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Evaluating the marginal utility principle for long-term hydropower scheduling

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ABSTRACT

The conversion of the potential energy of dammed water into hydropower depends on both reservoir storage and release, which are the major difficulties in hydropower reservoir operation. This study evaluates the marginal utility principle, which determines the optimal carry-over storage between periods, for long-term hydropower scheduling. Increasing marginal cost and decreasing marginal return are two important characteristics that determine the marginal utility principle in water supply. However, the notion of decreasing marginal return is inapplicable in hydropower scheduling. Instead, the carry-over storage from one period has an increasing marginal contribution to the power generation in the next period. Although carry-over storage incurs an increasing marginal cost to the power generation in the current period, the marginal return is higher than the marginal cost. The marginal return from the carry-over storage further increases in the multi-period case. These findings suggest saving as much carry-over storage as possible, which is bounded by the operational constraints of storage capacity, environmental flow, and installed capacity in actual hydropower scheduling. The marginal utility principle is evaluated for a case study of the Three Gorges Reservoir, and the effects of the constraints are discussed. Results confirm the theoretical findings and show that the marginal return from carry-over storage is larger than the marginal cost. The operational constraints help determine the optimal carry-over storage.

1. Introduction

Reservoir operation involves a sequential decision making process. A release decision is made at the beginning of one period based on hydrological forecast and is implemented to satisfy multiple operational objectives. For example, Wang et al. [21] investigated hydropower operation of cascade reservoirs considering hydraulic connections; Null and Lund [15] incorporated environmental flow into reservoir operation to restore fish habitat; Zhao and Zhao [28] analyzed water supply under streamflow forecast uncertainty. Over time, release decisions in subsequent periods are dynamically updated with improved forecast information and are implemented. Wu et al. [22] and Chen and Chau [1] respectively presented hydrological forecasting based on artificial intelligence and hydrological models. Zhao et al. [27] illustrated the quantification of predictive uncertainty of streamflow forecasts. Xu et al. [23] explored how to efficiently use dynamically updated hydrological forecasts in reservoir operation.

Operating rules, including standard operating policies and hedging rules, and optimization models, such as mathematical programming and heuristic algorithms, have been proposed to aid in reservoir operation. For instance, Tu et al. [19] and Eum et al. [5] respectively presented applications of hedging rules to deal with water supply and hydropower issues; while Cheng et al. [2] and Giuliani et al. [7] respectively applied nonlinear programming and many-objective evolutionary algorithm to optimize reservoir operations. Decision making involving reservoir operation is complex because of many issues. The effects of market demands of hydropower and water resources on decision making were analyzed in Olivares and Lund [16]. The application of probabilistic forecast to reservoir operation was presented by Sankarasubramanian et al. [17]. The integration of operating rules, optimization models and algorithms were investigated by Georgakakos et al. [6] for the joint operation of multiple reservoirs in California and by Lu et al. [12] for the operation of cascade reservoirs on the Yangtze River.

Despite the complexities, decision making of reservoir operation can be conceptualized as a simple two-stage model: either releasing stored water from a reservoir for beneficial use in the

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current period or retaining water for future use. This two-stage conceptualization complements complex optimization models. Derivations from the two-stage model aid in making operation decisions. For example, hedging operations, which reduce current release in anticipation of water shortage, are common in water resource management. Using two-stage models, Draper and Lund [4] addressed the rationality behind hedging and attributed hedging operations to the economic characteristic of diminishing marginal utility. A multi-period model can be formulated as a recursive two-stage model, as in dynamic programming. Based on two-stage formulations, Zhao et al. [26,31] and Zhao and Zhao [29] developed improved dynamic programming algorithms for reservoir operation.

Draper and Lund [4] proposed the marginal utility principle to characterize the optimality condition of decisions in the two-stage model. This principle is derived from the utility function, which is also called the objective function in optimization. In water supply, the utility function is simple because it depends only on water use. Specifically, water released for current use and carry-over storage retained for future use exhibit a diminishing marginal utility. Draper and Lund [4], You and Cai [24], and Zhao et al. [25] derived the marginal utility principle by elaborating the marginal utilities of release and carry-over storage. The concavity of the utility function assures that the marginal utility principle is both sufficient and necessary for optimal water supply decisions. In flood control, Zhao et al. [30] proposed the marginal utility principle by formulating flood risks and by considering the utility functions of current and future safety margins.

However, the utility function is more complicated in hydropower operations. The formulations of Liu et al. [10], Cheng et al. [2], and Tilmant et al. [18] indicated that the conversion of the potential energy of dammed water depends on both reservoir storage and release. Moreover, Zhao et al. [31] observed that carry-over storage may exhibit increasing marginal utility, instead of diminishing marginal utility, because an increase in this storage creates a higher water head and enables future release to generate more power. Does the marginal utility principle apply to hydropower problems? Are the optimal decisions in hydropower problems determined by this principle? This study addresses these two research questions and builds on Zhao et al. [31] by elaborating both the marginal cost and marginal return of carry-over storage in one-, two-, and multi-period cases. In addition to theoretical analyses, a case study of the long-term hydropower scheduling of the Three Gorges Reservoir is conducted. The effects of the marginal utility principle and reservoir operational constraints on optimal hydropower scheduling decisions are elaborated.

2. Marginal utility principle and hydropower scheduling

Consider a reservoir operation problem with a study horizon of T periods. The time period is denoted by subscript t. An optimization model can be formulated as follows:

$$\max \quad \sum_{t=1}^{T} g_t(s_t, s_{t+1}, r_t, q_t). \tag{1}$$

In Eq. (1), the total utility from period 1 to period T is maximized. The single-period utility $g_t()$ is formulated in a general form. The explanatory variables include reservoir storage s_t at the beginning of period t, storage s_{t+1} at the end of period t (the beginning of the next period $t+1$), release r_t during period t, and inflow q_t in period t. A two-stage formulation of the multi-period model in Eq. (1) is presented as follows:

$$G_t(s_t) = \max \quad g_t(s_t, s_{t+1}, r_t, q_t) + G_{t+1}(s_{t+1}). \tag{2}$$

In Eq. (2), s_{t+1} is also called the carry-over storage from period t to period $t+1$, which is the decision variable in the optimization model. $G_t(s_t)$ and $G_{t+1}(s_{t+1})$ are the maximum cumulative utility functions from period T to periods t and $t+1$, respectively. The recursion in Eq. (2) from period $T-1$ to period 1 optimizes the reservoir operation in Eq. (1).

The marginal utility principle for water supply can be derived through Eq. (2). First, the utility function is simplified as $f_t(r_t)$ depending on release-only conditions (please refer to You and Cai [24] for justifications of the simplification). The two-stage model is reformulated as

$$F_t(s_t) = \max \quad f_t(r_t) + F_{t+1}(s_{t+1}). \tag{3}$$

In Eq. (3), the single-period and maximum cumulative utility functions are denoted by f and F, respectively, in order to distinguish them from the utility functions g and G in hydropower scheduling. In Eq. (3), $f_t()$ is a concave function of r_t ($t = 1, 2, \ldots, T$), that is,

$$\begin{cases} \dfrac{df_t}{dr_t} > 0 \\ \dfrac{d^2 f_t}{dr_t^2} < 0 \end{cases}. \tag{4}$$

This equation indicates that release r_t in period t has a diminishing marginal contribution to the water supply utility f_t. Zhao et al. [26] derived that the concavity of $f_t()$ $(t = T, T-1, \ldots, t+1)$ leads to the concavity of $F_{t+1}()$, that is,

$$\begin{cases} \dfrac{dF_{t+1}}{ds_{t+1}} > 0 \\ \dfrac{d^2 F_{t+1}}{ds_{t+1}^2} < 0 \end{cases}. \tag{5}$$

Eq. (5) indicates that the carry-over storage retained for future use exhibits a diminishing marginal contribution to the maximum cumulative utility.

r_t in Eq. (4) and s_{t+1} in Eq. (5) are constrained by the water balance relationship, that is, $r_t + s_{t+1} = s_t + q_t$. This relationship suggests that

$$r_t = s_t + q_t - s_{t+1}. \tag{6}$$

Incorporating Eq. (6) into Eq. (4),

$$\begin{cases} \dfrac{df_t}{ds_{t+1}} = \dfrac{df_t}{dr_t}\dfrac{dr_t}{ds_{t+1}} = -\dfrac{df_t}{dr_t} < 0 \\ \dfrac{d^2 f_t}{ds_{t+1}^2} = \dfrac{d^2 f_t}{dr_t^2}\left(\dfrac{dr_t}{ds_{t+1}}\right)^2 = \dfrac{d^2 f_t}{dr_t^2} < 0 \end{cases}. \tag{7}$$

Evidently, $\frac{df_t}{ds_{t+1}}$ is the opposite of $\frac{df_t}{dr_t}$ and has a negative value. Thus, retaining s_{t+1} for the future incurs a cost. $\frac{d^2 f_t}{ds_{t+1}^2}$ is negative, indicating an increase in marginal cost as s_{t+1} increases.

