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Metastable spontaneous breaking of $N = 2$ supersymmetry

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ABSTRACT

We show that contrary to the common lore it is possible to spontaneously break $N = 2$ supersymmetry even in simple theories without constant Fayet–Iliopoulos terms. We consider the most general $N = 2$ supersymmetric theory with one hypermultiplet and one vector multiplet without Fayet–Iliopoulos terms, and show that metastable supersymmetry breaking vacua can arise if both the hyper-Kähler and the special-Kähler geometries are suitably curved. We then also prove that while all the scalars can be massive, the lightest one is always lighter than the vector boson. Finally, we argue that these results also directly imply that metastable de Sitter vacua can exist in $N = 2$ supergravity theories with Abelian gaugings and no Fayet–Iliopoulos terms, again contrary to common lore, at least if the cosmological constant is sufficiently large.

1. Introduction

It is by now well understood that the difficulty of achieving metastability for vacua leading to spontaneous supersymmetry breaking has a simple and universal origin related to Goldstone's theorem applied to supersymmetry. Indeed, since the Goldstino fermion must be massless, its sGoldstini scalar superpartners have masses that are entirely controlled by supersymmetry breaking effects and cannot be adjusted through supersymmetric mass terms. More precisely, it turns out that the average mass of these sGoldstini is entirely controlled by the geometry of the scalar manifold and the data of the local gauge symmetries, if present [1–3] (see also [4]). This fact is true not only in rigid supersymmetry but also in local supersymmetry, and its consequences have already been extensively investigated in a number of situations [5,6]. The general outcome is that one can infer a simple upper bound on the mass of the lightest scalar, which depends only on the geometric data of the theory and is increasingly stringent in theories with increasing number of supercharges. In some special classes of theories, this upper bound is vanishing or even negative, and therefore results in no-go theorems forbidding metastable spontaneous supersymmetry breaking [7,8].

In $N = 1$ theories, the situation is quite simple and clear, especially in the rigid limit [9]. In theories with only chiral multiplets, the upper bound on the lightest mass is controlled by the sectional curvature of the scalar manifold along the supersymmetry breaking direction. This implies that in renormalizable theories with flat geometry, there are always two massless scalars (corresponding to the so-called pseudo-moduli of O'Raifeartaigh models), while in non-renormalizable non-linear sigma-models all the scalars can be massive if the sectional curvature can be positive. In theories involving also vector multiplets, the situation is qualitatively similar but the vector multiplets can give quantitatively important effects. As a result, all the scalars can in general be massive, already in renormalizable theories (corresponding to the absence of pseudo-moduli in gauged O'Raifeartaigh models) and clearly also in non-renormalizable ones, even without Fayet–Iliopoulos terms.

In $N = 2$ theories, the situation is more interesting and less clear, even in the rigid limit [10]. In theories with only hypermultiplets, supersymmetry breaking stationary points are possible only for curved geometries and the upper bound on the lightest mass happens to always vanish, as a consequence of the structure of the sectional curvature of hyper-Kähler manifolds. This implies that there is always at least one tachyonic scalar, and thus that the vacuum cannot be metastable. In theories with only Abelian vector multiplets, a similar result holds true, and again it is impossible to get metastable supersymmetry breaking vacua. On the other hand, in more general theories with both hyper- and vector multiplets, only partial results concerning the possible metastability of supersymmetry breaking vacua exist. One general result in this direction has been presented in [13], where it was shown that for theories admitting an $SU(2)_R$ symmetry and a supercurrent conservation law involving a linear superconformal anomaly multiplet, it is impossible to construct a consistent non-linear realization of $N = 2$ supersymmetry. This suggests that in this class of theories there should be an unavoidable obstruction against having a non-supersymmetric stationary point at all or more plausibly against it to be metastable, originating from the presence of the

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$SU(2)_R$ symmetry or the existence of the linear superconformal anomaly multiplet. The common lore is that in order to spontaneously break supersymmetry, one needs constant Fayet–Iliopoulos terms, which spoil the $SU(2)_R$ symmetry in the vector multiplet sector (see [11,12] for examples). However, the $SU(2)_R$ symmetry can also be spoiled by the lack of isometries on the scalar manifold in the hypermultiplet sector, and this might provide an alternative to Fayet–Iliopoulos terms.

