Journal of King Saud University – Computer and Information Sciences (2016) xxx, xxx–xxx

King Saud University
Journal of King Saud University – Computer and Information Sciences
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Stock portfolio selection using Dempster–Shafer evidence theory

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Received 11 March 2016; revised 31 May 2016; accepted 5 July 2016

KEYWORDS

Stock portfolio selection;
Ranking;
Dempster–Shafer evidence theory;
Ant Colony Optimization;
Fuzzy Delphi method

Abstract Markowitz's return–risk model for stock portfolio selection is based on the historical return data of assets. In addition to the effect of historical return, there are many other critical factors which directly or indirectly influence the stock market. We use the fuzzy Delphi method to identify the critical factors initially. Factors having lower correlation coefficients are finally considered for further consideration. The critical factors and historical data are used to apply Dempster–Shafer evidence theory to rank the stocks. Then, a portfolio selection model that prefers stocks with higher rank is proposed. Illustration is done using stocks under Bombay Stock Exchange (BSE). Simulation is done by Ant Colony Optimization. The performance of the outcome is found satisfactory when compared with recent performance of the assets.

1. Introduction

Many factors directly or indirectly influence stock markets and make movements of asset prices very uncertain and unpredictable. Selection of portfolio may include two stages (Markowitz, 1952). Firstly, performance of different securities is observed with beliefs about their future performances. Secondly, with relevant beliefs about future performances a proper choice of portfolio is made. In modern portfolio theory (MPT) of investment, the main focus is given toward the maximization of expected return of portfolio for a given amount of portfolio risk, or equivalently minimizing the portfolio risk for a given level of expected return, by carefully choosing the investment proportions of various securities. In Markowitz (1952), Markowitz has quantified return as the mean and risk as the variance of the portfolio of the securities. The twin objectives of investors – profit maximization and risk minimization – are thus quantified. Though this theory has been widely accepted and adopted by various researchers, it is criticized since last few years. As in MPT the efficiency of market is considered to be the basic assumption, obtaining information about markets every time is costly and time consuming

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Peer review under responsibility of King Saud University.

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http://dx.doi.org/10.1016/j.jksuci.2016.07.001
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(Grossman and Stiglitz, 1980). Another problem in MPT is its computational burden caused by the quadratic utility functions and covariance matrix when the number of stocks increases (Yunusoglu and Selim, 2013). It also does not give importance to real investors' preferences (Xidonas et al., 2009). It is also found that investors prefer portfolios that lie behind the efficient frontier of Markowitz's model even though they are dominated by other portfolios with respect to expected return and risk. So some additional criteria must be added to the classical risk-return framework.

Thus, portfolio selection is proved to be a multi-dimensional problem and to resolve this inherent multi-criteria nature of this problem multi-criteria decision making (MCDM) approach has been adopted by many (Xidonas et al., 2011; Edwards et al., 2007; Abdollahzadeh, 2002; Siskos et al., 1993). Though all of these researches tried to bring efficiency in portfolio construction models, it is very hard to develop an effective portfolio especially in uncertain dynamic environment. As a result a much growing interest in applying artificial intelligence and soft computing techniques in stock selection and portfolio construction has been noticed in the last few years. Some researchers have used the efficient learning capability in artificial neural networks (ANN) for the selection of stocks and construction of portfolios (Adebiyi et al., 2012; Fernández and Gómez, 2007; Ko and Lin, 2008; Olatunji et al., 2011) whereas other researchers have used genetic algorithm (GA) for the portfolio optimization (Chen and Lin, 2009; Chen et al., 2009; Jiao et al., 2007; Chen et al., 2010). The application of fuzzy logic and fuzzy set theory has also become popular in recent years due to its uncertainty handling capability and the efficiency in bringing the vagueness in investors' preferences in portfolio construction (Bermudez et al., 2007; Bilbao-Terol et al., 2006; Fasanghari and Montazer, 2010; Tiryaki and Ahlatcioglu, 2005; Huang, 2008; Bhattacharyya et al., 2011; Bhattacharyya et al., 2014; Bhattacharyya et al., 2009; Bhattacharyya and Kar, 2011).

Portfolio selection process involves two stages. In the first stage, some suitable stocks are selected and then in the second stage percentage of total investment for each stock is identified. The Dempster–Shafer (DS) evidence theory is popular for its capability of dealing with uncertain and incomplete information but its use remained unnoticed in stock selection and portfolio recommendation.

In this research the Dempster–Shafer (DS) evidence theory is applied for the first time for the selection of stocks.

- This has considerably reduced the required number of expert interactions and the overall complexity of the model which was the major problem in most of the recent researches.
- At the same time another level of uncertainty handling mechanism is incorporated in the portfolio selection model.

The proposed work has two phases:

Phase I.
Four well known metrics, price to earning ratio (P/B), price to sales ratio (P/S), long-term debt to equity ratio (LTDER) and earn per share (EPS) are decided. Like other fundamental metrics, values of these factors give an indication about the future performance of stocks. The 10 years' (2003–04 to 2012–13) historical data on stocks from BSE of these factors function as a collection of evidences for the support or denial of the assumption that the respective stock is going to give good performance in future. Thus, these four factors individually act as evidences. Based upon these evidences, a degree of belief (or mass value) is assigned to the hypothesis *'Stock will perform good'* or *'Stock will perform poor'* for every stock registered under BSE. These mass values of individual evidences are then combined using Dempster's rule of combination to give a final belief about the performance of individual stocks. Well known semivariance to return ratio (S/R) of individual stocks is used to measure their performance.

Phase II.
Top 10 securities are identified based on their final mass values and then a portfolio is suggested by considering those top 10 securities. Ant Colony Optimization (ACO) is used for the construction of the portfolio. The return of the portfolio is found to be satisfactory when compared with the performance of different stocks in the year 2013–14 and 2014–15.

The brief structure of this article is depicted in Fig. 1.

Rest of the discussion is organized as follows. In Section 2, identification of critical factors and selection of evidences for the proposed DS theory based stock selection model are explained. DS evidence theory and its application in ranking of stocks is elaborated in Section 3. Section 4, suggests a portfolio selection model. In Section 5, results of the proposed model is compared and finally some concluding remarks are specified in Section 6.

2. Identification of critical factors and selection of evidences

Value investors believe that there is no *right way* to analyze stocks due to the presence of multi-dimensional uncertainties. Knowledge of ins and outs of any company's financial numbers can significantly help investors in the selection of stocks. Successful investors in history like George Soros and Warren Buffet, have preferred fundamentals including companies financial and operational data for their investment decisions. In BSE there are many important factors used by stock market experts for the evaluation and selection of stocks. By thorough literature survey and with the help of various experts' opinions initially, 10 metrics (ratios) namely earn per share (EPS), price to earning ratio (P/E), payout ratio(PR), price to sales ratio (P/S), long term debt to equity ratio (LTDER), price to book value (P/B), current ratio (CR), price to cash flow ratio (P/CF), profit margin(PM) and accounts receivable turnover (ART) are identified. But to reduce the complexity in the proposed model number of factors needed to be reduced. To select most important factors from tacit knowledge of experts, a questionnaire survey is conducted. The questions were about the importance of these 10 factors in stock selection and for that a 1–10 point scale is used. A higher point indicates higher importance. Questionnaire were distributed to 65 domain experts but 40 of them successfully completed the survey. To select the critical factors from this survey the Fuzzy Delphi method (Hsu and Yang, 2000) is applied. The Fuzzy Delphi method and its application to the proposed model is discussed below.