For the two-stage model in Eq. (3), Eqs. (5) and (7) indicate that s_{t+1} contributes to F_{t+1} at the cost of f_t. As s_{t+1} increases, the marginal return $\frac{dF_{t+1}}{ds_{t+1}}$ of s_{t+1} decreases (i.e., the blue dashed line in Fig. 1a),[1] whereas the marginal cost $\frac{df_t}{ds_{t+1}}$ increases (i.e., the red solid line in Fig. 1a). A decreasing marginal return and an increasing marginal cost produce the optimal carry-over storage s^*_{t+1}. The marginal return is obviously higher than the marginal cost when $s_{t+1} < s^*_{t+1}$, suggesting that s_{t+1} should be increased. The marginal return is lower than the marginal cost when $s_{t+1} > s^*_{t+1}$; that is, reducing s_{t+1} generates a net utility gain. Therefore, Draper and Lund [4] and You and Cai [24] proposed the following marginal utility principle

[1] For interpretation of color in Fig. 1, the reader is referred to the web version of this article.

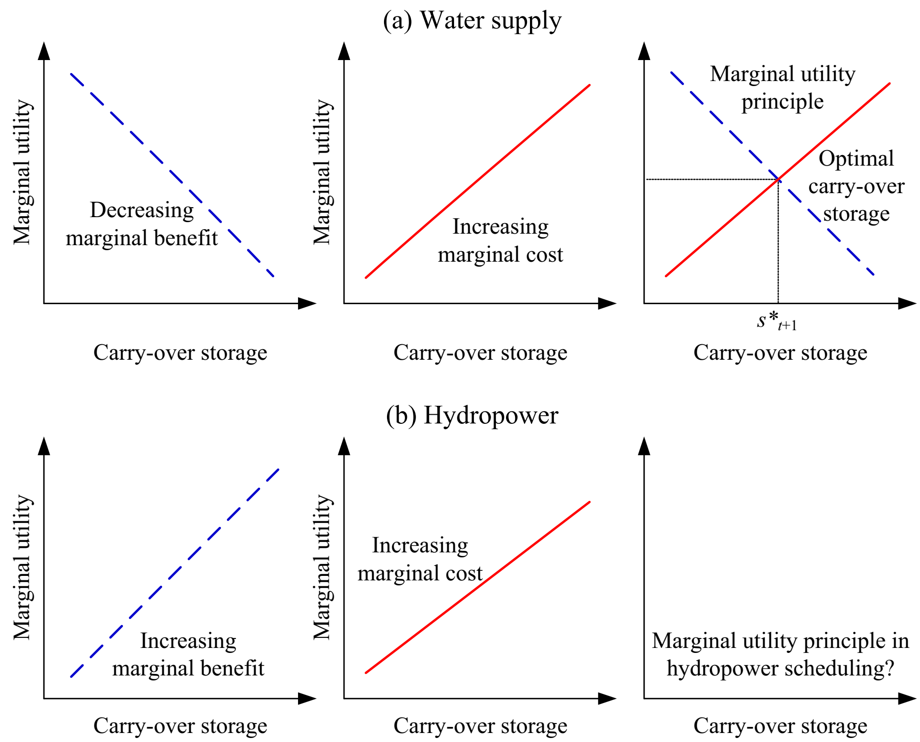

Fig. 1. Schematic of the marginal utility principle in (a) water supply and (b) hydropower scheduling.

for the two-stage model: the marginal return is equal to the marginal cost at the optimal carry-over storage s^*_{t+1}.

The marginal utility principle in water supply is conceptually simple because both the single-period and maximum cumulative utility functions are concave. However, the utility function in hydropower problems is more complicated, as shown in the following:

$$g_t(s_t, s_{t+1}, r_t, q_t) = \eta \times \left[\frac{SSR(s_t) + SSR(s_{t+1})}{2} - SDR\right] \times r_t. \quad (8)$$

In Eq. (8), the power generation $g_t(s_t, s_{t+1}, r_t, q_t)$ in period t is formulated as the product of the efficiency coefficient η, water head $\frac{SSR(s_t)+SSR(s_{t+1})}{2} - SDR$, and reservoir release r_t. The present study focuses on maximizing power generation in long-term hydropower scheduling. Therefore, unnecessary complexities in short-term power generation are not considered, such as fluctuations in electricity price (please refer to Cheng et al. [3] for more details), vibration zones in hydropower turbines (please refer to the formulations in Ma et al. [14]), and variations in power generation efficiency with storage and release (please refer to the case study by Li et al. [9]). In Eq. (8), the water head is the mean of the difference between the upstream and downstream water levels, which are determined by the stage–storage relationship (SSR) and the stage–discharge relationship (SDR), respectively. SDR is simplified as a constant.

For Eq. (8), Zhao et al. [31] derived the following:

$$\begin{cases} \frac{\partial g_t}{\partial r_t} > 0 \\ \frac{\partial^2 g_t}{\partial r_t^2} < 0 \end{cases}, \quad (9)$$

and

$$\frac{\partial^2 g_t}{\partial s_t \partial r_t} > 0. \quad (10)$$

In Eq. (9), the diminishing marginal contribution of r_t to g_t implies that the marginal cost of power generation during period t (the red solid line in Fig. 1b) increases with the carry-over storage s_{t+1}. However, in Eq. (10), the complementarity between s_t and r_t suggests that s_{t+1} can generate an increasing marginal return (i.e., the blue dashed line in Fig. 1b), instead of a decreasing marginal return. More specifically, increasing s_{t+1} can make release $r_{t+1}, r_{t+2}, \ldots, r_T$ generate more power.

The marginal utility principle in water supply is attributable to the characteristics of diminishing marginal return and increasing marginal cost (Fig. 1a). However, in hydropower, increasing marginal return and marginal cost complicate the marginal utility principle (Fig. 1b). Is the marginal return larger or smaller than the marginal cost? Is there a high or low threshold where the marginal return is larger or smaller than the marginal cost? In the next section, both the marginal return and marginal cost of carry-over storage are investigated, and the marginal utility principle for hydropower scheduling is explored.

3. Theoretical analysis

This section analyzes one-, two-, and multi-period hydropower scheduling. Both the marginal return and marginal cost of carry-over storage are analytically derived in one- and two-period cases. The marginal cost is compared with the marginal return to explore the marginal utility principle. Furthermore, implications are provided for multi-period hydropower scheduling.

3.1. One-period formulation

A schematic of the reservoir operation during period t is shown in Fig. 2. The reservoir system in the said period is represented by a box. The carry-over storage s_t from the previous period $t-1$ and inflow q_t during period t are the inputs to the reservoir system, whereas the carry-over storage s_{t+1} to the next period and release r_t during period t are the outputs. Power generation depends on these four interrelated variables. The water balance relationship

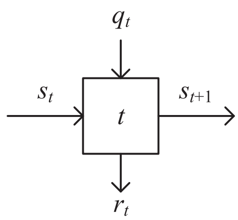

Fig. 2. Schematic of one-period reservoir operation (the reservoir system is represented by a box, and system inputs and outputs are represented by arrow lines).

indicates that $r_t = s_t + q_t - s_{t+1}$. Incorporating this equation into Eq. (8),

$$g_t(s_t, s_{t+1}) = \eta \times \left[\frac{SSR(s_t) + SSR(s_{t+1})}{2} - SDR\right] \times (s_t + q_t - s_{t+1}). \tag{11}$$

In Eq. (11), power generation is formulated as a function of the carry-over storage s_t and s_{t+1}. The inflow q_t is not treated as an explanatory variable in $g_t(s_t, s_{t+1})$ because q_t is a given input for the optimization model. As was illustrated in the literature review by Labadie [8] and a case study of hydropower by Turgeon [20], to determine carry-over storage between different periods is one of the most important issues in real-world reservoir operation.

