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Radiative E1 decays of X(3872)

Tian-Hong Wang, Guo-Li Wang *

Department of Physics, Harbin Institute of Technology, Harbin 150001, China

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ABSTRACT

Radiative E1 decay widths of X(3872) are calculated through the relativistic Salpeter method, with the assumption that X(3872) is the $\chi_{c1}(2P)$ state, which is the radial excited state of $\chi_{c1}(1P)$. We first calculated the E1 decay width of $\chi_{c1}(1P)$. The result is in agreement with experimental data excellently. Then we calculated the width of X(3872) with the assignment $\chi_{c1}(2P)$. Results are: $\Gamma(X(3872) \to \gamma J/\psi) = 33.0$ keV, $\Gamma(X(3872) \to \gamma\psi(2S)) = 146$ keV and $\Gamma(X(3872) \to \gamma\psi(3770)) = 7.09$ keV. The ratio $Br(X(3872) \to \gamma\psi(2S))/Br(X(3872) \to \gamma J/\psi) = 4.4$ agrees with experimental data by BaBar, but is larger than the new up-bound reported by Belle recently. With the same method, we also predicted the decay widths, $\Gamma(\chi_{b1}(1P)) \to \gamma\Upsilon(1S) = 30.0$ keV, $\Gamma(\chi_{b1}(2P)) \to \gamma\Upsilon(1S) = 5.65$ keV and $\Gamma(\chi_{b1}(2P)) \to \gamma\Upsilon(2S) = 15.8$ keV, from which we get the full widths: $\Gamma(\chi_{b1}(1P)) \sim 85.7$ keV and $\Gamma(\chi_{b1}(2P)) \sim 66.5$ keV.

1. Introduction

X(3872) was discovered by Belle Collaboration [1] in 2003 through the channel $B^{\pm} \to K^{\pm} J/\psi\pi^+\pi^-$. The mass reported by Belle is $M = 3872.0 \pm 0.6 \pm 0.5$ MeV, and the full width has an upper limit $\Gamma < 2.3$ MeV at 90% C.L. Later the existence of this particle was confirmed by CDF [2], D0 [3], and BaBar [4] Collaborations.

The radiative decay channel $X(3872) \to J/\psi\gamma$ [5] indicates that this particle has positive C-parity. The most possible J^{PC} of X(3872) is 1^{++}, which is favored by the analysis of the decay angular distribution [6]. But the dipion mass distribution and tripion mass distribution in $X(3872) \to J/\psi\pi^+\pi^-$ and $X(3872) \to J/\psi\pi^+\pi^-\pi^0$ [5] favor a ρ resonance and a ω resonance, respectively, which indicates a large isospin breaking. The mass of X(3872) is 50–100 MeV smaller than the predictions of potential models. To understand these puzzles, many assignments of X(3872) were proposed, besides the traditional charmonium state assignment [7–10], there are the assignments of a molecular state [11–16], a hybrid charmonium [17], a diquark–antidiquark state [18], cusp effect or virtual state [19,20] (for a review, see e.g. Ref. [21]).

If X(3872) is a 1^{++} state, then there are two most possible natural assignments, a molecular state or a traditional charmonium $\chi_{c1}(2P)$. Molecular state model predicts the value of $Br(B^+ \to X(3872)K^+)/Br(B^0 \to X(3872)K^0)$ is about 10% [22], while the experimental value is $0.5 \pm 0.3 \pm 0.05$ [23]. This model meets more serious problems when used to calculate radiative decays. From Ref. [11] (see Table 1), one can get the ratio $\Gamma_{\psi'\gamma}/\Gamma_{J/\psi\gamma} \sim 4 \times 10^{-3}$, while the experimental value by BaBar is 3.4 ± 1.4 [24]. In this Letter, we will not consider the possibility of X(3872) as a molecular state, but due to its E1 radiative decay, we consider the possibility of an ordinary charmonium state.

Because E1 radiative decays will play a fundamental role in determination of the nature of X(3872), in this Letter, we just calculate the radiative E1 decay widths of X(3872) by assigning it as the $\chi_{c1}(2P)$ state and give the results. Although there is a discrepancy in the mass values of experiments and models, as Ref. [8] proposed, this is due to additional effects, such as coupled-channel effect. For the large isospin breaking, charmonium model can also give a good explanation [25].