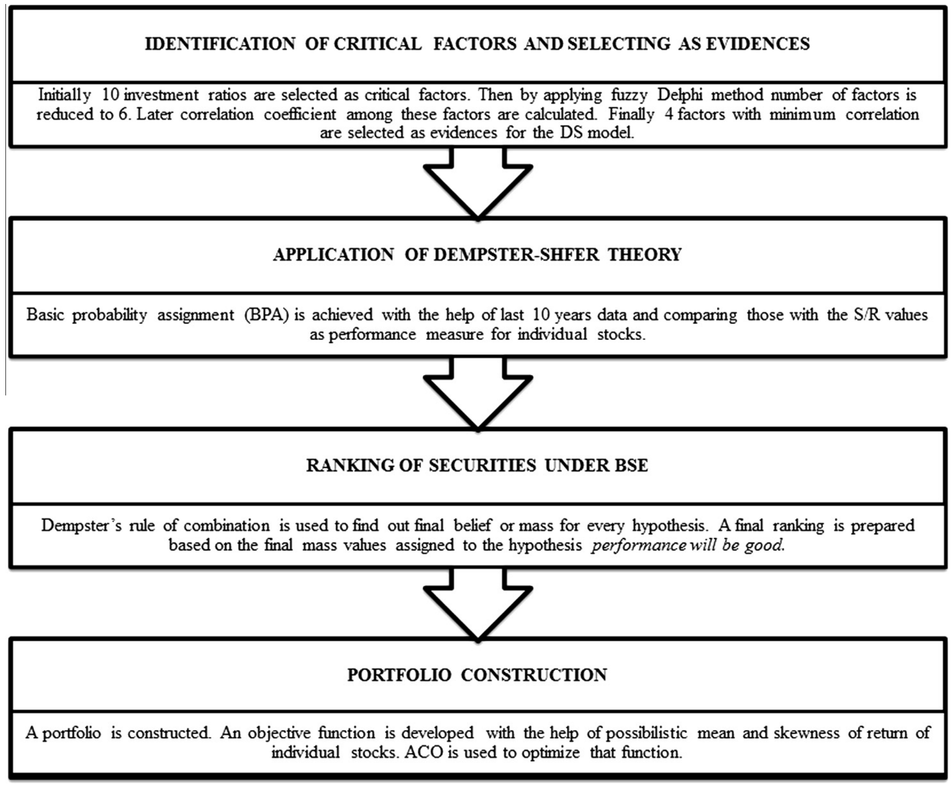

Figure 1 Proposed model for portfolio construction using the DS theory.

2.1. Fuzzy Delphi method

Traditional Delphi method was integrated with fuzzy set theory to improve the ambiguity and vagueness of the Delphi method (Murry et al., 1985) where membership degree is used to establish the membership function of each participant. Later max–min and fuzzy integration algorithms were developed by introducing the fuzzy theory into the Delphi method (Ishikawa et al., 1993) to predict the prevalence of computers in the future. In another research triangular fuzzy number is applied to the Delphi method to incorporate expert opinion (Hsu and Yang, 2000). The two terminal points of triangular fuzzy number represents the maximum and minimum values of experts' opinions and to derive the statistically unbiased effect and avoid the impact of extreme values, the geometric mean is taken as the membership degree of the triangular fuzzy numbers. This method is successfully implemented to construct key performance appraisal indicators for mobility of service industries (Kuo and Chen, 2008). It is noticed from this research that besides its simplicity this model can encompass all the expert opinions in one investigation. The Fuzzy Delphi method is also successfully applied in the determination of appraisal criteria for employees' performance evaluation based on MCDM technique (Falsafi et al., 2011). The main advantage of this method in collecting group decision lies in that every expert opinion will be considered and integrated to achieve the consensus of group decisions (Kuo and Chen, 2008). Uncertain and subjective messages in human thinking can also be induced in this model. It also reduces the investigation time and cost.

For the selection of critical factors for stock evaluation the fuzzy Delphi method proposed by Hsu and Yang (2000) is applied in this research to denote expert consensus with geometric mean. The process is explained as follows:

2.1.1. Representing expert opinions by triangular fuzzy number

All expert opinions collected from questionnaire are organized into estimates and then the triangular fuzzy number $\widetilde{T}_F$ is created as follows:

$$\left.\begin{array}{l} \widetilde{T}_F = (L_F, M_F, U_F) \\ L_F = min(X_{F_i}) \\ M_F = \sqrt[n]{\prod_{i=1}^{n} X_{F_i}} \\ U_F = max(X_{F_i}) \end{array}\right\} \quad (1)$$

where i denotes the ith expert, $i = 1, 2, 3, \ldots, n$ and X_{F_i} indicates evaluation value of the ith expert for factor F; The bottom of all experts' evaluation scores for factor F is represented by L_F; U_F indicates the ceiling of all the experts' evaluation scores for the factor F and M_F represents the geometric mean of all the experts' evaluation scores for the factor F.

2.1.2. Selection of factors

To denote the expert group consensus on the importance value of the 10 previously identified factors, the geometric mean M_F of each factor's triangular fuzzy number is used. This is explicitly done to avoid the impact of extreme values. The geometric

Table 1 Factors with geometric mean.

Sl No.	Factor	Geometric Mean (M_F)
1	Earn per share	7.17
2	Price to earning ratio	8.43
3	Payout ratio	7.00
4	Current ratio	6.02
5	Price to cash flow ratio	6.59
6	Price to sales ratio	8.35
7	Price to book value ratio	8.26
8	Profit margin	6.52
9	Long term debt to equity ratio	8.20
10	Accounts receivable turnover	5.81

mean calculated for these 10 factors are mentioned in Table 1. For threshold value σ the selection criteria are decided as below:

If $M_F \geqslant \sigma = 7,$ the factor is accepted.

If $M_F < \sigma = 7,$ the factor is rejected.

So finally 6 factors, EPS, P/E, PR, P/S, P/B and LTDER satisfied the threshold criteria and were selected for further consideration.

2.2. Selection of evidences for the proposed model

The DS evidence theory is used for stock selection and ranking in this model where the historical values of few critical factors of stocks are treated as evidences of their performance. But in the DS theory all evidences used are required to be conditionally independent. There is no significant evidence in the stock market which can conclude that the 6 factors selected above are conditionally dependent or not. So to check the dependencies of these factors, historical data of last three years (2012–13, 2013–14 and 2014–15) are used. S/R is considered as one of the most effective performance indicator of stocks in various stock exchanges. To determine the influence of a factor in the overall performance of any stock, value of the factor in any financial year is divided by the S/R value of that particular stock for the same year. The result is termed as 'impact score' as this score indicates the level of impact of any factor in the overall performance of any stock. For example, the value of P/E for Reliance Industries Ltd. was 13.67 and S/R value was 0.66 in FY 2014–15. So the impact score of P/E for Reliance Industries Ltd. in FY 2014–15 is calculated as 20.66. In this way impact scores of all 6 previously selected factors are calculated for all 30 registered stocks under BSE for FY 2012–13, FY 2013–14 and FY 2014–15. Now the dependencies of these factors are determined through the following steps.

