$

$$b' = \frac{3}{2}\alpha b, \tag{47}$$

where the definition of α is the same as in (31). Similarly, we can prove that there does not exist a strict critical point in this system. Note that a critical point in this system must be a de Sitter one if it exists because of the definition of the variable b. If we define different variables, the result may become different.

We see that neither for the case of exponential potential nor the inverse power law potential the attractor does not exist in the tachyon-DGP system. In fact, it was shown that for a positive potential the Born–Infeld type scalar has a critical point only if it has a nonvanishing minimum, which corresponds to the de Sitter attractor of the system [21]. Our results can be treated as examples of the above conclusion, for no minimum appears at an exponential potential or an inverse power law potential.

Like the case of an exponential potential, a quasi-attractor appears again in this system, which suggests the universality of the quasi-attractor behavior.

Following the discussions of the last subsection, we plot 3-dim phase portraits in the subspace x–y–l in the branch $\theta = 1$ and $\theta = -1$ respectively, and their the projections on x–y, x–l and y–l planes.

Fig. 7 displays the evolution of a tachyon attached to a DGP brane in the phase space x–y–l. The converging orbits indicate the same final state for different initial conditions. However, we showed that there was no strict attractor in this system. It is just a quasi-attractor. Different orbits correspond to different initial conditions. In this case, the slow-roll parameter $\alpha \approx 5.5 \times 10^{-4} \ll 1$ at the quasi-attractor. Therefore, the case is very similar to what happens in slow-roll inflation and the universe is effectively a de Sitter one. Fig. 7 and the following Fig. 8 describe the evolution of the universe from $s = -0.6$ to $s = 3$.

For the branch $\theta = -1$, we have almost the same conclusion. The phase portraits of x–y–l and its projections are displayed in Figs. 9 and 10, respectively. We set the same parameters in Figs. 9 and 10 as in the branch $\theta = 1$.

From Fig. 9, the quasi-attractor appears again as we expected. In branch $\theta = -1$, $\alpha \approx 0.029 \ll 1$ at the quasi-attractor. The corresponding density and pressure read,

$$\frac{\rho_T}{\rho_c} = 2.20, \tag{48}$$

and

$$\frac{p_T}{\rho_c} = -2.15 \approx -\frac{\rho_T}{\rho_c}. \tag{49}$$

We see that tachyon finally evolves as cosmological constant. However, the evolution of the universe is not determined by the tachyon only, but by the joint effect of the tachyon and geometric term. In the following we will study the evolution of the virtual dark energy, which carries the total effect of the tachyon and the geometric effect and determines the destiny of the universe.