This Letter is organized as follows. In Section 2, we solve the instantaneous Bathe–Salpeter (BS) equation (Salpeter equation) [26,27], and get wave functions of initial and final states. Then within Mandelstam formalism [28], we calculate the transition matrix element. In Section 3, we compare our results with other theoretical predictions and experimental data, some predictions and discussions are also given in this section.

2. E1 decay of X(3872) with $\chi_{c1}(2P)$ charmonium assignment

The wave function of 1^{++} sate is,

$$\varphi_{1^+}(q_\perp) = i\varepsilon_{\mu\nu\alpha\beta} P^\nu q_\perp^\alpha \epsilon_1^\beta \big[\varphi_1 M\gamma^\mu + \varphi_2 \not{P}\gamma^\mu + \varphi_3 \not{q}_\perp\gamma^\mu + \varphi_4 \not{P}\gamma^\mu \not{q}_\perp/M\big]/M^2, \tag{1}$$

* Corresponding author.
E-mail addresses: thwang.hit@gmail.com (T.-H. Wang), gl_wang@hit.edu.cn (G.-L. Wang).

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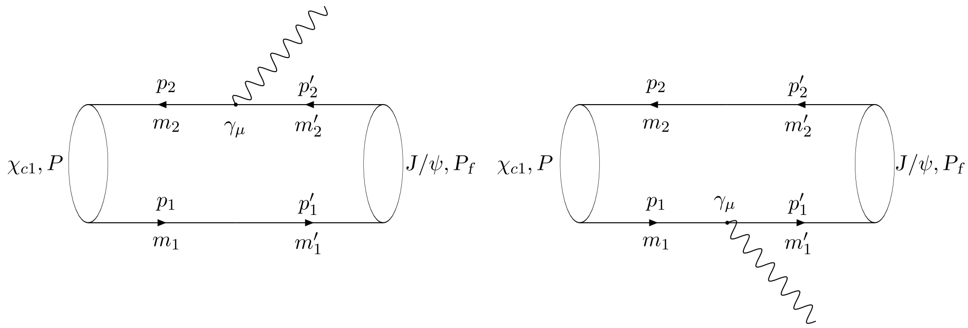

Fig. 1. Feynman diagram for the transition: $\chi_{c1} \to J/\psi + \gamma$.

where $\epsilon_{\mu\nu\alpha\beta}$ is the totally antisymmetric tensor. ϵ_1 is the polarization vector of the meson while M is its mass. P and q are the total momentum and relative momentum of constitute quark and antiquark, respectively, which are defined as:

$$p_1 = \alpha_1 P + q, \quad \alpha_1 = \frac{m_1}{m_1 + m_2},$$
$$p_2 = \alpha_2 P - q, \quad \alpha_2 = \frac{m_2}{m_1 + m_2}, \tag{2}$$

where p_1, p_2 are the momenta of quark and antiquark, respectively. $m_1 = m_2$ is the mass of constitute quarks. φ_is are functions of $q_\perp^2$. $q_\perp$ has the form: $q_\perp^\mu = q^\mu - (P \cdot q/M^2)P^\mu$. Because there are two constrain conditions [29], φ_3, φ_4 can be expressed by φ_1, φ_2 [29]. The wave function above has a different form with that in [29], but they are equivalent to each other. We show a general wave function form for 1^+ state, which means quark and antiquark inside the meson can have different masses. If we consider charmonium 1^{++} state, the quark and antiquark have the same mass, then φ_3 will disappear [29].