The effects of s_t and s_{t+1} on g_t are measured by partial derivatives:

(1) $\frac{\partial g_t}{\partial s_t}$ indicates the marginal contribution of s_t to g_t:

$$\frac{\partial g_t}{\partial s_t} = \eta \frac{SSR'(s_t)}{2}(s_t + q_t - s_{t+1}) + \eta\left[\frac{SSR(s_t) + SSR(s_{t+1})}{2} - SDR\right]. \tag{12}$$

In Eq. (12), $SSR'(s_t) > 0$ (i.e., the reservoir water level increases with storage), $s_t + q_t - s_{t+1} = r_t > 0$ (i.e., the reservoir release is greater than zero), and $\frac{SSR(s_t)+SSR(s_{t+1})}{2} - SDR > 0$ (i.e., the reservoir water level is higher than the water level in the downstream river channel). Therefore, $\frac{\partial g_t}{\partial s_t} > 0$.

(2) $\frac{\partial^2 g_t}{\partial s_t^2}$ illustrates the trend of $\frac{\partial g_t}{\partial s_t}$ as s_t increases:

$$\frac{\partial^2 g_t}{\partial s_t^2} = \eta SSR'(s_t) + \eta\frac{SSR''(s_t)}{2}(s_t + q_t - s_{t+1}). \tag{13}$$

The stage–discharge relationship is important in Eq. (13). This study follows the formulations in Lund [13] and approximates this relationship using a power function to determine whether $\frac{\partial^2 g_t}{\partial s_t^2}$ is positive or negative.

$$SSR(s_t) = as_t^b + c \quad (a > 0, 0 < b < 1, c > 0). \tag{14}$$

$SSR()$ is a monotonically increasing and concave function, that is, $SSR'() > 0$ and $SSR''() < 0$. Based on Eq. (14), $SSR'(s_t) = abs_t^{b-1}$ and $SSR''(s_t) = ab(b-1)s_t^{b-2}$. Given that $SSR''(s_t) < 0$ $(0 < b < 1)$ and $s_t + q_t - s_{t+1} < s_t$ (i.e., the reservoir release is less than the storage), $\eta\frac{SSR''(s_t)}{2}(s_t + q_t - s_{t+1}) > \eta\frac{SSR''(s_t)}{2}s_t$. Therefore, $\frac{\partial^2 g_t}{\partial s_t^2} > \eta abs_t^{b-1} + \eta\frac{ab(b-1)}{2}s_t^{b-1} = \eta\frac{ab+ab^2}{2}s_t^{b-1} > 0$.

(3) $\frac{\partial g_t}{\partial s_{t+1}}$ measures the marginal cost of s_{t+1} to g_t:

$$\frac{\partial g_t}{\partial s_{t+1}} = \eta\frac{SSR'(s_{t+1})}{2}(s_t + q_t - s_{t+1}) - \eta\left[\frac{SSR(s_t) + SSR(s_{t+1})}{2} - SDR\right]. \tag{15}$$

In Eq. (15), $\eta\frac{SSR'(s_{t+1})}{2}(s_t + q_t - s_{t+1}) < \eta\frac{SSR'(s_{t+1})}{2}s_{t+1}$ $(s_t + q_t - s_{t+1} < s_{t+1}$, that is, the reservoir release is less than the storage). Supposing that the reservoir water level is equal to the downstream water level when storage is zero, the following is then derived:

$$\frac{\partial g_t}{\partial s_{t+1}} < \eta\frac{abs_{t+1}^{b-1}}{2}s_{t+1} - \eta\left(\frac{as_t^b + as_{t+1}^b}{2}\right) = -\eta\frac{a(1-b)s_{t+1}^b}{2} - \eta\frac{abs_t^b}{2} < 0.$$

(4) $\frac{\partial^2 g_t}{\partial s_{t+1}^2}$ represents the trend of $\frac{\partial g_t}{\partial s_{t+1}}$ as s_{t+1} increases:

$$\frac{\partial^2 g_t}{\partial s_{t+1}^2} = -\eta SSR'(s_{t+1}) + \eta\frac{SSR''(s_{t+1})}{2}(s_t + q_t - s_{t+1}). \tag{16}$$

$SSR'(s_{t+1}) > 0$ and $SSR''(s_{t+1}) < 0$. Therefore, $\frac{\partial^2 g_t}{\partial s_{t+1}^2} < 0$.

In summary, the partial derivatives in Eqs. (12), (13), (15), and (16) exhibit the following characteristics:

$$\begin{cases} \frac{\partial g_t}{\partial s_t} > 0 & (17.1) \\ \frac{\partial^2 g_t}{\partial s_t^2} > 0 & (17.2) \\ \frac{\partial g_t}{\partial s_{t+1}} < 0 & (17.3) \\ \frac{\partial^2 g_t}{\partial s_{t+1}^2} < 0 & (17.4) \end{cases}. \tag{17}$$

As can be observed, with a fixed s_{t+1}, the carry-over storage s_t from the previous period contributes to power generation g_t (Eq. 17.1), and the marginal return increases as s_t increases (Eq. 17.2). Moreover, under a fixed s_t, the carry-over storage s_{t+1} to the next period incurs a cost to g_t (Eq. 17.3). The marginal cost increases as s_{t+1} increases (Eq. 17.4).

In the two-stage reservoir operation (Eq. (2)), one basic issue is determining the carry-over storage s_{t+1}, which links period t to subsequent periods. s_{t+1} incurs an increasing marginal cost to the power generation in period t, as indicated in Eqs. (17.3) and (17.4). However, s_{t+1} contributes to the power generation in the subsequent periods $t+1$ to T, as implied by Eqs. (17.1) and (17.2). The following derivations compare the marginal return with the marginal cost.

3.2. Two-period derivation

Fig. 3 illustrates a two-period hydropower scheduling. The reservoir inflow q_t and q_{t+1} during the two periods are the given inputs. Both the initial storage s_t at the beginning of period t and the ending storage s_{t+2} at the end of period $t+1$ are also fixed. Thus, the focus is on the carry-over storage s_{t+1} from period t to period $t+1$.

The power generation in the two periods is calculated as follows:

$$\begin{cases} g_t(s_t, s_{t+1}) = \eta \times \left[\frac{SSR(s_t) + SSR(s_{t+1})}{2} - SDR\right] \times (s_t + q_t - s_{t+1}), \\ g_{t+1}(s_{t+1}, s_{t+2}) = \eta \times \left[\frac{SSR(s_{t+1}) + SSR(s_{t+2})}{2} - SDR\right] \times (s_{t+1} + q_{t+1} - s_{t+2}). \end{cases} \tag{18}$$

The marginal utilities of s_{t+1} in the two periods are assessed as follows:

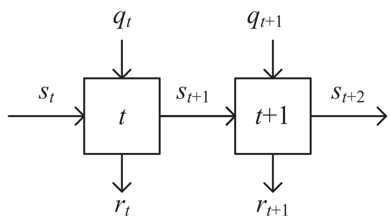

Fig. 3. Schematic of two-period reservoir operation (the carry-over storage s_{t+1} from period t to period $t + 1$ is crucial in maximizing the power generation in the two periods).

$$\begin{cases} \frac{\partial g_t}{\partial s_{t+1}} = \eta \frac{SSR'(s_{t+1})}{2}(s_t + q_t - s_{t+1}) - \eta \left[\frac{SSR(s_t)+SSR(s_{t+1})}{2} - SDR\right], \\ \frac{\partial g_{t+1}}{\partial s_{t+1}} = \eta \frac{SSR'(s_{t+1})}{2}(s_{t+1} + q_{t+1} - s_{t+2}) + \eta \left[\frac{SSR(s_{t+1})+SSR(s_{t+2})}{2} - SDR\right]. \end{cases} \tag{19}$$

According to Eq. (17), the marginal return $\frac{\partial g_{t+1}}{\partial s_{t+1}}$ of s_{t+1} in period $t + 1$ occurs at the marginal cost $\frac{\partial g_t}{\partial s_{t+1}}$ in period t.

A two-period hydropower scheduling aims to maximize the power generation in these two periods. The optimization model is as follows:

$$\max \quad g_t(s_t, s_{t+1}) + g_{t+1}(s_{t+1}, s_{t+2}). \tag{20}$$

In Eq. (20), s_{t+1} is the decision variable. The optimal s^*_{t+1} depends on both the marginal return and the marginal cost. If the marginal return is larger than the marginal cost, that is, $\frac{\partial g_t}{\partial s_{t+1}} + \frac{\partial g_{t+1}}{\partial s_{t+1}} > 0$, then s_{t+1} should be increased as a net utility gain can be generated. Otherwise, reducing s_{t+1} and consuming more water in period t are beneficial.