2.2.1. Generating correlation coefficient matrix of evidences

The correlation coefficient of two random variables is a measure to determine the degree of their linear independence. If two variables X and Y have N scalar observations each, then the Pearson correlation coefficient is defined as

$$\rho(X,Y) = \frac{1}{N-1}\sum_{i=1}^{N}\left(\frac{\overline{X_i - \mu_X}}{\rho_X}\right)\left(\frac{Y_i - \mu_Y}{\rho_Y}\right) \tag{2}$$

where μ_X and ρ_X are the mean and standard deviation of X, respectively and μ_Y and ρ_Y are the mean and standard deviation of the variable Y. We can also define the correlation coefficient in terms of the covariance of X and Y as follows

$$\rho(X,Y) = \frac{cov(X,Y)}{\rho_X \rho_Y} \tag{3}$$

The correlation coefficient matrix of two random variables is generated as

$$M = \begin{pmatrix} \rho(X,X) & \rho(X,Y) \\ \rho(Y,X) & \rho(Y,Y) \end{pmatrix} \tag{4}$$

Considering the 30 registered stocks as 30 observations, 6 factors as random variables and corresponding impact scores as their values, correlation coefficient matrices are calculated using Eq. (4) for the three financial years. Table 2 shows these correlation coefficient values for FY 2014–15.

Table 2 gives an indication about the dependencies of factors among each other. Higher value indicates higher dependency and lower value indicates lower dependency.

Figs. 2–4 show the level of dependencies of factors in the form of bar graphs for the three financial years.

From these three figures it is noticed that correlation coefficient values among P/E and P/B are high in all the three years. This clearly indicates that P/E and P/B are highly dependent with each other. In the same way PR and EPS are also found to be highly dependent. As these factors are highly dependent, if P/E and P/B are used as evidences in the DS evidence theory it may lead to a result of super estimate. The same is true for PR and EPS. But the dependencies of P/E (or P/B), P/S, LTDER and EPS (or PR) among each other are relatively low. So all of these factors can be used as evidences for the DS synthesis of the proposed DS stock selection model. In this proposed research finally historical values of P/B, P/S, LTDER and EPS for all the stocks are selected as evidence.

P/B ratio of a company is used to compare market value of the stock to its book value. It is also known as price to equity ratio and is defined as:

$$P/B = \frac{Stock\ Price}{Total\ Assets - Intangible\ Assets\ and\ Liabilities} \tag{5}$$

A lower P/B sometimes suggests that the stock is undervalued. However, it could also mean that something is fundamentally wrong with the company. Subramanyam and Venkatachalam (1998) and Barbee et al. (1996) show that P/B and return of a stock are very much interrelated because it aggregates current and past earnings and it helps to explain the variation of market value indirectly.

Table 2 Correlation coefficients of factors for FY 2014-15.

Factors	P/E	P/B	P/S	LTDER	EPS	PR
P/E	1	0.57	0.051	0.36	0.19	0.25
P/B	0.57	1	0.17	−0.08	0.39	0.50
P/S	0.051	0.17	1	0.24	0.02	0.22
LTDER	0.36	−0.08	0.24	1	−0.10	−0.07
EPS	0.19	0.39	0.027	−0.10	1	0.85
PAYOUT	0.25	0.50	0.22	−0.07	0.85	1

Please cite this article in press as: Mitra Thakur, G.S. et al., Stock portfolio selection using Dempster–Shafer evidence theory. Journal of King Saud University – Computer and Information Sciences (2016), http://dx.doi.org/10.1016/j.jksuci.2016.07.001

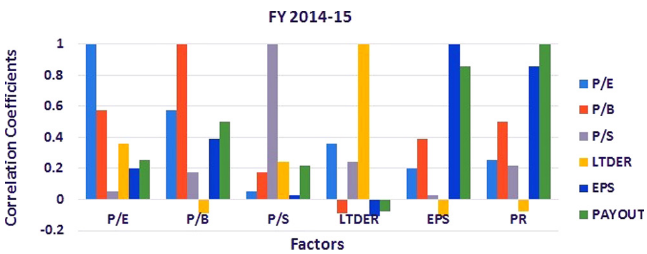

Figure 2 Dependencies of factors in FY 2014-15.

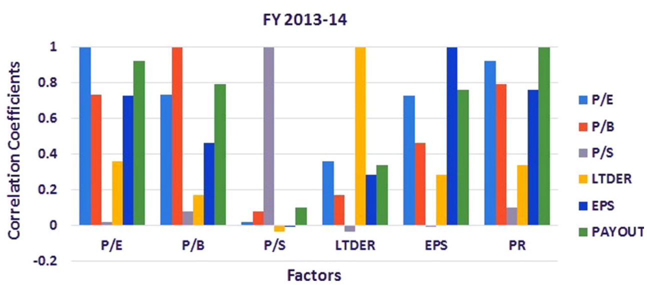

Figure 3 Dependencies of factors in FY 2013-14.

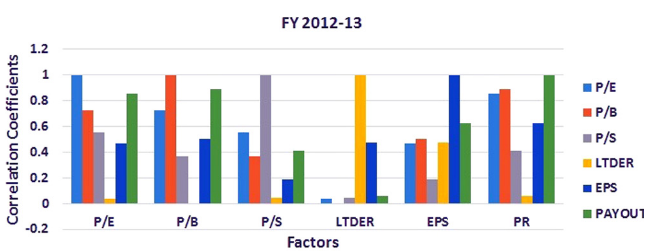

Figure 4 Dependencies of factors in FY 2012-13.

P/S is a valuation ratio that compares a company's stock price to its revenues. It is calculated by diving the company's stock market capitalization by its total sale. The relationship between volume of sales and stock prices are examined by many (Subramanyam and Venkatachalam, 1998; Ying, 1966). Ying (1966) shows that less price indicates a small volume whereas an increase in volume is indicated either by a high increase in price or a high decrease in price in future.

$LTDER$ is a ratio of total liabilities of a stock to its shareholders equity. It is a leverage ratio and it measures the degree to which the assets are financed by the debts and the shareholders equity and is calculated by the total liabilities divided by shareholders equity. Subramanyam and Venkatachalam (1998) states that long-term debt to equity ratio also influences the stock market indirectly. Bowman (1980) demonstrates that debt to equity ratio is an important variable for risk and security of any stock and also shows that higher values of this ratio are indicating high risk in future and vice versa.

EPS is the portion of a company's profit allocated to each outstanding share of common stock. Earning per share serves as indicator of a company's profitability and it is calculated as:

$$EPS = \frac{Net\ Income - Dividends\ on\ Preferred\ Stock}{Average\ Outstanding\ of\ Shares} \quad (6)$$

With these four factors, *semi-variance to return ratio* (S/R) is also used in this work as a measure of performance of stocks. As most of the investors would like to get maximum return with minimum risk, possible lower value of this S/R ratio indicates good performance of stocks.

Historical data for these factors of all thirty registered companies under BSE are collected from www.capitaline.in. These four factors function as evidences to assign basic probabilities to the hypothesis set in the proposed model.