The wave function of 1^{--} state is [30],

$$\begin{aligned}\varphi_{1^-}(q'_\perp) = {} & q'_\perp \cdot \epsilon_2\big[f_1(q'_\perp) + \not{P}_f f_2(q'_\perp)/M_f + \not{q}'_\perp f_3(q'_\perp)/M_f \\ & + \not{P}_f \not{q}'_\perp f_4(q'_\perp)/M_f^2\big] + M_f \not{\epsilon}_2 f_5(q'_\perp) \\ & + \not{\epsilon}_2 \not{P}_f f_6(q'_\perp) + (\not{q}'_\perp \not{\epsilon}_2 - q'_\perp \cdot \epsilon_2) f_7(q'_\perp) \\ & + (\not{P}_f \not{\epsilon}_2 \not{q}'_\perp - \not{P}_f q'_\perp \cdot \epsilon_2) f_8(q'_\perp)/M_f,\end{aligned} \tag{3}$$

where M_f, P_f and ϵ_2 are the mass, momentum and polarization vector of the meson, respectively. Again, if we consider charmonium, the constitute quark and antiquark inside the mason have the equal mass. Because there are four constrain equations [30], f_7 and f_2 will disappear, and f_1 and f_8 can be expressed by f_3, f_4, f_5 and f_6. Here we will not present the details of solving BS equation, which can be found in Ref. [31].

We just give the Cornell potential which is applied when solving BS equation:

$$V(\vec{q}) = V_s(\vec{q}) + \gamma_0 \otimes \gamma^0 V_v(\vec{q}), \tag{4}$$

$$V_s(\vec{q}) = -\left(\frac{\lambda}{\alpha} + V_0\right)\delta^3(\vec{q}) + \frac{\lambda}{\pi^2}\frac{1}{(\vec{q}^2 + \alpha^2)^2}, \tag{5}$$

$$V_v(\vec{q}) = -\frac{2}{3\pi^2}\frac{\alpha_s(\vec{q})}{\vec{q}^2 + \alpha^2}, \tag{6}$$

$$\alpha_s(\vec{q}) = \frac{12\pi}{25}\frac{1}{\ln(a + \frac{\vec{q}^2}{\Lambda_{QCD}})}. \tag{7}$$

Here λ, α, e, V_0 and Λ_{QCD} are parameters. By fitting the mass spectra of 1^{++}, 1^{--} masons, we can find the best-fit values of these parameters: $a = e = 2.7183$, $\alpha = 0.06$ GeV, $\lambda = 0.2$ GeV, $m_c = 1.7553$ GeV, $m_b = 5.13$ GeV, $\Lambda_{QCD} = 0.26$ GeV ($c\bar{c}$), 0.20 GeV ($b\bar{b}$) (see [29]). For 1^{++} state, $V_0 = -0.452$ GeV ($c\bar{c}$), -0.521 GeV ($b\bar{b}$), for 1^{--} state, $V_0 = -0.465$ GeV ($c\bar{c}$), -0.570 GeV ($b\bar{b}$). Here α is the effective gluon mass. Since the potential we chose is a phenomenological one and the gluon mass as a parameter is not running, the value of α here is lower than the usual chosen especially when it is running close to the infrared limit.

Wave functions above are constructed based on the quantum number J^P or J^{PC} of mesons. For example, J^P of every term in Eq. (3) is 1^- (or 1^{--} for equal mass system). One can see that there is S wave and D wave mixing automatically, especially for the third state ($\psi(3770)$), which is D wave dominating, but mixing with a small part of S wave. This can be seen clearly in spherical polar coordinates [32].

The relativistic transition amplitude of 1^{++} state decaying to a photon and a 1^{--} state (see Fig. 1) can be written in terms of BS wave function:

$$T = \langle P_f\epsilon_2, k\epsilon|S|P\epsilon_1\rangle = \frac{(2\pi)^4 e e_q}{\sqrt{2^3\omega_\gamma E E_f}}\delta^4(P_f + k - P)\epsilon^\xi M_\xi, \tag{8}$$

where ϵ, ϵ_1 and ϵ_2 are the polarization vectors of the photon, initial meson and final meson, respectively. P, P_f and k are the momenta of initial meson, final meson and photon, respectively. $e_q = \frac{2}{3}$ for charm quark and $e_q = -\frac{1}{3}$ for bottom quark are the charges in unit of e. M^ξ is the matrix element of the electromagnetic current, which according to Refs. [28,32], in the leading order (the order of $\alpha = \frac{e^2}{4\pi}$, also neglect terms contain ψ^{+-}, ψ^{-+} and ψ^{--}, which contribute less than 1%) can be written as:

$$\begin{aligned}M^\xi = e e_q \int \frac{d\vec{q}}{(2\pi)^3}\mathrm{Tr}\bigg[&\frac{\not{P}}{M}\bar{\varphi}'^{++}(q_\perp + \alpha_2 P_{f\perp})\gamma^\xi \varphi^{++}(q_\perp) \\ & - \bar{\varphi}'^{++}(q_\perp - \alpha_1 P_{f\perp})\frac{\not{P}}{M}\varphi^{++}(q_\perp)\gamma^\xi\bigg],\end{aligned} \tag{9}$$

where φ^{++} is the positive part of BS equation. $P_{f\perp}$ and $\bar{\varphi}^{++}$ are defined as $P_{f\perp}^\mu = P_f^\mu - (P \cdot P_f/M^2)P^\mu$ and $\gamma_0(\varphi^{++})^\dagger\gamma_0$, respectively.

For X(3872), the positive energy part of wave function has the form:

$$\varphi_{1^{++}}^{++} = i\varepsilon_{\mu\nu\alpha\beta}P^\nu q_\perp^\alpha \epsilon_1^\beta\left(A_1\gamma^\mu + A_2\gamma^0\gamma^\mu + A_3\gamma^0\gamma^\mu \not{q}_\perp\right), \tag{10}$$

where A_1, A_2, A_3 are defined as:

$$A_1 = \frac{1}{2}\left(\frac{\varphi_1}{M} + \frac{\omega}{m}\frac{\varphi_2}{M}\right), \tag{11}$$

$$A_2 = \frac{1}{2}\left(\frac{\varphi_1}{M} + \frac{\omega}{m}\frac{\varphi_2}{M}\right)\frac{m}{\omega}, \tag{12}$$

Table 1
E1 decay widths of $\chi_{c1}(1P)$ and $\chi_{c1}(2P)$. In Ref. [8], Barnes and Godfrey (labeled by B&G) have made the impulse, nonrelativistic, zero recoil, and dipole approximations. In Ref. [11], three method are adopted: one has the same approximations as B&G, but compute with a improved potential (labeled by Swanson1); one has no approximation (labeled by Swanson2); the last one is molecular model (labeled by Swanson3). Our values inside the parentheses are for the cases that the mass of 3923 MeV for X(3872) is chosen.

Ref.	$\Gamma^{\chi_{c1}(1P)}_{J/\psi\gamma}$ (keV)	$\Gamma^{\chi_{c1}(2P)}_{J/\psi\gamma}$ (keV)	$\Gamma^{\chi_{c1}(2P)}_{\psi(2S)\gamma}$ (keV)	$\Gamma^{\chi_{c1}(2P)}_{\psi(3770)\gamma}$ (keV)
This work	306	33.0 (33.3)	146 (182)	7.09 (9.83)
PDG [34]	320			
Li and Chao [7]		45	60	
Swanson1 [11]		71	95	6.5
Swanson2 [11]		139	94	6.4
Swanson3 [11]		8	0.03	0
B&G [8]		11.0	63.9	3.7
Eitchen et al. [35]		110	180	25
Dong et al. [36]		1–2	5–6	

$$A_3 = \frac{1}{2}\left(\frac{\varphi_1}{M} + \frac{\omega}{m}\frac{\varphi_2}{M}\right)\frac{1}{\omega}. \tag{13}$$

The positive energy part of wave function for 1^{--} state can be written as:

$$\begin{aligned}\varphi^{++}_{1^{--}} &= B_1\not\epsilon_2 + B_2\not\epsilon_2\not P_f + B_3\not P_f\not\epsilon_2\not q'_\perp + B_4 q'_\perp\cdot\epsilon_2 \\ &\quad + B_5 q'_\perp\cdot\epsilon_2\not P_f + B_6 q'_\perp\cdot\epsilon_2\not q'_\perp + B_7 q'_\perp\cdot\epsilon_2\not P_f\not q'_\perp,\end{aligned} \tag{14}$$

where the expressions of B_1 to B_7 can be found in Ref. [33].