We find that when the universe is approaching the quasi-attractor the EOS of the virtual dark energy can cross the phantom divide in the branch $\theta = -1$. A concrete numerical example of this crossing behaviors is displayed in Fig. 11, in which we take the parameter set as $\mu/A = 0.25$, $\Omega_{dm0} = 0.3$, $\Omega_{r_c} = 0.2$.
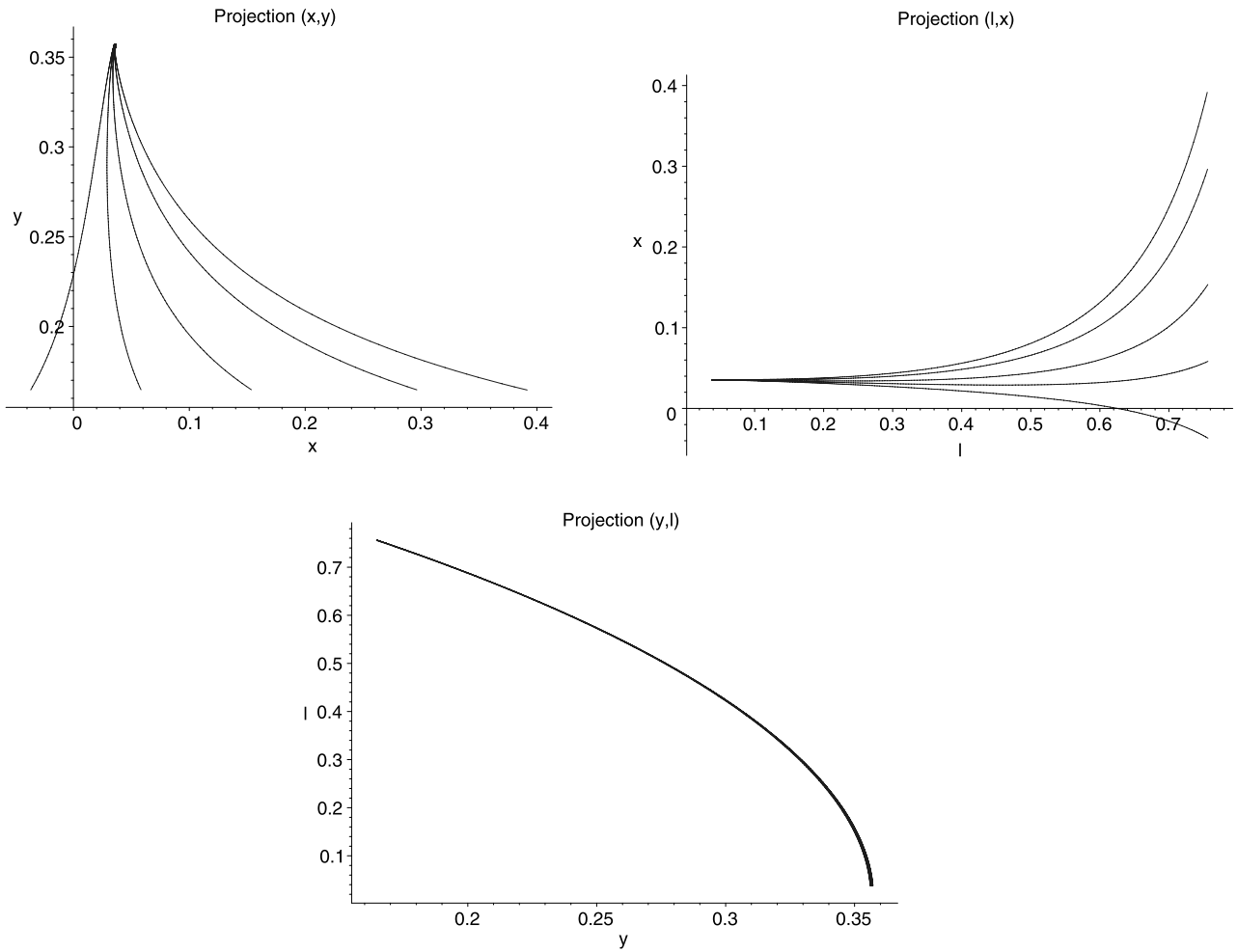
Projection (x,y)

Projection (l,x)

Projection (y,l)

Fig. 8. The projections of Fig. 7 on x–y, x–l, l–y planes, respectively.
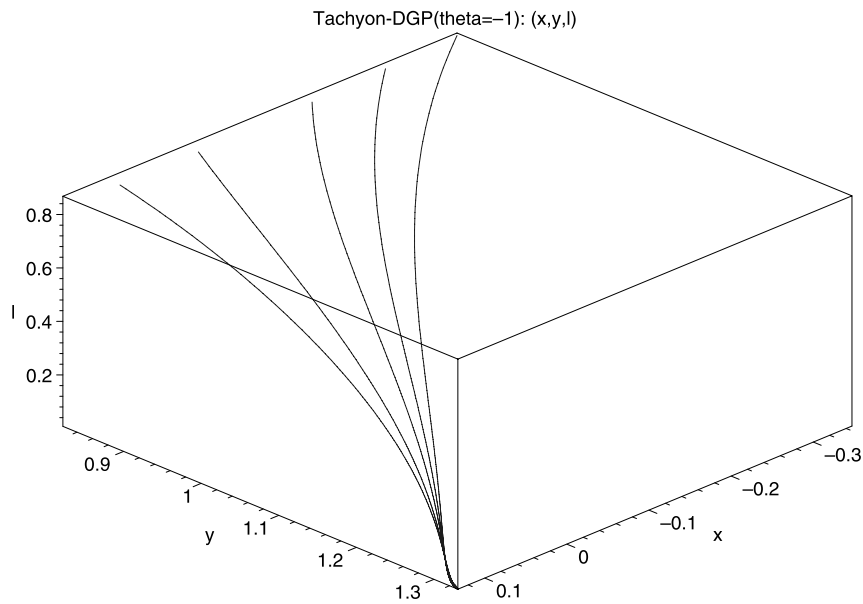
Tachyon-DGP(theta=−1): (x,y,l)