The aim of this Letter is to assess whether the possibility of having metastable supersymmetry breaking in $N=2$ theories is really linked to the presence of Fayet–Iliopoulos terms. For this we shall study in full generality the simplest class of such theories for which no-go theorems based on the sGoldstino masses do not exist so far, namely theories involving just one hypermultiplet and one vector multiplet with an Abelian gauge symmetry.

2. General setup

Let us consider the most general $N=2$ supersymmetric theory involving only one hypermultiplet and one vector multiplet without Fayet–Iliopoulos terms. This has the form of a gauged non-linear sigma-model on a target space that is the product of an arbitrary four-dimensional hyper-Kähler manifold admitting a triholomorphic isometry and an arbitrary two-dimensional special-Kähler manifold. The bosonic part of the action, describing the four real scalars q^u belonging to the hypermultiplet and the complex scalar z plus the real vector A_μ belonging to the vector multiplet, is given by [14–21] (see [10] for a recent review on the rigid case):

$$\mathcal{L} = -\frac{1}{4}\rho F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}\theta F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{2}g_{uv}D_\mu q^u D^\mu q^v - g_{z\bar{z}}\partial_\mu z\partial^\mu\bar{z} - V. \tag{2.1}$$

In this expression $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$ and $D_\mu q^u = \partial_\mu q^u + k^u A_\mu$. Moreover, g_{uv} denotes the metric of the hyper-Kähler manifold, k^u a triholomorphic Killing vector on it and P^i the three associated Killing potentials. Similarly, $g_{z\bar{z}}$ denotes the metric of the special-Kähler manifold, while ρ and θ are the real and imaginary parts of the corresponding holomorphic gauge kinetic function, such that in particular $\rho = g_{z\bar{z}}$. Finally

$$V = g_{uv}k^u k^v|z|^2 + \frac{1}{2}\rho^{-1}|\vec{P}|^2. \tag{2.2}$$

Happily, it turns out that there exists a general local parametrization for the two kinds of manifolds that are involved in this construction, in terms of two harmonic functions f and l of three and two real variables, respectively. It is then possible to construct a general theory based on arbitrary choices for these two harmonic functions. The trivial choices of constant f and l correspond to flat spaces while less trivial choices of non-constant f and l correspond to curved spaces.

Any four-dimensional hyper-Kähler manifold admitting a triholomorphic isometry can be locally described with coordinates $q^u = x_i, t$ and a Ricci-flat metric of the Gibbons–Hawking form [22–24]:

$$ds^2 = g_{uv}\,dq^u dq^v = f\,d\vec{x}^2 + f^{-1}(dt + \vec{\omega}\cdot d\vec{x})^2. \tag{2.3}$$

This depends on a single real function $f = f(\vec{x})$ of the three variables x_i, which must be harmonic and therefore satisfies the three-dimensional Laplace equation:

$$\Delta f = 0. \tag{2.4}$$

The three functions ω_i are determined, modulo an irrelevant ambiguity, by the following equation, whose integrability is guaranteed by the Laplace equation:

$$\vec{\nabla}\times\vec{\omega} = \vec{\nabla} f. \tag{2.5}$$

The three closed Kähler forms, which satisfy $dJ_i = 0$ thanks to the above equation defining ω_i, are given by (see for instance [25])

$$J_i = (dt + \vec{\omega}\cdot d\vec{x})\wedge dx_i - \frac{1}{2}f\epsilon_{ijk}\,dx_j\wedge dx_k. \tag{2.6}$$

Finally, the isometry acts as a simple shift of the t coordinate by some real parameter ξ, and the associated Killing vector reads

$$k = \xi\,\partial_t. \tag{2.7}$$

In this parametrization, the components g_{uv} and g^{uv} of the metric and its inverse are easily worked out, and their positivity requires $f > 0$. The components $(J_i)_{uv}$ of the three Kähler forms are easily verified to satisfy the quaternionic algebra $(J_i)^u{}_w(J_j)^w{}_v = -\delta_{ij}\delta^u_v + \epsilon_{ijk}(J_k)^u{}_v$. Finally, it is also straightforward to verify that this Killing vector (2.7), whose only non-vanishing component is $k^t = \xi$, is triholomorphic, and that the corresponding Killing potentials P^i, defined by $\nabla_u P_i = -(J_i)_{uv}k^v$, read:

$$\vec{P} = \xi\vec{x}. \tag{2.8}$$

Any two-dimensional special-Kähler manifold can be locally described with special complex coordinate z and a metric of the following form:

$$ds^2 = 2g_{z\bar{z}}\,dz\,d\bar{z} = 2l\,|dz|^2. \tag{2.9}$$

This depends on a single real function $l = l(z,\bar{z})$ of the two variables $z,\bar{z}$, which must be a harmonic function corresponding to the real part of a holomorphic function related to the prepotential and therefore satisfies the two-dimensional Laplace equation:

$$\partial\bar{\partial}l = 0. \tag{2.10}$$

In this parametrization, the unique non-trivial components of the metric and its inverse are given by $g_{z\bar{z}} = l$ and $g^{z\bar{z}} = l^{-1}$. Positivity of the metric requires $l > 0$.

It is worth emphasizing that the above general constructions can also be obtained in an algebraic way, using superfields. In the hyper-Kähler case, one can consider an $N=2$ single tensor multiplet, which consists of a linear multiplet L plus a chiral multiplet Q from the $N=1$ perspective and automatically incorporates a shift symmetry [26] (see also [27]). The most general $N=2$ kinetic Lagrangian for such a multiplet is then obtained from a potential $H = H(L, Q, \bar{Q})$ which must be a harmonic function: $H_{LL} + H_{Q\bar{Q}} = 0$. After switching to a description in terms of four real scalars, one then finds a Gibbons–Hawking space, with H_{LL} mapping to the harmonic function f and $\mathrm{Re}\,H_{LQ}$, $\mathrm{Im}\,H_{LQ}$ mapping to the two non-trivial components of $\vec{\omega}$ (see for instance Appendix C of [28]). In the special-Kähler case, one can use an $N=2$ vector multiplet, which consists of a chiral multiplet Φ plus a vector multiplet V from the $N=1$ perspective. The most general $N=2$ kinetic Lagrangian for such a multiplet involves a potential $F = F(\Phi)$ which must be a holomorphic function: $F_{\bar{\Phi}} = 0$. Keeping complex coordinates, one then directly finds the special-Kähler space in the above-described form, with $\mathrm{Im}\,F_{\Phi\Phi}$ mapping to the harmonic function l.

Summarizing, with the above local parametrization of the two components of the scalar manifold, the data defining the model are the following:

$$g_{uv}=\begin{pmatrix} f+f^{-1}\omega_1^2 & f^{-1}\omega_1\omega_2 & f^{-1}\omega_1\omega_3 & f^{-1}\omega_1 \\ f^{-1}\omega_2\omega_1 & f+f^{-1}\omega_2^2 & f^{-1}\omega_2\omega_3 & f^{-1}\omega_2 \\ f^{-1}\omega_3\omega_1 & f^{-1}\omega_3\omega_2 & f+f^{-1}\omega_3^2 & f^{-1}\omega_3 \\ f^{-1}\omega_1 & f^{-1}\omega_2 & f^{-1}\omega_3 & f^{-1} \end{pmatrix},$$

$$k^u=\begin{pmatrix}0\\0\\0\\ \xi\end{pmatrix}, \tag{2.11}$$

$$g_{z\bar z}=l, \qquad \rho=l, \tag{2.12}$$

$$V=\xi^2\left[f^{-1}|z|^2+\frac{1}{2}l^{-1}|\vec x|^2\right]. \tag{2.13}$$

3. Vacua and masses

Let us now look for Poincaré invariant vacuum states of the above theory, defined by constant expectation values for the six independent scalar fields $q^I=x_i,t,z,\bar z$. To start, we compute the first derivative V_I and find that $V_t=0$ while

$$V_i=\xi^2\left[-f^{-2}|z|^2f_i+l^{-1}x_i\right], \tag{3.14}$$

$$V_z=\xi^2\left[-\frac{1}{2}l^{-2}|\vec x|^2 l_z+f^{-1}\bar z\right]. \tag{3.15}$$