3. DS evidence theory and its application to the proposed model

The DS-theory was introduced by Dempster (1967) and then was extended by Shafer (1976). It is an extension of classical probability theory by generalization of the Bayesian theory of subjective probability. Being a mathematical framework for representation of uncertainty, the DS theory combines the degrees of belief derived from independent items of evidences. The DS theory is successfully applied (Hong-dong et al., 2008; Maseleno and Hasan, 2012; Zhang et al., 2007) in various kinds of problem under uncertainty. However no such contribution in portfolio selection problem is noticed. DS theory mainly deals with four concepts: frame of discernment, basic probability assignment (BPA), the belief or mass function and the plausibility. Frame of discernment is considered to be a finite set of mutually exclusive and exhaustive hypotheses. Assume that $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$ is the frame of discernment. Let X is an arbitrary subset of Θ. The mass function over Θ can be described as $m : 2^{\Theta} \rightarrow [0, 1]$, such that $m(\phi) = 0$ and $\sum_{X \subseteq \Theta} m(X) = 1$. $m(X)$ is the value of basic probability for a given set X of interest. Shafer defined the concept of Belief (bel) as $Bel(X) = \sum_{Y \subseteq X} m(Y)$ and Plausibility (Pl) as $pl(X) = \sum_{Y \cap X \neq \phi} m(Y)$ and assigned each set of hypotheses an interval $[belief, plausibility]$ within which the degree of belief of each hypothesis must lie. Basic probability assignment can be viewed as determining a set of probability distribution P over Θ such that $Bel(X) \leqslant P(X) \leqslant pl(X)$. Dempster's rule of combination for combining two sets of masses m_1 and m_2 is

$$m_3(Z) = \frac{\sum_{X \cap Y = Z} m_1(X) m_2(Y)}{\sum_{X \cap Y = \phi} m_1(X) m_2(Y)} \quad (7)$$

3.1. Basic probability assignment (BPA)

By analyzing historical data and consulting with 35 domain experts, for each stock, threshold values for each of these five ratios discussed are decided. For all these factors the values higher than these threshold values are treated as the presence of evidence. As lower S/R values indicate good performance of stocks, a threshold value 0.05 has been set as a performance bar for the stocks under BSE by considering their performances over the last decade. Hence, if S/R of any stock is less than 0.05 then only the performance of the stock will be treated as good. As the model is proposed for short-term investment period, the presence of any particular evidence in any particular year is here treated to support or deny the performance of any stock in its next year. Hence the presence of evidence in any particular year supports or denies the hypothesis of the corresponding next year.

Now, say, for any particular stock, during last financial years in t different years any particular evidence is present. Now the S/R values are checked for corresponding next t

different financial years. For example for any particular stock if any evidence is present in 2004–05, S/R value of 2005–06 is checked for that stock. Now consider that in these t different years G times S/R value was below 0.05 and it was above that value P times. As last 10 years' data are considered for this model, the value of $G - P$ will be between -10 to $+10$. Now based on this $G - P$ value, BPA for hypothesis *performance will be good (PG)*, *performance will be poor (PP)* and *performance will be good or poor (PG, PP)* is assigned as below:

$$\begin{cases} \text{If } G-P>2 & \text{then the } BPA(PG)=\frac{|G-P|}{10} \\ \text{If } G-P<-2 & \text{then the } BPA(PP)=\frac{|G-P|}{10} \\ \text{If } \mid G-P\mid=0 & \text{then the } BPA(PG,PP)=0.8 \\ \text{If } \mid G-P\mid=1 & \text{then the } BPA(PG,PP)=0.7 \\ \text{If } \mid G-P\mid=2 & \text{then the } BPA(PG,PP)=0.6 \end{cases} \tag{8}$$

From the above assignment of probability it is clear that when $G - P$ is greater than $+2$, performance of the stock is satisfactory in most of the cases when the evidence is present and thus basic probability is assigned accordingly toward the hypothesis *Performance will be good*. In the same way when $G - P$ is less than -2, performance of the stock is not satisfactory in most of the cases when the evidence is present and thus basic probability is assigned accordingly toward the hypothesis *Performance will be poor*. When the value of $G - P$ is a value between -2 to $+2$, we can easily conclude that the performance of the stock is very fluctuating and uncertain. So the belief toward the hypothesis *Performance will be good or poor* becomes strong and basic probabilities are assigned in support of this.

For further clarification let us consider the BPA for Dr. Reddy's Laboratories Ltd., one of the registered companies under BSE. Table 3 shows S/R values for 2004–05 to 2013–14 and Table 4 shows the values of the four factors over the period 2003–04 to 2012–13. From Table 4 we can find that in six different years (2012–13, 2011–12, 2010–11, 2009–10, 2008–09 and 2005–06) EPS was above the threshold value 50. Hence EPS evidence was considered to be present in these six different years. Now from Table 3 we can see that in corresponding next six years (2013–14, 2012–13, 2011–12, 2010–11, 2009–10 and 2006–07) S/R was less than 0.05 for five times and once it was above that. So the value of G will be 5 and P will be 1 and BPA assigned toward the hypothesis Performance will be good in the presence of the EPS evidence for Dr. Reddy's Laboratories Ltd. is 0.4.

3.2. Application of Dempster's rule of combination

In further explanation of the proposed model the example of Dr. Reddy's Laboratories Ltd. is extended. Table 5 shows the BPA for different hypotheses in the presence of four different evidences with their standard values for Dr. Reddy's Laboratories Ltd.

In the same way belief values are assigned for all other 29 registered companies under BSE. Once the belief values are assigned, in the next phase Dempster's combination rule is applied to calculate the final masses for all the companies. Final mass calculation for Dr. Reddy's Laboratories Ltd. is explained as an example. From Table 4 we can conclude that all the four evidences were present in 2012–13. Let us now consider EPS to be the first evidence and m_1 be the mass function to assign belief value to the hypothesis based on this evidence. So from Table 5, $m_1(PG) = 0.4$ and $m_1(\Theta) = 1 - 0.4 = 0.6$ where $m_1(\Theta)$ represents the belief in the rest of the hypotheses of the frame of discernment. Now consider P/B ratio to be the second evidence and m_2 be the mass function to assign belief value to the hypothesis based on this evidence. Again from Table 5, $m_2(PG, PP) = 0.6$ and $m_2(\Theta) = 1 - 0.6 = 0.4$. Now these two evidences are combined to generate new mass m_3 as mentioned in Table 6.

Table 5 BPA for Dr. Reddy's Laboratories Ltd.

Evidences	Hypotheses		
	PG	PP	(PG, PP)
EPS	0.4		
P/B			0.6
LTDER			0.6
P/S	0.3		

Table 6 Mass combination considering first two evidences.

Combining m_1 and m_2	$m_2(PG, PP) = 0.6$	$m_2(\Theta) = 0.4$
$m_1(PG) = 0.4$	$PG = 0.24$	$PG = 0.16$
$m_1(\Theta) = 0.6$	$(PG, PP) = 0.36$	$\Theta = 0.24$

Table 3 10 years S/R for Dr. Reddy's Laboratories Ltd.

Period	2013–14	2012–13	2011–12	2010–11	2009–10	2008–09	2007–08	2006–07	2005–06	2004–05
S/R	0.0195	0.0062	0.0178	0.0732	−0.0576	0.0069	0.0217	0.0467	0.0579	0.0139

Table 4 10 years value of four factors for Dr. Reddy's Laboratories Ltd.