Fig. 9. The evolution of a tachyon on a DGP in the branch $\theta = -1$, where the parameters and initial conditions are the same as in the branch $\theta = 1$.

Fig. 11 clearly displays that the EOS crosses -1 at $s \sim -0.3$. At the same time we plot the corresponding deceleration parameter in Fig. 11, which is determined by the total fluids in the universe.

Fig. 12 illuminates the evolution of the deceleration parameter for a tachyon on a DGP in the branch $\theta = -1$ with the same parameters in the above figure.
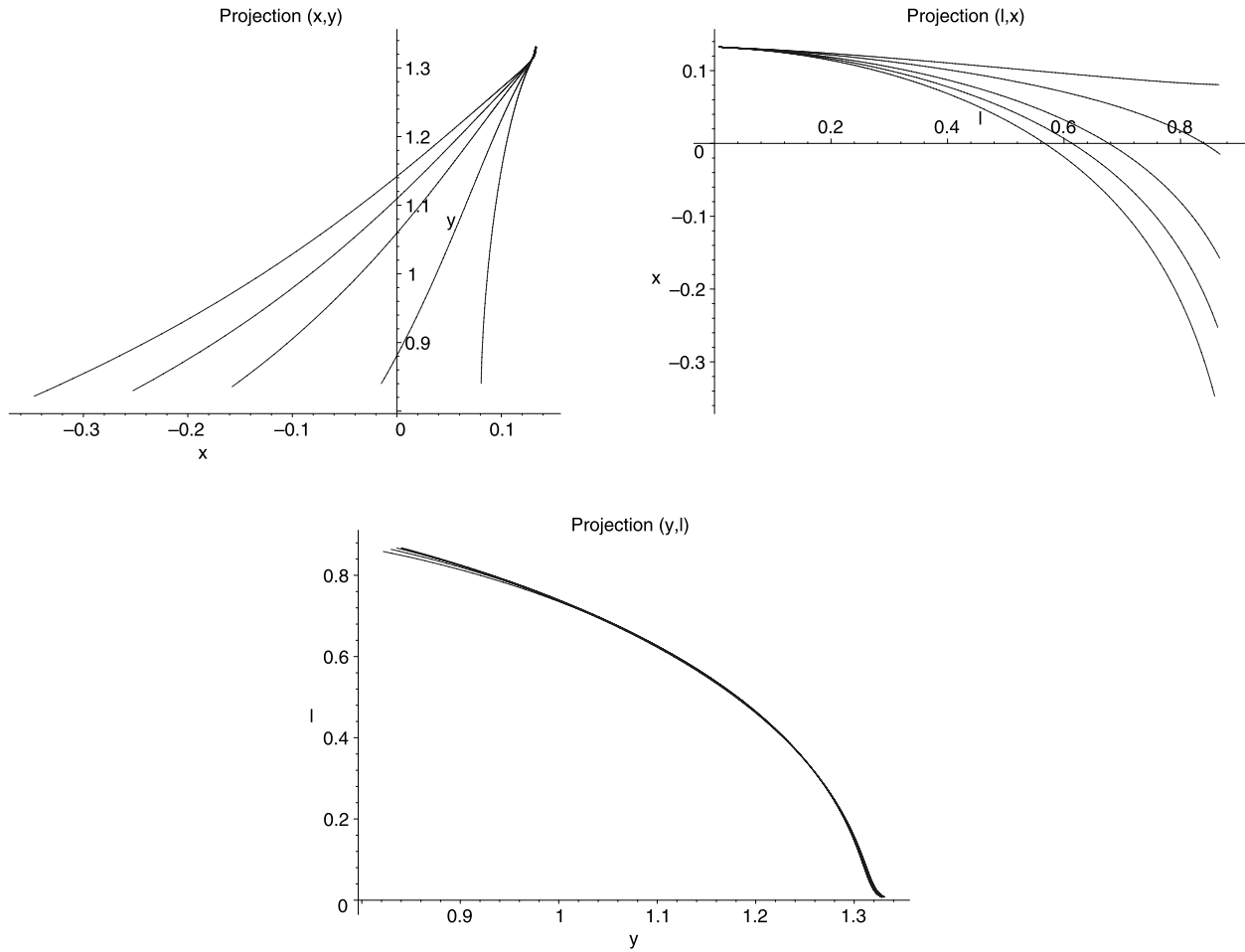
Projection (x,y)