3. Numerical results and discussions

We first calculated the decay width of $\chi_{c1}(1P)$ to J/ψ and γ. The result 306 keV shown in Table 1 agrees with the experimental value 320 keV very well. This shows that our method can be used to describe radiative decay. For X(3872), with the 2^3P_1 charmonium assumption, we calculated decay widths of three channels. We first solved the instantaneous BS equation by setting the parameter $V_0 = -0.452$ GeV. The mass of $\chi_{c1}(2P)$ is 3.923 GeV [29], which is about 50 MeV larger than that of X(3872). This is the common character of all potential models, which may be due to the coupled-channel effect. The results which we got by using this wave function are included in parentheses in Table 1. To make the mass of $\chi_{c1}(2P)$ equal to 3872 MeV, we solved the BS equation by setting $V_0 = -0.516$ GeV, and keeping other parameters unchanged. (This also causes a mass decrease of 50 GeV for other states. Here we just want to get the wave function of $\chi_{c1}(2P)$ when its mass is 3872 MeV. To make the spectrum agree with experimental data, we have to modify our coupled equations, especially the potential, which is our future work.) Decay widths with this set of parameters are those outside parentheses. We can see that the value of $\Gamma^{\chi_{c1}(2P)}_{J/\psi\gamma}$ is almost unchanged, while the values of $\Gamma^{\chi_{c1}(2P)}_{\psi(2S)\gamma}$ and $\Gamma^{\chi_{c1}(2P)}_{\psi(3770)\gamma}$ are reduced by nearly 20% and 30%, respectively.

One can see that our $\Gamma_{J/\psi\gamma} = 33.0$ keV is of the same order with that of Li and Chao [7], B&G [8] and Swanson1 [21], but much larger than 8 keV of Swanson3 [11] (molecular model) and 1–2 keV of Dong [36] (molecular and $c\bar{c}$ mixture). Our $\Gamma_{\psi'\gamma} =$ 146 keV is about 2.5 times larger than that of Li and Chao [7] and B&G [8], but approximately equals to that of Eitchen [35], which has considered the influence of open-charm channels. The results in Swanson2 [11] have used a improved potential and included no zero recoil and dipole approximation which used in Swanson3 [11] and B&G [8]. But as B&G [8] did, the wave function and meson mass are calculated by adding spin-dependent interaction in the Hamiltonian. In this Letter, we started from BS equation, which is relativistic covariance. By using instantaneous approximation, we get coupled Salpeter equations, which has included the relativistic effects automatically.

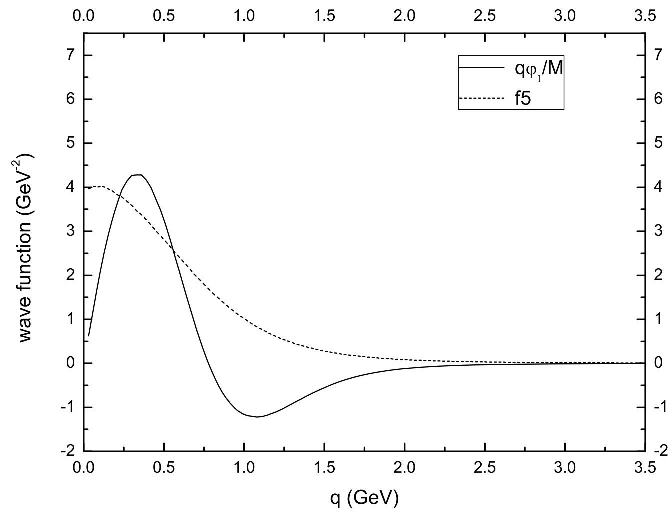

Fig. 2. Radial wave function $\frac{|\vec{q}|}{M}\varphi_1$ of X(3872) and f5 of J/ψ.