For the matrix of second derivatives $V_{I\bar J}$, we instead find $V_{tt}=0$ and $V_{it}=0$ while

$$V_{ij}=\xi^2\left[-f^{-2}|z|^2\left(f_{ij}-2f^{-1}f_if_j\right)+l^{-1}\delta_{ij}\right], \tag{3.16}$$

$$V_{z\bar z}=\xi^2\left[-\frac{1}{2}l^{-2}|\vec x|^2\left(l_{z\bar z}-2l^{-1}|l_z|^2\right)+f^{-1}\right], \tag{3.17}$$

$$V_{zz}=\xi^2\left[-\frac{1}{2}l^{-2}|\vec x|^2\left(l_{zz}-2l^{-1}l_z^2\right)\right], \tag{3.18}$$

$$V_{iz}=\xi^2\left[-f^{-2}\bar z f_i-l^{-2}x_il_z\right]. \tag{3.19}$$

Finally, we also need to compute the vielbein $e_I{}^P$ that allows us to locally trivialize the metric as $g_{I\bar J}=e_I{}^P\delta_{P\bar Q}(e^\dagger)^{\bar Q}{}_{\bar J}$ and thus canonically normalize the scalar fields. One finds a block diagonal result given by:

$$e_i{}^p=\begin{pmatrix} f^{1/2} & 0 & 0 & f^{-1/2}\omega_1 \\ 0 & f^{1/2} & 0 & f^{-1/2}\omega_2 \\ 0 & 0 & f^{1/2} & f^{-1/2}\omega_3 \\ 0 & 0 & 0 & f^{-1/2}\end{pmatrix}, \tag{3.20}$$

$$e_z{}^z=l^{1/2}. \tag{3.21}$$

The possible vacua correspond to the stationary points of the potential V. The stationarity conditions $V_I=0$ determining them are easy to analyze. We see that whenever f or l are constant and at least one of the factors of the scalar manifold is flat, stationarity implies vanishing values for all the fields and unbroken supersymmetry with vanishing vacuum energy. To get non-trivial supersymmetry-breaking stationary points, we thus need both of the functions f and l to be non-trivial and thus both factors of the scalar manifold to be curved. In that case the value of the fields is non-vanishing and the stationarity conditions imply that:

$$f_i=f^2l^{-1}|z|^{-2}x_i, \tag{3.22}$$

$$l_z=2l^2f^{-1}|\vec x|^{-2}\bar z. \tag{3.23}$$

Using these results we can then simplify the unnormalized mass matrix $V_{I\bar J}$, and finally compute the physical mass matrix associated to canonically normalized fields as $m^2_{I\bar J}=(e^{-1})_I{}^PV_{P\bar Q}(e^{-1\dagger})^{\bar Q}{}_{\bar J}$.

Although the form of $e_I{}^P$ depends on ω_i in the entries related to the would-be Goldstone mode t, the final result for $m^2_{I\bar J}$ does not depend on ω_i. One finds that $m^2_{tt}=0$ and $m^2_{it}=0$ while

$$m^2_{ij}=\xi^2\left[-f^{-3}|z|^2f_{ij}+2l^{-2}|z|^{-2}x_ix_j+f^{-1}l^{-1}\delta_{ij}\right], \tag{3.24}$$

$$m^2_{z\bar z}=\xi^2\left[-\frac{1}{2}l^{-3}|\vec x|^2l_{z\bar z}+4f^{-2}|\vec x|^{-2}|z|^2+f^{-1}l^{-1}\right], \tag{3.25}$$

$$m^2_{zz}=\xi^2\left[-\frac{1}{2}l^{-3}|\vec x|^2l_{zz}+4f^{-2}|\vec x|^{-2}\bar z^2\right], \tag{3.26}$$

$$m^2_{iz}=\xi^2\left[-f^{-1/2}l^{-3/2}|z|^{-2}\bar zx_i-2f^{-3/2}l^{-1/2}|\vec x|^{-2}\bar zx_i\right]. \tag{3.27}$$