Projection (l,x)

Projection (y,l)

Fig. 10. The projections of Fig. 9 on $x - y$, $x - l$, $l - y$ planes, respectively.
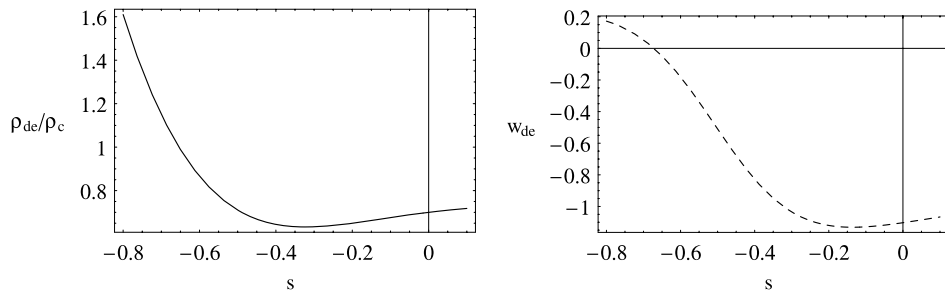
Fig. 11. Density of virtual dark energy and its EOS in the branch $\theta = -1$ for an inverse power law potential. Left panel: The evolution of the density of virtual dark energy as a function of s. Right panel: The evolution of the EOS for the virtual dark energy as a function of s.

From Figs. 11 and 12, one sees the EOS of effective dark energy crosses -1. Also, the deceleration parameter is consistent with observations.

From the conclusion which we obtained in the last subsection, the crossing does not appear in the $\theta = 1$ branch.

4. Conclusions and discussions

In this Letter, the dynamics of a tachyon attached to a DGP brane is studied. Two kinds of potentials of the tachyon field, exponential potential and inverse power law potential, are explored, respectively.

In the investigation of tachyon-DGP, we find the quasi-attractor behavior. Traditionally, if a dynamical system does not permit crit-
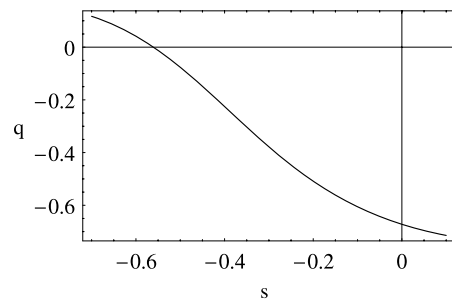
Fig. 12. The deceleration parameter as a function of s corresponding to Fig. 11.

ical points, we just stop, imagining that the orbits in this system must be disordered and never converge. However, we find that the orbits with different initial conditions converge even though there is no real critical points. This quasi-attractor is full of vitality, which can appear in both of the two branches, and for both of the two kinds of potentials. The converging evolution of the orbits in the phase portraits offers a new view on the cosmological constant problem and coincidence problem.

In the branch $\theta = -1$, we find that the EOS of the virtual dark energy, which is caused by the tachyon and geometric term, can cross the phantom divide for both of exponential potential and inverse power law potential. This provides a new theoretical possibility for the extraordinary observation of dark energy. We find that the geometric term plays a significant role in this crossing. Contrarily, the crossing behavior do not appear in the branch $\theta = 1$.