Factors	Threshold value	2012–13	2011–12	2010–11	2009–10	2008–09	2007–08	2006–07	2005–06	2004–05	2003–04
P/B	3	3.9	4.5	4.7	3.7	1.6	2.1	2.8	4.9	2.8	3.7
P/S	3	3.53	4.4	5.19	4.74	1.94	2.88	3.02	5.18	3.48	4.28
LTDER	0.5	0.51	0.33	0.36	0.18	0.39	0.63	0.64	0.52	0.38	0.27
EPS	50	98.57	113.62	74.51	53.81	52.78	33.32	28.27	58.82	10.64	2.8

So new mass m_3 for hypotheses can be concluded as

$$\left.\begin{array}{l} m_3(PG) = \frac{0.24+0.16}{1-0} = 0.4 \\ m_3(PG, PP) = \frac{0.36}{1-0} = 0.36 \\ m_3(\Theta) = \frac{0.24}{1-0} = 0.24 \end{array}\right\} \quad (9)$$

Now consider LTDER to be the new evidence and m_4 be the mass function to assign belief value to the hypothesis in the presence of this evidence. From Table 5, $m_4(PG, PP) = 0.6$, $m_4(\Theta) = 1 - 0.6 = 0.4$. So again these m_4 and m_3 are combined to generate mass m_5 for hypotheses as per Table 7.

Now following the above table mass m_5 can be concluded as follows:

$$\left.\begin{array}{l} m_5(PG) = \frac{0.24+0.16}{1-0} = 0.4 \\ m_5(PG, PP) = \frac{0.216+0.144+0.144}{1-0} = 0.504 \\ m_3(\Theta) = \frac{0.096}{1-0} = 0.24 \end{array}\right\} \quad (10)$$

Now consider another new evidence P/S and m_6 be the mass function to assign belief value to the hypothesis in the presence of this evidence. So again from Table 5 $m_6(PG) = 0.3$ and $m_6(\Theta) = 1 - 0.3 = 0.7$.

Now again combining m_6 and m_5 the final mass m_7 is generated as shown in Table 8.

Now following Eq. (7) and the above table final mass m_7 can be concluded as follows:

$$\left.\begin{array}{l} m_7(PG) = \frac{0.12+0.28+0.1512+0.0288}{1-0} = 0.58 \\ m_7(PG, PP) = \frac{0.3528}{1-0} = 0.3528 \\ m_7(\Theta) = \frac{0.0672}{1-0} = 0.0672 \end{array}\right\} \quad (11)$$

In this way final masses have been calculated for rest of the 29 companies registered under BSE. Table 9 shows the details of 30 companies based on their final mass values for hypothesis *performance will be good (PG)*.

4. Portfolio construction

The main objective of constructing a portfolio is to determine optimum investment ratios for the securities such that the overall return is maximized under a tolerable risk for a given period of investment. In this section a portfolio selection model has been proposed by selecting the top ten securities as enlisted in Table 9.

Table 7 Mass combination after considering first three evidences.

Combining m_3 and m_4	$m_4(PG, PP) = 0.6$	$m_4(\Theta) = 0.4$
$m_3(PG) = 0.4$	$PG = 0.24$	$PG = 0.16$
$m_3(PG, PP) = 0.36$	$(PG, PP) = 0.216$	$(PG, PP) = 0.144$
$m_3(\Theta) = 0.24$	$(PG, PP) = 0.144$	$\Theta = 0.096$

Table 8 Mass combination after considering all four evidences.

Combining m_5 and m_6	$m_6(PG) = 0.3$	$m_6(\Theta) = 0.7$
$m_5(PG) = 0.4$	$PG = 0.12$	$PG = 0.28$
$m_5(PG, PP) = 0.504$	$PG = 0.1512$	$(PG, PP) = 0.3528$
$m_5(\Theta) = 0.096$	$PG = 0.0288$	$\Theta = 0.0672$

Table 9 Rank of 30 stocks based on the proposed model.

Name of the stock	Final belief for the hypothesis PG	Rank
ITC Ltd.	0.996	1
State Bank of India	0.996	2
Hindustan Unilever Ltd.	0.995	3
Cipla Ltd.	0.988	4
Mahindra & Mahindra Ltd.	0.988	5
Hero Moto Corp Ltd.	0.970	6
Hindalco Industries Ltd.	0.960	7
HDFC Bank Ltd.	0.952	8
Infosys Ltd.	0.950	9
HDFC Ltd.	0.944	10
Tata Motors Ltd.	0.94	11
Sesa Sterlite Ltd.	0.895	12
Sun Pharmaceutical Inds. Ltd.	0.86	13
Maruti Suzuki India Ltd.	0.84	14
ONGC Ltd.	0.66	15
Tata Steel Ltd.	0.64	16
NTPC Ltd.	0.6	17
Wipro Ltd.	0.6	18
Dr. Reddy's Laboratories Ltd.	0.58	19
Tata Power Co. Ltd.	0.58	20
Reliance Industries Ltd.	0.5	21
GAIL (India) Ltd.	0.4	22
Bharat Heavy Electricals Ltd.	0.3	23
Axis Bank Ltd.	0	24
Bajaj Auto Ltd.	0	25
Bharti Airtel Ltd.	0	26
Coal India Ltd.	0	27
ICICI Bank Ltd.	0	28
Larsen & Toubro Ltd.	0	29
Tata Consultancy Services Ltd.	0	30

4.1. Construction of the objective function

The following notations are used in the construction of the constrained objective function.

- $\tilde{r}_i$: Fuzzy return of the ith asset;
- x_i: Fraction of the total investment allotted to the ith asset;
- μ_s: Weighted mean of asset semi-variances;
- r_f: Risk free return rate;
- r_p: Portfolio return;
- s_p: Skewness of the portfolio;
- v_p: Variance of the portfolio.

As an objective function here the ratio of the difference of fuzzy portfolio return and the risk free return to the weighted mean semivariance of the assets is used. Certainly, the higher value of the ratio will indicate the better investment; so the optimization target will be to maximize this ratio. Thus the objective function is formed as:

$$MAX \frac{E(\sum \tilde{r}_i x_i) - r_f}{\mu_s} \quad (12)$$

Table 10 Expected return, variance, skewness and semivariance of stocks.

Rank	Name of the stock	Return	Variance	Skewness	Semi-variance
1	ITC Ltd.	0.2801	0.0035	0.7523	0.000314
2	State Bank of India	0.0216	0.0006	0.1569	0.00098
3	Hindustan Unilever Ltd.	0.1443	0.0001	0.2391	0.00004
4	Cipla Ltd.	0.0437	0.0002	0.8258	0.00018
5	Mahindra & Mahindra Ltd.	0.0396	0.0009	0.0484	0.0027
6	Hero Moto Corp Ltd.	0.1379	0.0004	0.8057	0.00049
7	Hindalco Industries Ltd.	0.1337	0.0024	0.9996	0.00192
8	HDFC Bank Ltd.	0.0205	0.0005	−0.2192	0.00222
9	Infosys Ltd.	0.0356	0.0003	0.8648	0.0002
10	HDFC Ltd.	0.0408	0.0008	−0.7517	0.00435

where, after descending sort of portfolio, $\mu_s = \sum x_i s_i$, i.e. x_i is the ith weight in the descending order and s_i is the semi-variance of the ith ranked asset.