Based on Eq. (19), the sum of the marginal return and marginal cost is derived as follows:

$$\begin{aligned} \frac{\partial g_t}{\partial s_{t+1}} + \frac{\partial g_{t+1}}{\partial s_{t+1}} &= \eta \frac{SSR'(s_{t+1})}{2}(s_t + q_t + q_{t+1} - s_{t+2}) \\ &\quad - \eta \frac{SSR(s_t) - SSR(s_{t+2})}{2} \\ &= \eta \frac{SSR'(s_{t+1})}{2}(q_t + q_{t+1}) \\ &\quad + \frac{\eta}{2}\left[SSR(s_t) - SSR(s_{t+2}) - SSR'(s_{t+1})(s_t - s_{t+2})\right]. \end{aligned} \tag{21}$$

Eq. (21) shows that $\frac{\partial g_t}{\partial s_{t+1}} + \frac{\partial g_{t+1}}{\partial s_{t+1}}$ is reformulated as the sum of two components. Given that the total release in the two periods is $(q_t + q_{t+1}) + (s_t - s_{t+2})$, the two components respectively measure the effects of s_{t+1} on $q_t + q_{t+1}$ and $s_t - s_{t+2}$:

(1) The first component $\eta \frac{SSR'(s_{t+1})}{2}(q_t + q_{t+1})$ in Eq. (21) indicates the influence of s_{t+1} on the marginal productivity of $q_t + q_{t+1}$. Increasing s_{t+1} creates a higher water head, that is, $SSR'(s_{t+1}) > 0$, and enables q_t and q_{t+1} to generate more power. Thus, this component is positive. A larger value of $q_t + q_{t+1}$ augments the value of this component.

(2) The second component $\frac{\eta}{2}[SSR(s_t) - SSR(s_{t+2}) - SSR'(s_{t+1})(s_t - s_{t+2})]$ represents the effect of s_{t+1} on the marginal productivity of $s_t - s_{t+2}$. Applying the first-order Taylor series expansion to s_{t+1} yields $SSR(s_t) \approx SSR(s_{t+1}) + SSR'(s_{t+1})(s_t - s_{t+1})$ and $SSR(s_{t+2}) \approx SSR(s_{t+1}) + SSR'(s_{t+1})(s_{t+2} - s_{t+1})$. Therefore, $SSR(s_t) - SSR(s_{t+2})$ tends to be equal to $SSR'(s_{t+1})(s_t - s_{t+2})$, and the value of the second component is approximately zero. This outcome suggests that the effect of s_{t+1} on $s_t - s_{t+2}$ is secondary when compared with its effect on $q_t + q_{t+1}$.

Therefore, $\frac{\partial g_t}{\partial s_{t+1}} + \frac{\partial g_{t+1}}{\partial s_{t+1}}$ is generally positive. That is, increasing carry-over storage generates a net gain in the power generation in the two-period hydropower scheduling. A two-period water supply has a balanced level, at which the marginal return is equal to the marginal cost (Fig. 1a), of the carry-over storage. However, such a balanced level may not occur in hydropower scheduling. A larger s_{t+1} is preferable.

3.3. Implications for multi-period hydropower scheduling

Analyzing both one- and two-period hydropower scheduling provides implications for the multi-period model (Fig. 4). Based on Eq. (2), a recursive two-stage formulation of the multi-period model is derived as follows:

$$G_t(s_t) = \max \quad g_t(s_t, s_{t+1}) + G_{t+1}(s_{t+1}). \tag{22}$$

The above equation is similar to Eq. (20), except that $g_{t+1}(s_{t+1}, s_{t+2})$ (i.e., the power generation in period $t + 1$) is replaced by $G_{t+1}(s_{t+1})$ (i.e., the maximum cumulative power generation from period T to period $t + 1$). Unlike the two-period model where the marginal return of the carry-over storage s_{t+1} can be derived from $g_{t+1}(s_{t+1}, s_{t+2})$, the marginal return of s_{t+1} cannot be derived analytically because $G_{t+1}(s_{t+1})$ cannot be expressed as a closed form.

The two-period model (Eq. (20)) is linked to the multi-period model (Eq. (22)). Note that $G_{t+1}(s_{t+1})$ is derived from the recursive computation at period $t + 1$:

$$G_{t+1}(s_{t+1}) = \max \quad g_{t+1}(s_{t+1}, s_{t+2}) + G_{t+2}(s_{t+2}). \tag{23}$$

$G_{t+1}(s_{t+1})$ obviously accounts for the power generations in periods $t + 1, t + 2, \ldots, T$ (Fig. 4). In the two-period model, s_{t+2} is fixed, and the carry-over storage s_{t+1} contributes to the power generation in period $t + 1$ only (Fig. 3). Under the multi-period model, $s_{t+2}, s_{t+3}, \ldots, s_{T-1}$ are decision variables, whose values can increase with s_{t+1}. Therefore, an increase in s_{t+1} contributes to the power generation in period $t + 1$; increases of $s_{t+2}, s_{t+3}, \ldots, s_{T-1}$ with s_{t+1} further augments the power generations in the subsequent periods. Given that the marginal return is higher than the marginal cost in the two-period case, the marginal return of the carry-over storage in a multi-period hydropower scheduling is expectedly even larger than the marginal cost.

4. Hydropower scheduling of the Three Gorges Reservoir

The above theoretical analyses are conducted for unconstrained hydropower scheduling. The operational constraints of storage capacity, installed hydropower capacity, and environmental flow have important roles in real-world reservoir operation. In this section, the Three Gorges Reservoir is studied to investigate the marginal utility principle in constrained hydropower scheduling.

4.1. Setting of the case study

The Three Gorges Reservoir is one of the largest water resource projects in the world. This reservoir controls the streamflow of the upper Yangtze River and has a drainage area of 1 million km^2 and a multi-annual mean flow of 4.51×10^{11} m^3. The reservoir is mainly operated for flood control to protect the downstream area from flood risks in the rainy season from early May to late September. Liu et al. [11] illustrated that during this period, the reservoir storage is generally maintained at the flood-control storage level of 171.5×10^8 m^3 in preparation for upstream floods. In September and October, the reservoir is refilled to the normal storage level of 393×10^8 m^3. In the dry season from November to April, the reservoir produces hydropower and releases environmental flow to sustain the downstream riverine ecosystem. Subsequently, the

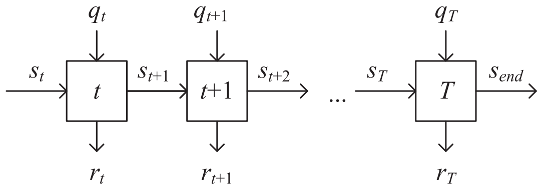

Fig. 4. Schematic of multi-period reservoir operation.

reservoir storage gradually decreases from 393×10^8 m^3 to 171.5 $\times$ 10^8 m^3, and the corresponding operating rules were explored by Liu et al. [10].

This study focuses on hydropower scheduling during the dry season and divides the aforementioned six months into $T = 18$ ten-day periods (i.e., period 1 covers November 1–10, period 2 is from November 11–20, and so on). The total power generation is maximized by considering the operational constraints:

$$\max \quad \sum_{t=1}^{T} g_t,$$
$$s.t. \quad \begin{cases} g_t = \eta \times \left[\frac{SSR(s_t)+SSR(s_{t+1})}{2} - SDR(r_t)\right] \times r_t \times \Delta, \\ r_t \times \Delta = s_t + q_t \times \Delta - s_{t+1}, \\ \underline{s} \leqslant s_{t+1} \leqslant \bar{s}, \\ g_t \leqslant \bar{g}, \\ r_t \geqslant \underline{r}, \\ s_1 = s_{ini}, \\ s_{T+1} = s_{end}. \end{cases} \tag{24}$$

The optimization model above is illustrated in Fig. 5. The decision variables are selected as the carry-over storage s_{t+1} ($t = 1, 2, \ldots, T - 1$) between two contiguous periods. Based on the carry-over storage, reservoir release and power generation are determined by the water balance relationship and the utility function, respectively.

In Eq. (24), the objective function is the sum of single-period power generations, which are calculated as the product of the efficiency coefficient ($\eta = 9.0$), water head, reservoir release, and length of one period ($\Delta = 240$ h). The stage–storage relationship is approximated by a power function $SSR(s_t) = 177.34 \times s_t^{0.11} - 167.32$ $(171.5 \leqslant s_t \leqslant 393)$ (i.e., the units of s_t and SSR are 10^8 m^3 and m, respectively). The downstream water level is fixed at 65 m.