Acknowledgements

References

[1] P. Horava, E. Witten, Nucl. Phys. B 475 (1996) 94, arXiv:hep-th/9603142.
[2] R. Maartens, gr-qc/0312059.
[3] L. Randall, R. Sundrum, Phys. Rev. Lett. 83 (1999) 4690, arXiv:hep-th/9906064.
[4] G. Dvali, G. Gabadadze, M. Porrati, Phys. Lett. B 485 (2000) 208, hep-th/0005016;
G. Dvali, G. Gabadadze, Phys. Rev. D 63 (2001) 065007;
A. Lue, astro-ph/0510068.
[5] R.G. Cai, H.s. Zhang, JCAP 0408 (2004) 017, arXiv:hep-th/0403234;
E. Papantonopoulos, V. Zamarias, JCAP 0410 (2004) 001, arXiv:gr-qc/0403090;
S. del Campo, R. Herrera, Phys. Lett. B 653 (2007) 122, arXiv:0708.1460 [gr-qc];
H.S. Zhang, Z.H. Zhu, Phys. Lett. B 641 (2006) 405, arXiv:astro-ph/0602579.
[6] A.G. Riess, et al., Astron. J. 116 (1998) 1009, astro-ph/9805201;
S. Perlmutter, et al., Astrophys. J. 517 (1999) 565, astro-ph/9812133.
[7] Edmund J. Copeland, M. Sami, Shinji Tsujikawa, hep-th/0603057;
J.P. Uzan, arXiv:astro-ph/0605313.
[8] U. Alam, V. Sahni, T.D. Saini, A.A. Starobinsky, Mon. Not. Roy. Astron. Soc. 354 (2004) 275, arXiv:astro-ph/0311364;
U. Alam, V. Sahni, A.A. Starobinsky, JCAP 0406 (2004) 008, arXiv:astro-ph/0403687;
D. Huterer, A. Cooray, astro-ph/0404062;
Y. Wang, M. Tegmark, astro-ph/0501351;
Andrew R. Liddle, Pia Mukherjee, David Parkinson, Yun Wang, astro-ph/0610126.
[9] H. Zhang, arXiv:0909.3013 [astro-ph.CO]; see this article for a more complete reference list for this crossing behavior.
[10] Yi-Fu Cai, Emmanuel N. Saridakis, Mohammad R. Setare, Jun-Qing Xia, arXiv:0909.2776v1.
[11] H.S. Zhang, Z.H. Zhu, Phys. Rev. D 73 (2006) 043518;
H. Wei, R.G. Cai, Phys. Rev. D 73 (2006) 083002.
[12] H.S. Zhang, Z.H. Zhu, Phys. Rev. D 75 (2007) 023510, arXiv:astro-ph/0611834;