The ratios of E1 decay widths and the width of X(3872) $\to \pi^+\pi^- J/\psi$ detected by BaBar are [24]:

$$\frac{\mathrm{Br}(X(3872)\to\gamma J/\psi)}{\mathrm{Br}(X(3872)\to\pi^+\pi^- J/\psi)} = 0.33\pm0.12, \tag{15}$$

$$\frac{\mathrm{Br}(X(3872)\to\gamma\psi(2S))}{\mathrm{Br}(X(3872)\to\pi^+\pi^- J/\psi)} = 1.1\pm0.4. \tag{16}$$

Up to now, the widths and branch ratios of E1 decay channels have not been measured precisely. But the ratio can be drawn from Eqs. (15) and (16) [24]:

$$\frac{\mathrm{Br}(X(3872)\to\gamma\psi(2S))}{\mathrm{Br}(X(3872)\to\gamma J/\psi)} = 3.4\pm1.4. \tag{17}$$

With our results in Table 1 we get this ratio:

$$\frac{\mathrm{Br}(X(3872)\to\gamma\psi(2S))}{\mathrm{Br}(X(3872)\to\gamma J/\psi)} = 4.4, \tag{18}$$

which is very close to that of Eq. (17). In Refs. [7] and [8] this ratio is 1.3 and 6.1, respectively. We can see that models with charmonium assumption can predict this ratio correctly, while molecular model prediction is very small. In Ref. [36], a composite state which contains both molecular hadronic component and a $c\bar{c}$ component was considered. By changing the mixing angle, a correct ratio can be reached, but the decay widths are dramatically changed.

Recently Bhardwaj reported the new results of Belle on X(3872) at the QWG2010 conference [37], which is $\frac{\mathrm{Br}(X(3872)\to\gamma\psi(2S))}{\mathrm{Br}(X(3872)\to\gamma J/\psi)} < 2.1$. Our result with the $\chi_{c1}(2P)$ assignment is two times larger than this up-bound, so there is still long way to go to know the nature of X(3872).

The large ratio $\Gamma_{\psi'\gamma}/\Gamma_{J/\psi\gamma}$ can be understood by Figs. 2 and 3. For J/ψ, its wave function has no node, that is the numerical values of the wave function in the whole space are all positive (see Fig. 2), while for X(3872) and $\psi(2S)$, since they are the radial excited states of $\chi_{c1}(1P)$ and J/ψ, respectively, the wave functions have one node, that is, before the node the values of wave functions are positive, after the node the values are negative. So when we calculate the transition amplitude, we need to compute the overlap integral shown in Eq. (9). There exists dramatically cancellation in the overlap integral before the node and after the

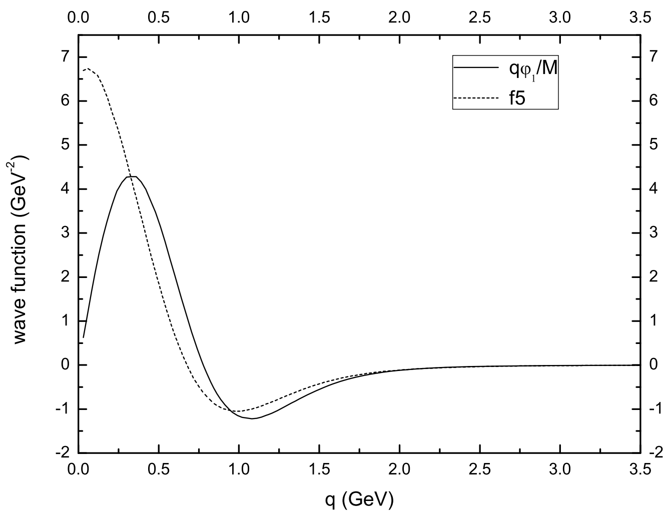

Fig. 3. Radial wave function $\frac{|\vec{q}|}{M}\varphi_1$ of X(3872) and f5 of $\psi(2S)$.