A fuzzy aggregation function is used to find the fuzzy returns of the securities from the statistical database of previous five years (2008–09 to 2012–13). If t_i means ith position of the data, the fuzzy return can be calculated as:

$$\tilde{r}_i = \left(min(r_i), \frac{\sum t_i r_i}{\sum t_i}, max(r_i) \right) \tag{13}$$

The following set of constraints is included in the model.

$$\left. \begin{array}{l} r_p > \alpha, \ v_p > \beta, \ s_p > \gamma \\ \sum_{i=1}^{n} x_i = 1, \ x_i \leqslant M, \ x_i > 0, \ \forall i \end{array} \right\} \tag{14}$$

Values for α, β, γ, M and m are decided based on the investor's preferences. For detail explanation on the constraints, readers can go through Bhattacharyya (2013).

Thus the final model for the portfolio optimization as discussed above can be summarized as below:

$$\left. \begin{array}{l} Maximize \ \frac{E\left(\sum \tilde{r}_i x_i\right) - r_f}{\mu_s} \\ \text{Subject to,} \\ r_p > \alpha, v_p > \beta, s_p > \gamma \\ \sum_{i=1}^{n} x_i = 1, \ x_i \leqslant M, \ x_i > 0, \ \forall i \end{array} \right\} \tag{15}$$

4.2. Optimization using ACO

In this section an algorithm is proposed and implemented to solve the model using ACO. ACO is a very popular meta-heuristic optimization technique basically inspired by the foraging behavior of biological ants (Dorigo et al., 2006; Deneubourg et al., 1990). The pseudo code of the proposed algorithm is shown below.

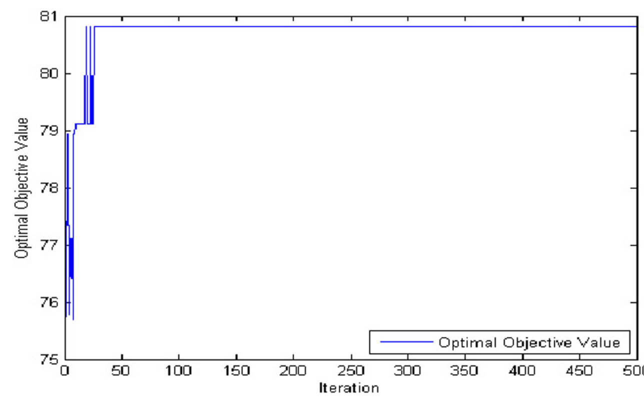

Figure 5 The convergence of objective value based on proposed ranking.

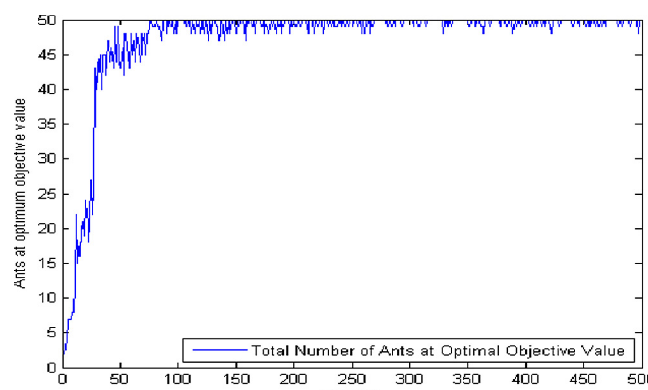

Figure 6 Ant accumulation at optimum solutions based on proposed ranking.

Table 11 Ratio allocation for the proposed portfolio.

ITC Ltd	SBI	Hindustan Unilever Ltd.	Cipla Ltd.	M & M Ltd.	Hero Moto Corp. Ltd.	Hindalco Industries Ltd.	HDFC Bank Ltd.	Infosys Ltd.	HDFC Ltd.
0.289	0.219	0.180	0.145	0.039	0.029	0.027	0.025	0.021	0.020

Table 12 Top 15 stocks under BSE.

Rank	Top 15 securities provided by the proposed model	Final mass value	Top 15 securities based on their performance (FY 2013–14)	S/R	Top 15 securities based on their performance (FY 2014–15)	S/R
1	ITC Ltd.	0.996	Sesa Sterlite Ltd.	−0.387	State Bank Of India	−40.34
2	State Bank Of India	0.996	NTPC Ltd.	−0.015	Infosys Ltd.	−40.14
3	Hindustan Unilever Ltd.	0.994	Hero Motocorp Ltd.	0.003	ITC Ltd.	0.018
4	Cipla Ltd.	0.988	Maruti Suzuki India Ltd.	0.008	HDFC Bank Ltd.	0.061
5	Mahindra & Mahindra Ltd.	0.988	Hindustan Unilever Ltd.	0.012	Hero Motocorp Ltd.	0.115
6	Hero Motocorp Ltd.	0.97	Cipla Ltd.	0.013	Hindustan Unilever Ltd.	0.121
7	Hindalco Industries Ltd.	0.96	State Bank Of India	0.013	Wipro Ltd.	0.141
8	HDFC Bank Ltd.	0.952	Bharat Heavy Electricals Ltd.	0.014	Maruti Suzuki India Ltd.	0.161
9	Infosys Ltd.	0.95	Wipro Ltd.	0.015	Tata Power Co. Ltd.	0.166
10	HDFC Ltd.	0.944	ITC Ltd.	0.016	Tata Motors Ltd.	0.176
11	Tata Motors Ltd.	0.94	Tata Power Co. Ltd.	0.017	TCS Ltd.	0.18
12	Sesa Sterlite Ltd.	0.895	Infosys Ltd.	0.018	Coal India Ltd.	0.181
13	Sun Pharmaceutical Inds. Ltd.	0.86	Hindalco Industries Ltd.	0.02	Mahindra & Mahindra Ltd.	0.187
14	Maruti Suzuki India Ltd.	0.84	Dr. Reddy'S Laboratories Ltd.	0.02	Cipla Ltd.	0.187
15	ONGC Ltd.	0.66	Sun Pharmaceutical Inds. Ltd.	0.023	Sun Pharmaceutical Inds. Ltd.	0.245

Table 13 Ratio allocation for the rank-irrelevant portfolio.

ITC Ltd	SBI	Hindustan Unilever Ltd.	Cipla Ltd.	M & M Ltd.	Hero Moto Corp. Ltd.	Hindalco Industries Ltd.	HDFC Bank Ltd.	Infosys Ltd.	HDFC Ltd.
0.028	0.092	0.201	0.173	0.023	0.152	0.035	0.105	0.178	0.016

Table 14 Comparison between rank-based and rank-irrelevant portfolio.