H.s. Zhang, H. Noh, Phys. Lett. B 679 (2009) 81, arXiv:0904.0067 [gr-qc];
R.G. Cai, H.S. Zhang, A. Wang, Commun. Theor. Phys. 44 (2005) 948, arXiv:hep-th/0505186;
Hongsheng Zhang, Zong-Hong Zhu, Modern Phys. Lett. A 24 (7) (2009) 541, arXiv:0704.3121.
[13] G.W. Gibbons, Phys. Lett. B 537 (2002) 1;
G. Shiu, I. Wasserman, Phys. Lett. B 541 (2002) 6;
T. Padmanabhan, T.R. Choudhury, Phys. Rev. D 66 (2002) 081301;
D.J. Liu, X.Z. Li, Phys. Rev. D 70 (2004) 123504, arXiv:astro-ph/0402063;
X.Z. Li, X.H. Zhai, Phys. Rev. D 67 (2003) 067501, arXiv:hep-ph/0301063;
X.Z. Li, D.J. Liu, J.G. Hao, arXiv:hep-th/0207146;
A. Frolov, L. Kofman, A. Starobinsky, Phys. Lett. B 545 (2002) 8;
M. Fairbairn, M.H.G. Tytgat, Phys. Lett. B 546 (2002) 1;
L. Kofman, A. Linde, JHEP 0207 (2002) 004;
M. Sami, Modern Phys. Lett. A 18 (2003) 691;
Y.S. Piao, R.G. Cai, X.M. Zhang, Y.Z. Zhang, Phys. Rev. D 66 (2002) 121301;
J.M. Cline, H. Firouzjahi, P. Martineau, JHEP 0211 (2002) 041;
M.C. Bento, O. Bertolami, A.A. Sen, Phys. Rev. D 67 (2003) 063511;
Y.S. Piao, Q.G. Huang, X.M. Zhang, Y.Z. Zhang, Phys. Lett. B 570 (2003) 1;
Z.K. Guo, Y.S. Piao, R.G. Cai, Y.Z. Zhang, Phys. Rev. D 68 (2003) 043508;
S. Nojiri, S.D. Odintsov, Phys. Lett. B 571 (2003) 1;
D.A. Steer, F. Vernizzi, Phys. Rev. D 70 (2004) 043527;
V. Gorini, A.Y. Kamenshchik, U. Moschella, V. Pasquier, Phys. Rev. D 69 (2004) 123512;
B.C. Paul, M. Sami, Phys. Rev. D 70 (2004) 027301;
J.M. Aguirregabiria, R. Lazkoz, Modern Phys. Lett. A 19 (2004) 927.
[14] T. Padmanabhan, Phys. Rev. D 66 (2002) 021301;
A. Feinstein, Phys. Rev. D 66 (2002) 063511;
J.S. Bagla, H.K. Jassal, T. Padmanabhan, Phys. Rev. D 67 (2003) 063504;
Gianluca Calcagni, Andrew R. Liddle, Phys. Rev. D 74 (2006) 043528;
D. Choudhury, D. Ghoshal, D.P. Jatkar, S. Panda, Phys. Lett. B 544 (2002) 231;
X.Z. Li, J.G. Hao, D.J. Liu, Chinese Phys. Lett. 19 (2002) 1584;
J.G. Hao, X.Z. Li, Phys. Rev. D 66 (2002) 087301;
M.R. Garousi, M. Sami, S. Tsujikawa, Phys. Rev. D 70 (2004) 043536;
V.H. Cardenas, Phys. Rev. D 73 (2006) 103512;
J.M. Aguirregabiria, R. Lazkoz, Phys. Rev. D 69 (2004) 123502;
E.J. Copeland, M.R. Garousi, M. Sami, S. Tsujikawa, Phys. Rev. D 71 (2005) 043003;
S.G. Shi, Y.S. Piao, C.F. Qiao, JCAP 0904 (2009) 027, arXiv:0812.4022 [astro-ph];
Y. Ping, L. Xu, H. Liu, Y. Shao, Internat. J. Modern Phys. D 17 (2008) 2017, arXiv:0801.0268 [hep-th].
[15] A. Sen, JHEP 0204 (2002) 048;
A. Sen, JHEP 0207 (2002) 065;
A. Sen, Internat. J. Modern Phys. A 20 (2005) 5513.
[16] L.P. Chimento, M. Forte, G.M. Kremer, M.G. Richarte, Phys. Rev. A 79 (2009) 083527, arXiv:0903.4144 [gr-qc];
L.P. Chimento, M.G. Richarte, arXiv:0905.3780 [astro-ph.CO].
[17] C. Deffayet, Phys. Lett. B 502 (2001) 199;
C. Deffayet, G. Dvali, G. Gabadadze, Phys. Rev. D 65 (2002) 044023;
C. Deffayet, S.J. Landau, J. Raux, M. Zaldarriaga, P. Astier, Phys. Rev. D 66 (2002) 024019.
[18] Y. Wang, P. Garnavich, ApJ 552 (2001) 445;
M. Tegmark, Phys. Rev. D 66 (2002) 103507;
Y. Wang, K. Freese, Phys. Lett. B 632 (2006) 449, astro-ph/0402208.
[19] G.W. Gibbons, Classical Quantum Gravity 20 (2003) S321, arXiv:hep-th/0301117.
[20] D.N. Spergel, et al., arXiv:astro-ph/0603449.
[21] J.G. Hao, X.Z. Li, Phys. Rev. D 68 (2003) 083514, arXiv:hep-th/0306033;
J.G. Hao, X.Z. Li, Phys. Rev. D 68 (2003) 043501, arXiv:hep-th/0305207.