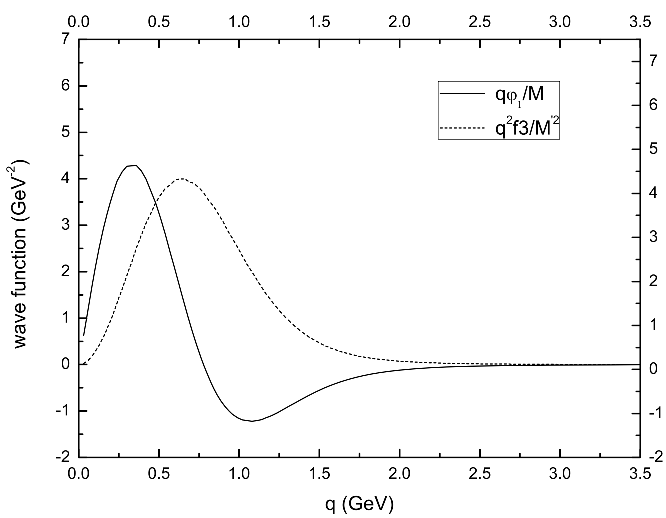

Fig. 4. Radial wave function $\frac{|\vec{q}|}{M}\varphi_1$ of X(3872) and $\frac{|\vec{q}|^2}{M'^2}$f3 of $\psi(3770)$.

node when we consider the decay X(3872) $\to \gamma J/\psi$ which can be seen from Fig. 2. This is the reason why the decay width 33.0 keV of this channel is much smaller (almost one order) than the width 306 keV of channel $\chi_{c1}(1P) \to \gamma J/\psi$. But for the decay X(3872) $\to \gamma\psi(2S)$, the two overlapping wave functions both have the node structures, see Fig. 3. So only in the region where one wave function is before the node, while the other is after the node, the overlapping integral gives negative contributions. And we can see from Fig. 3 that only a very small part of phase space will give negative contributions, so there is almost no cancellation when we calculate the transition amplitude. Finally we get a large decay width 146 keV for the channel of X(3872) $\to \gamma\psi(2S)$.

We have mentioned that the numerical values of $\Gamma^{\chi_{c1}(2P)}_{\psi(2S)\gamma}$ and $\Gamma^{\chi_{c1}(2P)}_{\psi(3770)\gamma}$ are very sensitive to the mass of X(3872) (see Table 1). This can be explained by different phase space and the node structure of wave functions. From Eq. (9) we can see that in the overlap integral the relative momentum $\vec{q}_\perp$ of final state has a shift $\alpha_2\vec{P}_f$ or $-\alpha_1\vec{P}_f$. When we change the mass of $\chi_{c1}(2P)$ from 3872 to 3923 MeV, the node position in the wave functions has almost no change, but the value of $|\vec{P}_f|$ will change obviously due to different phase space, for example, from 181 MeV to 230 MeV for $\chi_{c1}(2P) \to \psi(2S)\gamma$, which also changes the overlap integral. Finally we got much different values of decay width $\Gamma^{\chi_{c1}(2P)}_{\psi(2S)\gamma}$. Similar conclusion can be obtained for the case of $\chi_{c1}(2P) \to \gamma\psi(3770)$ (see Fig. 4). But for $\chi_{c1}(2P) \to \gamma J/\psi$, the relative small mass of J/ψ results in similar values of $|\vec{P}_f|$ for both cases, 695 MeV and 736 MeV, so the decay widths are similar for both cases. The two-body decay width can be written as: $\Gamma = \frac{1}{8\pi M}\frac{|\vec{P}_f|}{M}\bar{\Sigma}|T|^2$. So the pure change caused by the change of phase space is 23.8% for $\chi_{c1}(2P) \to \psi(2S)\gamma$ and 3.2% for $\chi_{c1}(2P) \to \gamma J/\psi$. From Table 1 the total change of the two processes is 24.7% and 0.9% respectively, which means most of the change for $\chi_{c1}(2P) \to \psi(2S)\gamma$ comes from phase space while for $\chi_{c1}(2P) \to \gamma J/\psi$ the larger contribution comes from the change of matrix element.

Table 2
E1 decay widths of $\chi_{b1}(1P)$ and $\chi_{b1}(2P)$.