The operational constraints of reservoir storage capacity, installed hydropower capacity, and environmental flow are considered: (1) The lower and upper bounds of storage are $\underline{s} = 171.5 \times 10^8$ m^3 and $\bar{s} = 393.0 \times 10^8$ m^3, respectively. (2) The lower bound of release, that is, the environmental flow, is set as $\underline{r} = 5000$ m^3/s. (3) The upper bound of power generation in one period is $\bar{g} = 18.20$ GW $\times$ 240 h $=$ 4368 GW h (i.e., the installed hydropower capacity of the Three Gorges Reservoir is 18.20 GW). In addition, the initial and ending storages are $s_{ini} = 393.0 \times 10^8$ m^3 and $s_{end} = 171.5 \times 10^8$ m^3, respectively, which are the boundary conditions for hydropower scheduling.

The optimization model is solved as follows by dynamic programming:

$$G_t(s_t) = \max \quad g_t(s_t, s_{t+1}) + G_{t+1}(s_{t+1})$$
$$s.t. \quad \begin{cases} \bar{s} \leqslant s_{t+1} \leqslant \bar{s} \\ g_t \leqslant \bar{g} \\ s_t + q_t\Delta - s_{t+1} \geqslant \bar{r}\Delta \end{cases} \tag{25}$$

As shown in Eq. (25), dynamic programming determines not only the optimal $s^*_{t+1}(t = 1, 2, \ldots, T - 1)$ but also the utility function $G_t(s_t)(t = 1, 2, \ldots, T - 1)$ for s_t. For example, in the recursion at period $t + 1$, dynamic programming derives the maximum cumulative power generation $G_{t+1}(s_{t+1})$ from periods T to $t + 1$ (Eq. (23)). For Eq. (25), the marginal cost and marginal return of s_{t+1} can be evaluated using $g_t(s_t, s_{t+1})$ and $G_{t+1}(s_{t+1})$, respectively. This characteristic makes dynamic programming particularly suitable for analyzing the marginal utility principle. Other optimization algorithms, such as nonlinear programming and heuristic algorithms, can also solve Eq. (24) but do not provide the utility function.

For Eq. (25), the state of the decision variable s_{t+1} is within $[171.5, 393.0] \times 10^8$ m^3. This range is discretized into 4,431 values with an interval width of 0.05×10^8 m^3. The storage capacity constraint is incorporated into the computation of dynamic programming through this method. The constraint of the installed hydropower capacity is considered by setting $g_t(s_t, s_{t+1}) = \begin{cases} g_t(s_t, s_{t+1}) & (g_t(s_t, s_{t+1}) \leqslant \bar{g}) \\ \bar{g} & (g_t(s_t, s_{t+1}) > \bar{g}) \end{cases}$. Specifically, reducing s_{t+1}, which increases reservoir release, contributes to power generation until the installed capacity is reached. The environmental flow constraint is accounted for by disregarding s_{t+1} values, which generate $s_t + q_t\Delta - s_{t+1} < \underline{r}\Delta$.

4.2. Results of two-period hydropower scheduling

The optimal decision $s^*_{t+1}(t = 1, 2, \ldots, T - 1)$ under the multi-annual mean flow condition is determined. Decisions involving both reservoir release and power generation are also calculated. The results exhibit certain important patterns, as shown in Fig. 6. The carry-over storage is maintained as high as possible in hydropower scheduling. The optimal storage is at the upper bound $\bar{s}$ in periods 1–6 when the inflow is higher than the environmental flow $\underline{r}$. The inflow is completely released because the reservoir cannot store more water. The reservoir storage is gradually depleted in periods 7–15 because the inflow only cannot satisfy the environmental flow. Meanwhile, the reservoir release is at the lower bound $\underline{r}$ to maintain a high storage level during these periods. The storage is still at a high level in periods 16–17 and is then depleted in period 18 to meet the ending storage constraint. Notably, the optimal decisions are computed under different inflow conditions, and similar results are obtained.

The marginal utility principle is investigated based on the optimal decisions. Given that $s^*_{t+1}(t = 1, 2, \ldots, T - 1)$ is optimal in multi-period hydropower scheduling, this equation is also optimal in the two-period case according to the principle of optimality. By fixing $s_t = s^*_t$ and $s_{t+2} = s^*_{t+2}$, the marginal cost $-\frac{\partial g_t}{\partial s_{t+1}}$ and marginal return $\frac{\partial g_{t+1}}{\partial s_{t+1}}$ of s_{t+1} are evaluated as follows, respectively:

(1) By setting $t = 1$, the utility of s_2 in periods 1 and 2 is evaluated, as shown in Fig. 7. In unconstrained optimization, the power generation $g_1(s^*_1, s_2)$ in period 1 decreases while $g_2(s_2, s^*_3)$ increases, as s_2 increases. This observation indicates that the carry-over storage s_2 contributes to the power generation in period 2 at the cost of the power generation in period 1. Moreover, the marginal cost of s_2 and the marginal return increase in periods 1 and 2, respectively. The plot illustrates that $\frac{\partial g_2}{\partial s_2} > -\frac{\partial g_1}{\partial s_2}$, which means that saving water in period 1 always generates a net gain in the total power generation. In the meantime the carry-over storage is constrained by the storage capacity, which makes $\bar{s}$ (the upper bound of storage) the optimal carry-over storage s^*_2. Notably, $\bar{s}$ is similarly optimal when $t = 2, 3, \ldots, 6$.

(2) The utility of s_{11} is analyzed by setting $t = 10$ (Fig. 8). In periods 10 and 11, the inflow volumes are less than the required environmental flow volume. Therefore, the reservoir storage

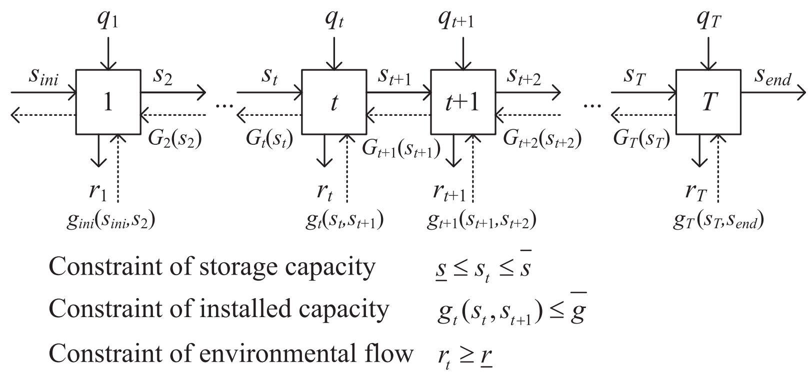

Fig. 5. Illustration of constrained multi-period hydropower scheduling (the solid lines represent the water balance relationship, while the dashed lines represent the recursive computation of power generation in dynamic programming).

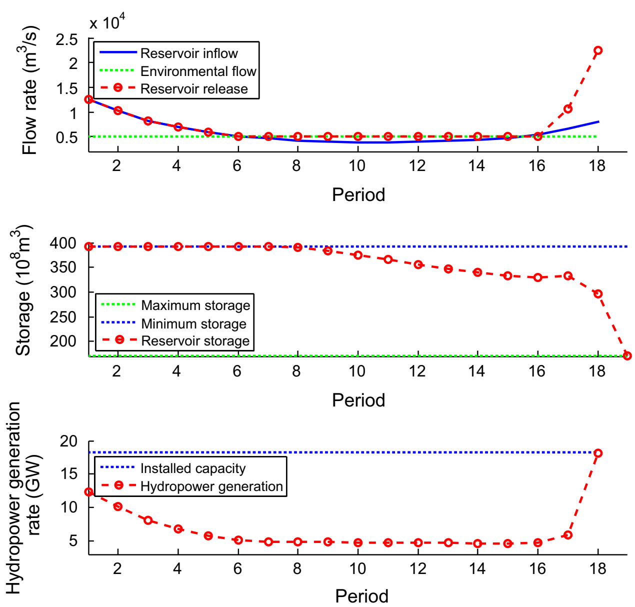

Fig. 6. Optimal hydropower scheduling decisions during the dry season (November to April) under the multi-annual mean flow condition.

in these two periods is reduced (Fig. 6). A high carry-over storage s_{11} increases the release in period 11 but cannot meet the environmental flow in period 10, while a low carry-over storage s_{11} increases the release in period 10 but cannot meet the environmental flow in period 11. The optimal carry-over storage s_{11}^* is thus determined by the environmental flow. At s_{11}^*, the marginal return is notably greater than the marginal cost. As shown in the subplots of unconstrained optimization in Fig. 8, a net utility gain occurs if more storage is carried over from period 10 to period 11. However, this operation is constrained by the environmental flow, which similarly affects hydropower scheduling when $t = 7, 8, \ldots, 15$.