Type of the portfolio	Portfolio return ($\sum \tilde{r}_i x_i$)	Portfolio risk (μ_s)	Risk-return ratio
Rank-based	0.1301	0.00067	0.0051
Rank-irrelevant	0.0820	0.00068	0.0082

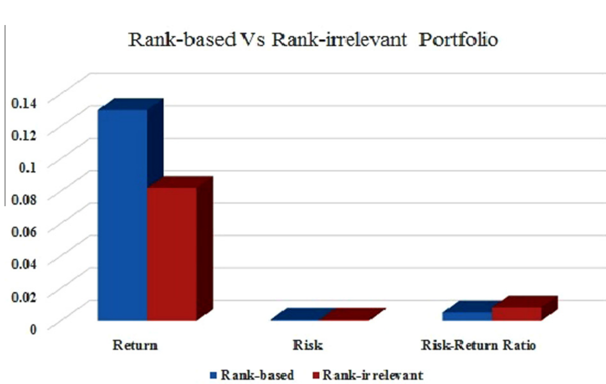

Figure 7 Ranked Vs rank-irrelevant portfolio.

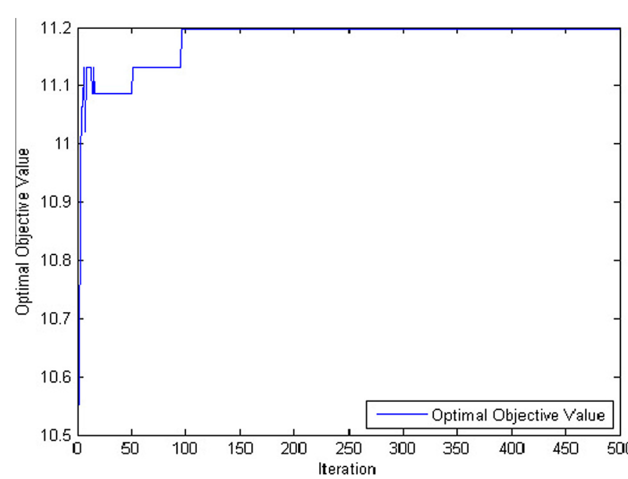

Figure 8 The convergence of objective value based on ranking using S/R.

Algorithm 1. ACO algorithm for portfolio optimization

Procedure ACO-Portfolio
  Generate N random solution nodes based on Eq. (15);
  Initialize the ACO;
  for ITERATION = 1 to I **do**
    for ANT = 1 to Γ **do**
      Select the start node randomly;
      for LIFETIME = 2 to L **do**
        Select next node based on the heuristic information and pheromone concentration in the path. Move to the next node only if it is better than the current node.
        Update pheromone on the selected path;
      end for
      Store the objective value and the path details of the final node reached by each ant;
    end for
    Identify the solution node where maximum number of ants have reached and consider that to be the optimum solution for the current iteration;
    Update the pheromone on the path of each ants who have reached this optimum solution;
    Evaporate the pheromone from all paths.
  end for
end procedure

Here the top 10 securities as enlisted in Table 9 are used to construct the portfolio. As $\tilde{r}_i$ is expressed as triangular fuzzy number, the Expected Return, Variance, Skewness and semi variances for last 5 years of these ten securities, as used in the implementation of the algorithm, are evaluated by the following theorem and are mentioned in Table 10.

Theorem 4.1. *Let $\widetilde{A} = (a, b, c)$ be a triangular fuzzy number. The weighted possibilistic mean, variance and skewness can be calculated as* Bhattacharyya (2013)*:*

$$\left.\begin{aligned} E(\widetilde{A}) &= \tfrac{1}{6}(a + 4b + c) \\ Var(\widetilde{A}) &= \tfrac{1}{18}(a^2 + b^2 + c^2 - ab - bc - ca) \\ Skew(\widetilde{A}) &= \frac{19(a^3+c^3)-8b^3-42b(a^2+c^2)+12b^2(a+c)-15(a^2c+ac^2)+60abc}{10\sqrt{2}\left(\sqrt{a^2+b^2+c^2-ab-bc-ca}\right)^3} \end{aligned}\right\} \tag{16}$$

When the algorithm is executed in MATLAB with the above dataset and considering other parameters as $r_f = 0.01$, $\beta = 0.5$, $\alpha = 0.05$, $\gamma = 0.001$, $M = 0.8$ and $\mu_s = 0.0016$, the maximum return is found as 0.1301. The proposed ratio allocation for this return is given in Table 11.

Fig. 5 shows convergence of the objective values as per the propose model and Fig. 6 depicts the accumulation of ants to the optimum objective values in each iteration. It is clear from these figures that proposed ACO algorithm can effectively solve the proposed portfolio model.

5. Result analysis

In this section performance of the proposed model is analyzed further in following four different phases.

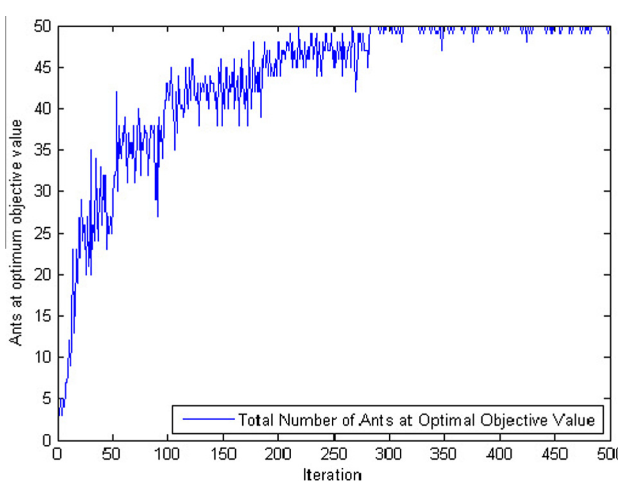

Figure 9 Ant accumulation at optimum solutions based on ranking using S/R.

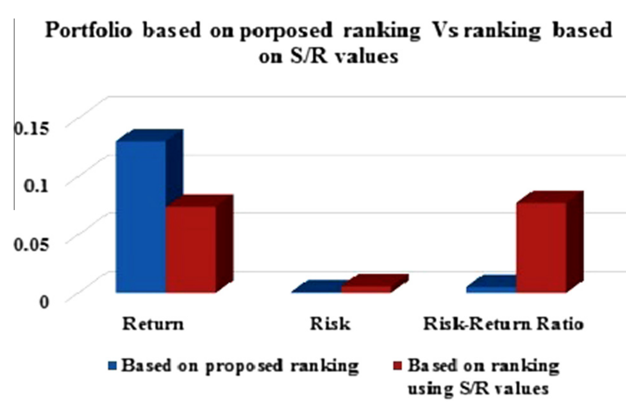

Figure 10 Portfolio based on proposed ranking Vs ranking based on S/R values.

Table 15 Comparison between proposed portfolio and the portfolio based on S/R values.

Type of the portfolio	Portfolio return ($\sum \tilde{r}_i x_i$)	Portfolio risk (μ_s)	Risk-return ratio
Based on proposed ranking	0.1301	0.00067	0.0051
Based on ranking using S/R values	0.0740	0.0057	0.0772

Please cite this article in press as: Mitra Thakur, G.S. et al., Stock portfolio selection using Dempster–Shafer evidence theory. Journal of King Saud University – Computer and Information Sciences (2016), http://dx.doi.org/10.1016/j.jksuci.2016.07.001

Table 16 Empirical comparison of recent researches with the proposed model.