Ref.	$\Gamma^{\chi_{b1}(1P)}_{\Upsilon(1S)\gamma}$ (keV)	$\Gamma^{\chi_{b1}(2P)}_{\Upsilon(1S)\gamma}$ (keV)	$\Gamma^{\chi_{b1}(2P)}_{\Upsilon(2S)\gamma}$ (keV)
This work	30.0	5.65	15.8
Kwong and Rosner [38]	32.8	9.31	15.9
Ebert et al. [39]	36.6	7.49	14.7
Fazio [9]	107		

Using the same method, we also calculated the radiative E1 decay widths of $\chi_{b1}(1P)$ and $\chi_{b1}(2P)$, and we show the results predicted by our method and other models in Table 2. One can see that the decay width $\Gamma(\chi_{b1}(1P) \to \gamma\Upsilon(1S)) = 30.0$ keV calculated by our method is about 3 times smaller than that of Refs. [9], but close to the values in Ref. [38] and Ref. [39], which are 32.8 keV and 36.6 keV, respectively. There are still no experimental data of these radiative decay widths, however, ratios are available. Particle Data Group [34] has listed the branching ratios: $\mathrm{Br}(\chi_{b1}(1P) \to \gamma\Upsilon(1S)) = (35 \pm 8) \times 10^{-2}$, $\mathrm{Br}(\chi_{b1}(2P) \to \gamma\Upsilon(1S)) = (8.5 \pm 1.3) \times 10^{-2}$, $\mathrm{Br}(\chi_{b1}(2P) \to \gamma\Upsilon(2S)) = (21 \pm 4) \times 10^{-2}$, so from this experimental data, we can get the ratio [9]:

$$\frac{\mathrm{Br}(\chi_{b1}(2P) \to \gamma\Upsilon(2S))}{\mathrm{Br}(\chi_{b1}(2P) \to \gamma\Upsilon(1S))} = 2.5 \pm 0.6. \tag{19}$$

Our result is

$$\frac{\mathrm{Br}(\chi_{b1}(2P) \to \gamma\Upsilon(2S))}{\mathrm{Br}(\chi_{b1}(2P) \to \gamma\Upsilon(1S))} = 2.8. \tag{20}$$

One can see that it's agreeable with the experimental value. The full decay widths of $\chi_{b1}(1P)$ and $\chi_{b1}(2P)$ can be estimated by the branching ratios and our predicted decay widths. The results are: $\Gamma_{\chi_{b1}(1P)} \sim 85.7$ keV, and $\Gamma_{\chi_{b1}(2P)} \sim 66.5$ keV.

In conclusion, we first calculated the radiative E1 decay width of $\chi_{c1}(1P)$. The excellent agreement between our result and experimental value shows that this method we used is good to deal with the charmonium radiative decays. Then with the traditional radial excited charmonium state $\chi_{c1}(2P)$ assignment for X(3872) we calculated the radiative E1 decay widths of this particle, $\Gamma(\mathrm{X}(3872) \to \gamma J/\psi) = 33.0$ keV, $\Gamma(\mathrm{X}(3872) \to \gamma\psi(2S)) = 146$ keV and $\Gamma(\mathrm{X}(3872) \to \gamma\psi(3770)) = 7.09$ keV. The value of $\Gamma_{\psi'\gamma}/\Gamma_{\psi\gamma}$ is 4.4, which is consistent with experimental result by BaBar, but is larger than the up-bound reported by Belle recently.

We also estimated the radiative E1 decay widths of the bottomonia states $\chi_{b1}(1P)$ and $\chi_{b1}(2P)$. Results are $\Gamma(\chi_{b1}(1P) \to \gamma\Upsilon(1S)) = 30.0$ keV, $\Gamma(\chi_{b1}(2P) \to \gamma\Upsilon(1S)) = 5.65$ keV, and $\Gamma(\chi_{b1}(2P) \to \gamma\Upsilon(2S)) = 15.8$ keV. The predicted ratio $\Gamma_{\Upsilon'\gamma}/\Gamma_{\Upsilon\gamma}$ of $\chi_{b1}(2P)$ is consistent with experimental data. The full decay widths of $\Gamma(\chi_{b1}(1P)) = 85.7$ keV and $\Gamma(\chi_{b1}(2P)) = 66.5$ keV (by the channel $\chi_{b1}(2P) \to \gamma\Upsilon(1S)$) are also estimated.

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