(3) By setting $t = 16$, the utility of s_{17} in periods 16 and 17 is explored, as shown in Fig. 9. The inflows q_{16} and q_{17} are greater than the environmental flow $\underline{r}$ in the two periods. However, the optimal release in period 16 is still $\underline{r}$. This outcome occurs because the marginal return of s_{17} in period 17 is greater than the marginal cost of s_{17} in period 16, that is, $\frac{\partial g_{17}}{\partial s_{17}} > -\frac{\partial g_{16}}{\partial s_{17}}$. Accordingly, the carry-over storage is set as high as possible, which creates a high water head for power generation in these two periods. The binding constraint of the environmental flow in period 16 helps determine the optimal carry-over storage.

(4) By setting $t = 17$, the utility of the carry-over storage s_{17} is evaluated for the last two periods. With a slight difference from the results in Figs. 7–9, Fig. 10 illustrates that the marginal return of s_{17} becomes zero beyond a specific threshold. This result occurs because of the constraint of the installed capacity, which represents the upper bound of the power generation in one period. The marginal return below the threshold is higher than the marginal cost. Above the threshold, the marginal return from the carry-over storage is smaller than the marginal cost. As shown in Fig. 10, the optimal carry-over storage is exactly at the threshold level. It is important to note that in unconstrained optimization, which disregards the constraint of installed capacity, the marginal return is still higher than the marginal cost when the carry-over storage exceeds the threshold.

In summary, the results of the four cases confirm the theoretical findings and illustrate that the marginal return of the carry-over storage is higher than its marginal cost in two-period hydropower scheduling. Therefore, increasing the carry-over storage, which yields a net gain in power generation, is favorable. Moreover, the reservoir operational constraints are crucial in determining the optimal carry-over storage: (1) The reservoir storage capacity sets the upper bound of the carry-over storage (Fig. 7) and constrains the increase in s_{t+1} ($t = 1, 2, \ldots, 6$) when ample inflow exists. (2) The environmental flow sets the lower bound of release and limits the increase in s_{t+1} ($t = 7, 8, \ldots, 15$) when inflow is limited (Figs. 8 and 9). (3) The installed hydropower capacity sets a threshold level for the beneficial use of carry-over storage (Fig. 10).

4.3. Results of multi-period hydropower scheduling

The marginal return from the carry-over storage is further evaluated in multi-period hydropower scheduling. In the two-period case, s_{t+1} contributes to the power generation of the next period $t + 1$, where the marginal return is measured using $\frac{\partial g_{t+1}(s_{t+1}, s_{t+2}^*)}{\partial s_{t+1}}$ (Figs. 7–10). In the multi-period case, s_{t+1} potentially contributes to the power generation in the subsequent periods $t + 1, t + 2, \ldots, T$. $\frac{dG_{t+1}(s_{t+1})}{ds_{t+1}}$ indicates a marginal return from a one unit increase in s_{t+1}. The marginal returns of the carry-over storage in both two- and multi-period hydropower scheduling are evaluated while considering operational constraints, as shown in Fig. 11.

Obviously, $\frac{dG_{t+1}(s_{t+1})}{ds_{t+1}}$ is equal to $\frac{\partial g_{t+1}(s_{t+1}, s_{t+2}^*)}{\partial s_{t+1}}$ when $t = 1$, 16, and 17. Generally, these outcomes are attributed to the operational constraints of storage capacity, installed hydropower capacity, and

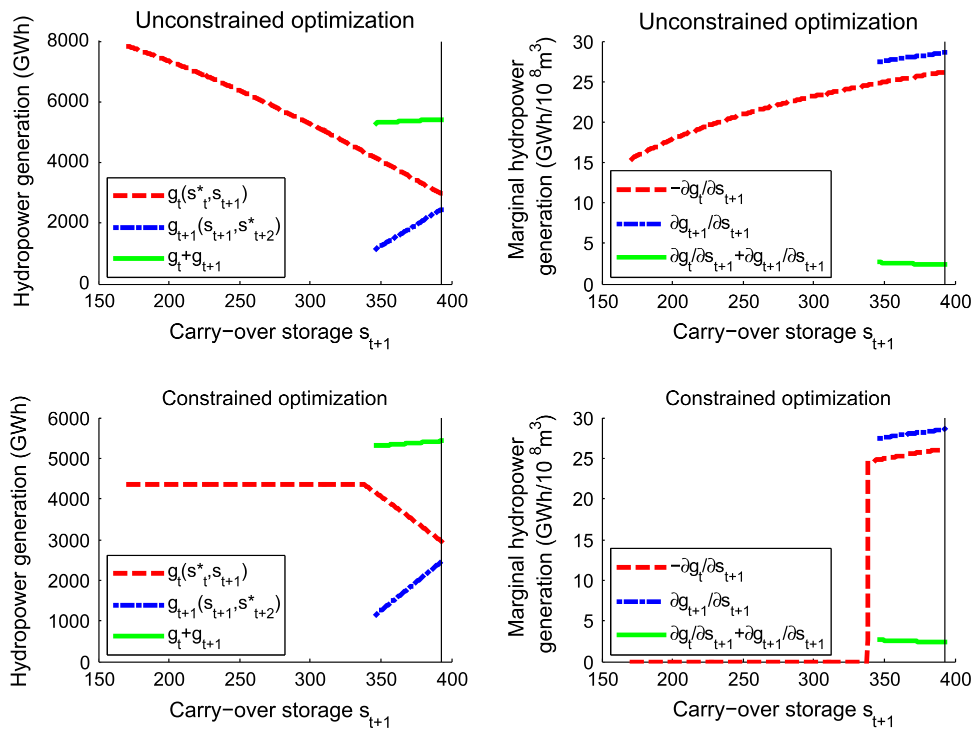

Fig. 7. Hydropower generation in periods 1 and 2 ($t = 1$) under unconstrained and constrained optimizations (the binding constraint of the storage capacity helps determine the optimal carried-over storage).

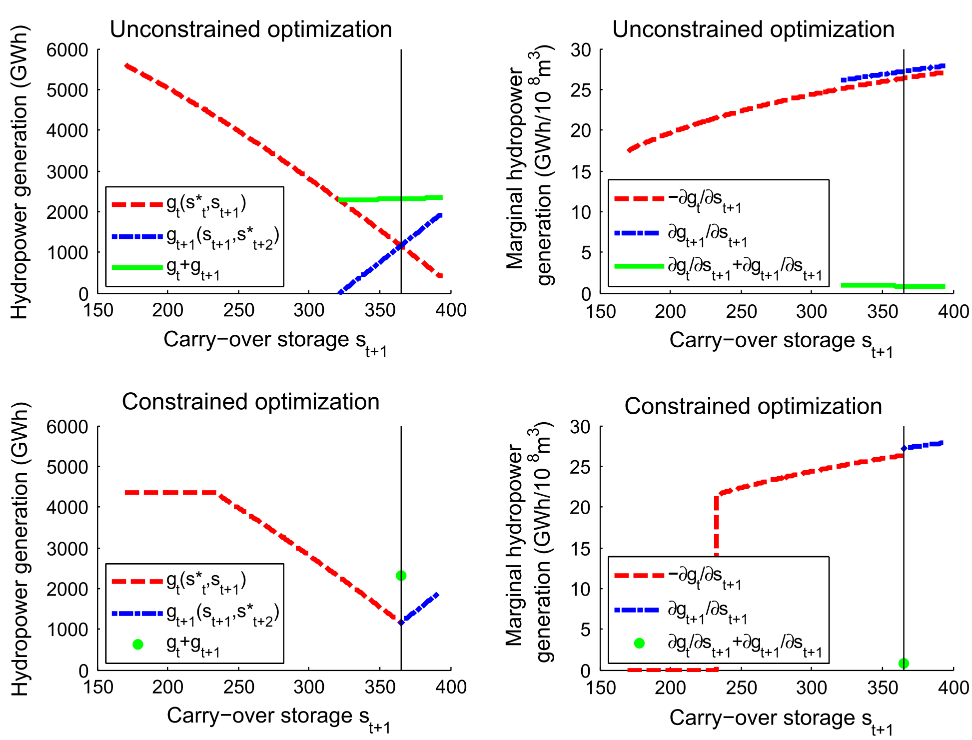

Fig. 8. Similar to Fig. 7, except for Periods 10 and 11 ($t = 10$) (the binding constraint of the environmental flow helps set the optimal carried-over storage).

ending storage. The constraints maintain s^*_{t+2} at a fixed level and makes s_{t+1} contribute to the power generation of period $t+1$ only. Therefore, $\frac{dG_{t+1}(s_{t+1})}{ds_{t+1}}$ and $\frac{\partial g_{t+1}(s_{t+1},s^*_{t+2})}{\partial s_{t+1}}$ are equal. On the other hand, $\frac{dG_{t+1}(s_{t+1})}{ds_{t+1}}$ is higher than $\frac{\partial g_{t+1}(s_{t+1},s^*_{t+2})}{\partial s_{t+1}}$ when $t = 10$ because s^*_{t+2} is not yet bound (release is bound by the environmental flow constraint instead). An increase in s_{t+1} can be carried over to the periods succeeding period $t+1$ and contributes to hydropower generation, thereby augmenting the marginal return from s_{t+1}.