Article	Stock selection approach	Portfolio return	Portfolio risk	Portfolio optimization approach	Optimization tool	Uncertainty handling tool	Real data source
Yunusoglu and Selim (2013)	Fuzzy rule-based expert System	No conventional return representation is used but compared the results with compound return, monthly average return, etc.	No conventional risk representation is used but evaluated the performance for different risk profiles; risk averse, risk neutral and risk prone	Maximizing the total weighted ratings of the stocks that are incorporated in the recommended portfolio	Mixed integer linear programing model	Fuzzy Sets and Fuzzy Logic	Istanbul Stock Exchange
Huang (2012)	Hybrid GA-Support Vector Regression (SVR) based model	Represented as surrogates generated by the SVR method	Not Addressed	Optimizing the set of input features and kernel parameters used by the SVR	Genetic Algorithm	Not Addressed	Taiwan Stock Exchange
Fasanghari and Montazer (2010)	Fuzzy rule-based expert System	Determined through questionnaires returned by investors	Determined through questionnaires returned by investors	Based on different investment criteria provided by investors	No optimization tool is used	Fuzzy Sets and Fuzzy Logic	Tehran Stock Exchange
Bhattacharyya et al. (2014)	Random	Return of each stock is considered as triangular fuzzy number and quantified as the mean of the fuzzy returns of stocks	Variance of the fuzzy returns of stocks	Maximization of the expected return as well as the skewness and minimization of the variance as well as the cross-entropy for the portfolio	Genetic Algorithm and multiple objective genetic algorithm (MOGA)	Fuzzy Sets, Fuzzy Cross Entropy	Bombay Stock Exchange
Nguyen and Lo (2012)	A novel generic robust ranking model	Not addressed separately	Not addressed separately	Finding optimal portfolio weight to maximize the average ranking for the worst realization of the ranking R that lies in a uncertainty set	Using a constraint generation method	Not Addressed	Dow Jones Industrial Average Index
Proposed article	A novel DS evidence theory based model	Return of each stock is considered as triangular fuzzy number and quantified as the mean of the fuzzy returns of stocks	Weighted mean Semivariance of the stocks	Maximizing the ratio of the difference of fuzzy portfolio return and the risk free return to the weighted mean semivariance of the assets	Ant Colony Optimization	Fuzzy Set theory and DS evidence theory	Bombay Stock Exchange

5.1. Effectiveness of the proposed Model:

In this article a rank preference based portfolio construction model is proposed. For this study last five years' historical data (FY 2008–09 to FY 2012–13) are used and the ranking is shown in Table 9. To check the reliability we have collected the data for next two financial years and then ranked the stocks in risk return frame work. A match for 10 companies in 2013–14 and a match of 11 companies in 2014–15 are found when that ranking is compared with the predicted top 15 companies using our proposed model. It promotes the stability of the ranking this system proposed in this article. However it would be appreciated to evaluate fresh ranking for each financial year for better assignment of stock in the portfolio evaluation process. Table 12 shows the details of these two rankings.

5.2. Comparing rank-based portfolio with rank-irrelevant portfolio:

In the proposed portfolio construction model, higher weightage is assigned to the stock having higher rank. The portfolio thus obtained is compared with the portfolio constructed without assigning any particular preference to any stock (alike the procedure of Markowitz (1952), Bhattacharyya et al. (2014), etc.). Table 13 shows the ratio allocation for rank-irrelevant portfolio.

Table 14 compares the return and risk of these two portfolios and Fig. 7 gives the graphical representation of this comparison.

It is clear from Fig. 7 that the proposed rank-based portfolio gives better return under comparatively lower risk in comparison with rank-irrelevant portfolio. This proves the robustness of the proposed portfolio model and ranking system.

5.3. Comparing the portfolio with proposed ranking and the ranking based on S/R values:

S/R ratio is one of the most popular ratios used by the investors for stock selection. In this stage another portfolio is constructed by considering top 10 stocks, based on their S/R values, under BSE for the year 2012–13 using the same ACO algorithm and the objective function. Figs. 8 and 9 show the convergence of the optimization and ant accumulations at optimum objective values respectively.

Table 15 compares the return and risk of these two portfolios and Fig. 10 gives the graphical representation of this comparison.

Fig. 10 demonstrates that portfolio based on the prosed ranking is capable of giving better return under lower risk. This gives an indication that if any investor had invested in BSE based on the predicted ranking at the end of 2012–13 he could get better return in 2013–14 and 2014–15. This assures applicability of the proposed model by ensuring better portfolio returns in short-term investment period.

5.4. Comparison of the proposed model with other existing models-An Empirical Study

Due to the inherent uncertainty in the stock market, selection of proper stock plays a vital role before the construction of investment portfolios. In literature many researches are found to address this challenging task, few of them mainly address stock selection problem, few give emphasis on portfolio construction and some researchers address both of these issues. In this section we have done an empirical comparison of 5 such recent researches with our proposed model. Though different tools and methodologies are used in these researches criteria like, stock selection approach, representation of portfolio return and risk, portfolio optimization approach, optimization tools used, tool used for handling uncertainty and data source, are addressed in this comparative study. Table 16 shows this comparison.

From this comparative study it is noticed that only in our proposed model all major issues regarding stock portfolio selection are addressed and solved effectively. Other major drawback of the researches using expert system approaches can be the complexity raised in these models due to the repeated expert interactions. For example, in his work Fasanghari and Montazer (2010) proposed a fuzzy expert system for the selection of superior stocks in Tehran Stock Exchange (TSE). In this work he identified 7 factors which influence the stock market and developed a rule base of total 932 rules for the selection of stocks. Though the outcome of the model is satisfactory, the major concern of this model is the development time and cost due to repetitive expert interactions. Fuzzy set theory is used in this model to deal with the uncertainty present in the rule base. But fuzzy set theory is more effective in dealing with vagueness rather than inherent uncertainty present in any model. To enhance the robustness of the proposed model, the DS evidence theory is used to deal with the uncertainty present in the historical performance of the stocks and the fuzzy set theory is used to deal with the vagueness. This increases the adaptability of the proposed model over the other existing models.

6. Conclusion

In this work a novel portfolio construction model is proposed where three major aspects of investment, investors' point of view toward stocks, previous performance of stocks and uncertainty in the market have been combined. Investors' point of view has been considered in terms of maximizing return and minimizing risk. The DS evidence theory is used in this model to incorporate the uncertainties present in the previous performance of stocks. Vagueness in the performance of stocks are dealt by considering fuzzy return and risk. Performance of the model is proved to be effective when compared with the recent performance of stocks. This model can significantly reduce the development time and cost incurred in other existing models due to repeated expert interactions.

Though this model is implemented here for BSE only, it can be applied for constructing portfolios in any Stock Exchanges around the world; however, selection of critical factors can vary in different stock exchanges. Though in this work a very effective objective function is considered and ACO is used due to its wide acceptability and effective performance for optimizing portfolios, researchers can use any type of valid objective function and any well-known optimization techniques like genetic algorithm (GA), particle swarm optimization (PSO), etc. for their purpose. To enhance the robustness of the model researchers can also think of hybridizing the DS evidence theory with other uncertainty handling tools like soft sets and rough sets.

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Please cite this article in press as: Mitra Thakur, G.S. et al., Stock portfolio selection using Dempster–Shafer evidence theory. Journal of King Saud University – Computer and Information Sciences (2016), http://dx.doi.org/10.1016/j.jksuci.2016.07.001