In conclusion, the marginal return of the carry-over storage in multi-period hydropower scheduling is at least equal to that in the two-period case based on the comparisons above.

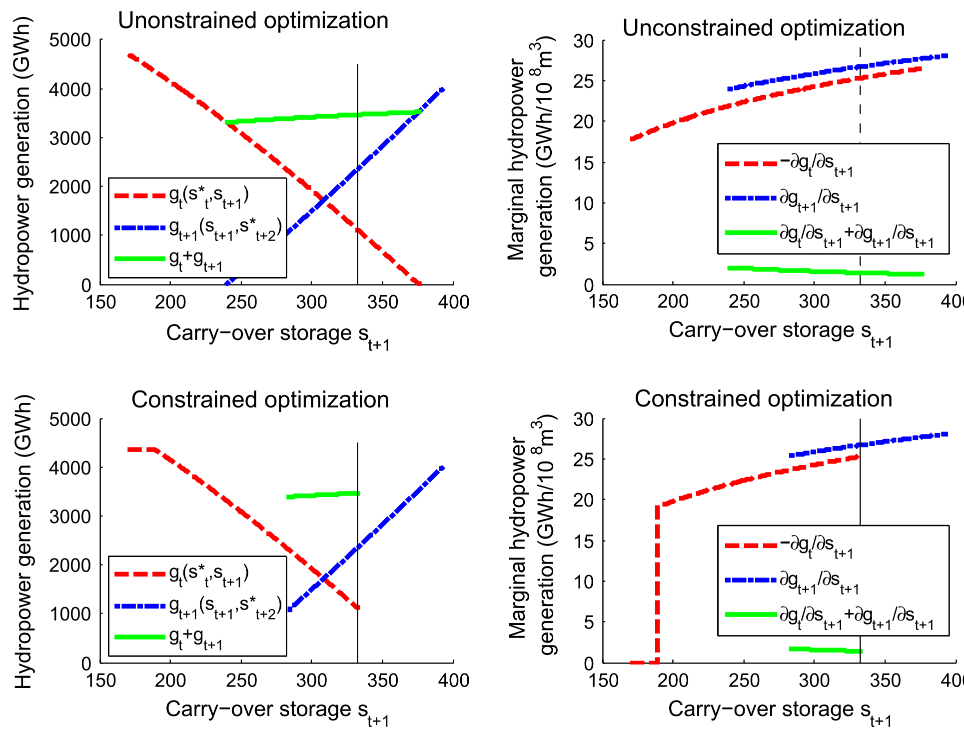

Fig. 9. Similar to Fig. 7, except for periods 16 and 17 ($t = 16$) (the binding constraint of the environmental flow in period 16 helps determine the optimal carried-over storage).

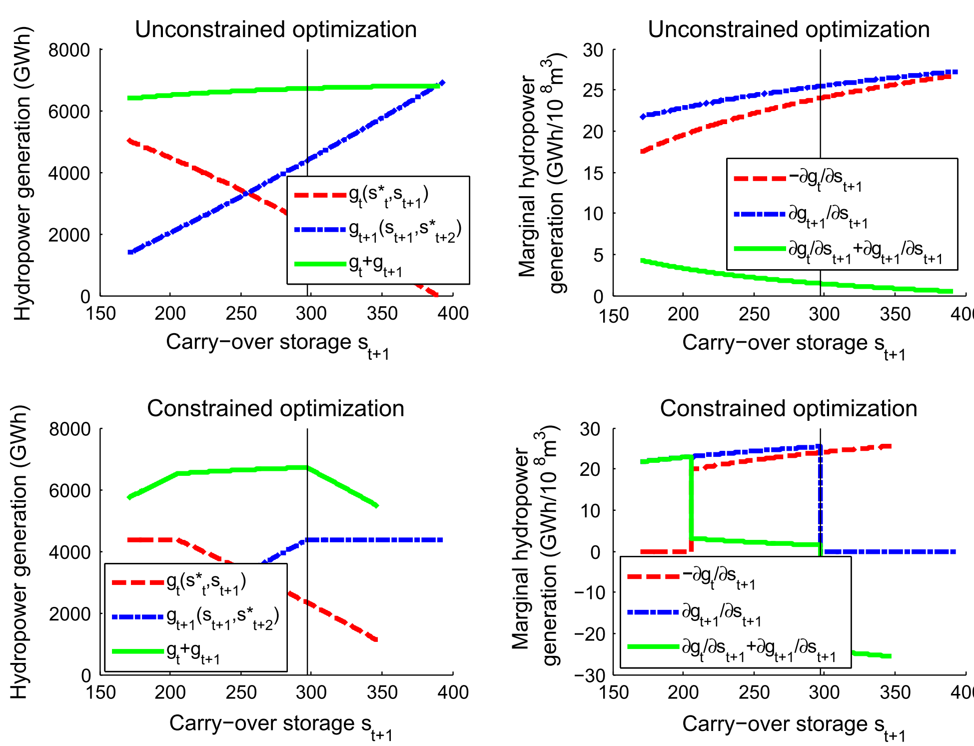

Fig. 10. Similar to Fig. 7, except for periods 17 and 18 ($t = 17$) (the binding constraint of the installed capacity in period 18 helps determine the optimal carried-over storage).

In two-period hydropower scheduling, the marginal return is higher than the marginal cost; in the multi-period case, the marginal return is even higher. Therefore, saving as much storage as possible for future periods is favorable. The optimal carry-over storage is influenced by operational constraints. The constraints of storage capacity and environmental flow limit the carry-over storage in subsequent periods. The constraint of installed hydropower capacity results in a threshold level for the carry-over storage, beyond which the marginal return becomes zero. In the multi-period case, the optimal carry-over storage is determined by the marginal utility principle and operational constraints.

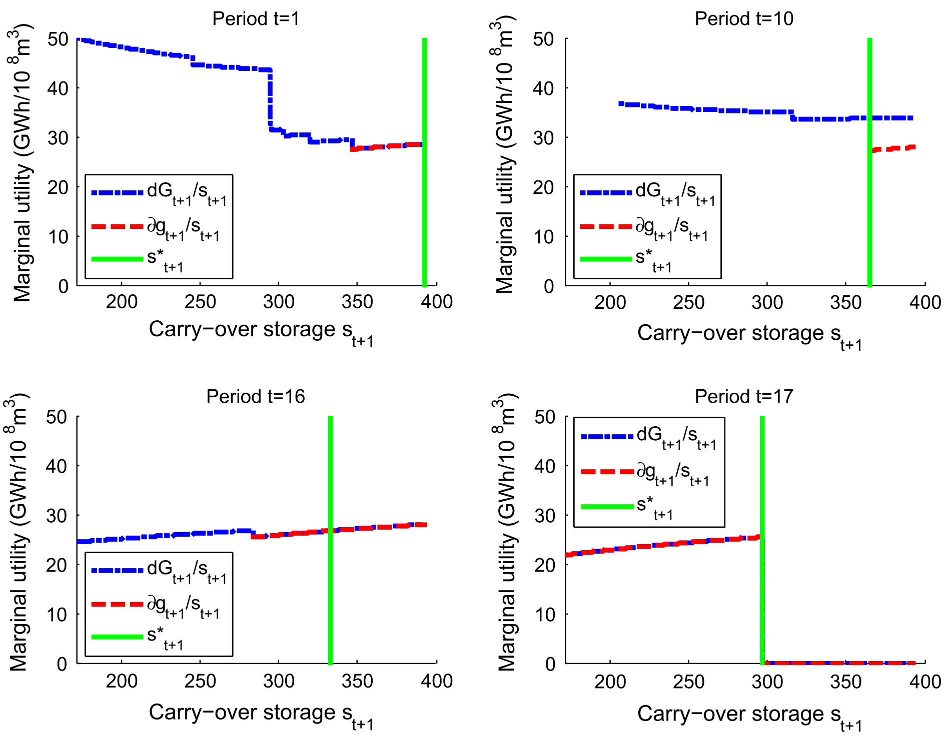

Fig. 11. Comparison of the marginal returns of the carry-over storage in the two- and multi-period hydropower scheduling when t = 1, 10, 16, and 17.

5. Discussion and conclusion

This study comprehensively analyzes the marginal utility principle for long-term hydropower scheduling. The main focus is on the carry-over storage between two consecutive periods. The marginal cost and marginal return are investigated in one-, two-, and multi-period cases. In the one-period case, the carry-over storage from the previous period has an increasing marginal contribution to power generation while the carry-over storage to the next period incurs an increasing marginal cost of power generation. Comparing the marginal cost and the marginal return in the two-period case, the latter is higher than the former because an increase in carry-over storage augments the marginal productivity of inflow. In the multi-period case, the marginal return is even higher because an increase in the carry-over storage contributes to the power generation in the next period and also in the subsequent periods.

Aside from subjecting it to theoretical analysis, the marginal utility principle is also evaluated in an actual case study, considering the operational constraints of storage capacity, installed hydropower capacity, and environmental flow. The theoretical findings of increasing marginal cost and marginal return of carry-over storage are confirmed. In both two- and multi-period cases, the marginal return is higher than the marginal cost. In particular, increasing the carry-over storage leads to a net gain in power generation. The optimal carry-over storage is subject to operational constraints. Storage capacity sets the upper bound of the carry-over storage, and environmental flow limits the carry-over storage to the next period. The installed hydropower capacity determines the threshold; if the carry-over storage exceeds this threshold, then the marginal return becomes zero. The marginal utility principle and the operational constraints determine the optimal hydropower scheduling decisions jointly.

This study explores the marginal utility principle in hydropower scheduling through theoretical analysis and an actual case study. In water supply, the characteristics of diminishing marginal return and increasing marginal cost result in the marginal utility principle. By contrast, the marginal return from the carry-over storage in hydropower scheduling does not diminish but increases instead. This difference is attributed to the complementarity between reservoir storage and release; that is, increasing the carry-over storage makes release more productive in power generation in the subsequent periods. Therefore, the marginal utility principle for hydropower scheduling is attributed to the characteristics of marginal return and marginal cost. In water supply, the marginal utility principle is applied to derive operating rules for decision making. Future efforts can be devoted to exploring the operating rules for hydropower scheduling based on the marginal utility principle and considering the time-varying electricity price in the analyses of marginal cost and marginal return.

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References

[1] Chen W, Chau KW. Intelligent manipulation and calibration of parameters for hydrological models. Int J Environ Pollut 2006;28:432–47.
[2] Cheng CT, Shen JJ, Wu XY. Short-term scheduling for large-scale cascaded hydropower systems with multivibration zones of high head. J Water Resour Plann Manage – ASCE 2012;138(3):257–67.
[3] Cheng CT, Shen JJ, Wu XY, Chau KW. Operation challenges for fast-growing China's hydropower systems and respondence to energy saving and emission reduction. Renew Sustain Energy Rev 2012;16:2386–93.

[4] Draper AJ, Lund JR. Optimal hedging and carryover storage value. J Water Resour Plann Manage – ASCE 2004;130(1):83–7.
[5] Eum HI, Kim YO, Palmer RN. Optimal drought management using sampling stochastic dynamic programming with a hedging rule. J Water Resour Plann Manage – ASCE 2011;137(1):113–22.
[6] Georgakakos AP, Yao H, Kistenmacher M, Georgakakos KP, Graham NE, Cheng FY, Spencer C, Shamir E. Value of adaptive water resources management in Northern California under climatic variability and change: reservoir management. J Hydrol 2012;412:34–46.
[7] Giuliani M, Herman JD, Castelletti A, Reed P. Many-objective reservoir policy identification and refinement to reduce policy inertia and myopia in water management. Water Resour Res 2014;50(4):3355–77.
[8] Labadie JW. Optimal operation of multireservoir systems: state-of-the-art review. J Water Resour Plann Manage 2004;130(2):93–111.
[9] Li X, Wei JH, Li TJ, Wang GQ, Yeh WWG. A parallel dynamic programming algorithm for multi-reservoir system optimization. Adv Water Resour 2014;67:1–15.
[10] Liu P, Cai XM, Guo SL. Deriving multiple near-optimal solutions to deterministic reservoir operation problems. Water Resour Res 2011:47.
[11] Liu P, Li LP, Guo SL, Xiong LH, Zhang W, Zhang JW, et al. Optimal design of seasonal flood limited water levels and its application for the Three Gorges Reservoir. J Hydrol 2015;2015(527):1045–53.
[12] Lu P, Zhou JZ, Wang C, Qiao Q, Mo L. Short-term hydro generation scheduling of Xiluodu and Xiangjiaba cascade hydropower stations using improved binary-real coded bee colony optimization algorithm. Energy Convers Manage 2015;91:19–31.
[13] Lund JR. Drought storage allocation rules for surface reservoir systems. J Water Resour Plann Manage – ASCE 2006;132(5):395–7.
[14] Ma C, Lian JJ, Wang JN. Short-term optimal operation of Three-gorge and Gezhouba cascade hydropower stations in non-flood season with operation rules from data mining. Energy Convers Manage 2013;65: 616–27.
[15] Null SE, Lund JR. Fish habitat optimization to prioritize river restoration decisions. River Res Appl 2012;28(9):1378–93.
[16] Olivares MA, Lund JR. Representing energy price variability in long- and medium-term hydropower optimization. J Water Resour Plann Manage – ASCE 2012;138(6):606–13.
[17] Sankarasubramanian A, Lall U, Souza FA, Sharma A. Improved water allocation utilizing probabilistic climate forecasts: short-term water contracts in a risk management framework. Water Resour Res 2009:45.
[18] Tilmant A, Arjoon D, Marques GF. Economic value of storage in multireservoir systems. J Water Resour Plann Manage 2014;140(3):375–83.
[19] Tu MY, Hsu NS, Tsai FTC, Yeh WWG. Optimization of hedging rules for reservoir operations. J Water Resour Plann Manage – ASCE 2008;134(1):3–13.
[20] Turgeon A. Stochastic optimization of multireservoir operation: The optimal reservoir trajectory approach. Water Resour Res 2007;43:W05420. http://dx.doi.org/10.1029/2005WR004619.
[21] Wang C, Zhou JZ, Lu P, Yuan L. Long-term scheduling of large cascade hydropower stations in Jinsha River, China. Energy Convers Manage 2015;90:476–87.
[22] Wu CL, Chau KW, Li YS. Methods to improve neural network performance in daily flows prediction. J Hydrol 2009;372:80–93.
[23] Xu W, Zhang C, Peng Y, Fu GT, Zhou HC. A two stage Bayesian stochastic optimization model for cascaded hydropower systems considering varying uncertainty of flow forecasts. Water Resour Res 2014;50(12):9267–86.
[24] You JY, Cai XM. Hedging rule for reservoir operations: 1. A theoretical analysis. Water Resour Res 2008;44(1).
[25] Zhao JS, Cai XM, Wang ZJ. Optimality conditions for a two-stage reservoir operation problem. Water Resour Res 2011:47.
[26] Zhao TTG, Cai XM, Lei XH, Wang H. Improved dynamic programming for reservoir operation optimization with a concave objective function. J Water Resour Plann Manage – ASCE 2012;138(6):590–6.
[27] Zhao TTG, Wang QJ, Bennett JC, Robertson DE, Shao QX, Zhao JS. Quantifying predictive uncertainty of streamflow forecasts based on a Bayesian joint probability model. J Hydrol 2015;528:329–40.
[28] Zhao TTG, Zhao JS. Joint and respective effects of long- and short-term forecast uncertainties on reservoir operations. J Hydrol 2014;517:83–94.
[29] Zhao TTG, Zhao JS. Improved multiple-objective dynamic programming model for reservoir operation optimization. J Hydroinform 2014;16(5):1142–57.
[30] Zhao TTG, Zhao JS, Lund JR, Yang DW. Optimal hedging rules for reservoir flood operation from forecast uncertainties. J Water Resour Plann Manage 2014;140(12).
[31] Zhao TTG, Zhao JS, Yang DW. Improved dynamic programming for hydropower reservoir operation. J Water Resour Plann Manage 2014;140(3):365–74.