The unnormalized mass of the vector field A_μ can be read off from the kinetic term of the hypermultiplet scalars and is given by $g_{uv}k^uk^v=\xi^2f^{-1}$. One then has to rescale this by $\rho^{-1}=l^{-1}$ to get the physical mass for the canonically normalized vector, finding

$$m_A^2=\xi^2f^{-1}l^{-1}. \tag{3.28}$$

A convenient way of parametrizing the above results is to introduce an angle θ that controls the relative orientation of the supersymmetry breaking direction between the hyper and the vector sectors. To do so, we consider the ratio of the two contributions in V and define at the vacuum point:

$$\tan^2\theta=\frac{1}{2}fl^{-1}|z|^{-2}|\vec x|^2. \tag{3.29}$$

Let us also introduce the direction v_i to which the vacuum point corresponds in the hypermultiplet field subspace, and similar the phase φ defined by the vacuum point in the vector multiplet field subspace, namely:

$$v_i=\frac{x_i}{|\vec x|}, \qquad \varphi=\arg z. \tag{3.30}$$

We then parametrize the overall scale of the fields at the vacuum point by an energy scale Λ defined as:

$$\Lambda^2=l|z|^2+\frac{1}{2}f|\vec x|^2. \tag{3.31}$$

In this way, the values of the fields are parametrized as:

$$x_i=\sqrt{2}f^{-1/2}\Lambda\sin\theta v_i, \qquad z=l^{-1/2}\Lambda\cos\theta e^{i\varphi}. \tag{3.32}$$

In addition, let us introduce the following dimensionless parameters associated to the second derivatives of the functions f and l:

$$a_{ij}=f^{-1}|\vec x|^2f_{ij}, \qquad b_{z\bar z}=l^{-1}|z|^2l_{z\bar z}, \qquad b_{zz}=l^{-1}z^2l_{zz}. \tag{3.33}$$

In this parametrization, the scalar masses $m^2_{I\bar J}$ can then be rewritten in the following very simple form:

$$m^2_{ij}=\left[\delta_{ij}+4\tan^2\theta v_iv_j-\frac{1}{2}\cot^2\theta a_{ij}\right]m_A^2, \tag{3.34}$$

$$m^2_{z\bar z}=\left[1+2\cot^2\theta-\tan^2\theta b_{z\bar z}\right]m_A^2, \tag{3.35}$$

$$m^2_{zz}=\left[2\cot^2\theta-\tan^2\theta b_{zz}\right]e^{-2i\varphi}m_A^2, \tag{3.36}$$

$$m^2_{iz}=\left[-\sqrt{2}(\cot\theta+\tan\theta)v_i\right]e^{-i\varphi}m_A^2. \tag{3.37}$$

Notice also that the vacuum energy is related to the vector mass and the scale defined by the expectation values of the fields:

$$V=\Lambda^2m_A^2. \tag{3.38}$$

4. Bounds on the scalar masses

We would now like to understand what kind of values can be achieved for the scalar masses m_i^2 corresponding to the eigenvalues of the mass matrix $m_{I\bar{J}}^2$. To this aim, we shall take the point of view that we choose some definite point corresponding to some values of $\vec{x}$ and z to be a priori the vacuum point, and then scan over all the possible forms of the functions f and l in the neighborhood of such a point. The condition that the chosen point should be a stationary point of V fixes the values of the first derivatives f_i and l_z. But the values of the functions f and l themselves as well as those of their second derivatives f_{ij} and $l_{z\bar{z}}$, l_{zz} are then arbitrary, except for the harmonicity constraints $\delta^{ij} f_{ij} = 0$ and $l_{z\bar{z}} = 0$. One may then scan over the two real parameters f and l and the seven independent real parameters among the f_{ij} and $l_{z\bar{z}}$, l_{zz}, and see what kind of masses one can achieve. In terms of the parametrization introduced at the end of previous section, this means in particular that we can scan over all the possible values of m_A^2, which controls the overall scale of the scalar masses, and θ, a_{ij}, $b_{z\bar{z}}$, b_{zz} which control instead the detailed form of the scalar mass matrix, with the only constraints being that:

$$\delta^{ij} a_{ij} = 0, \qquad b_{z\bar{z}} = 0. \tag{4.39}$$

To get an idea of whether it is possible or not to make all the eigenvalues positive, we may now look at the average values of the three blocks of the mass matrix, and reduce the original $(4+2)$-dimensional matrix to a simpler $(1+1)$-dimensional averaged matrix. More precisely, taking into account that we already know that there is one null eigenvalue in the hyper sector corresponding to the unphysical would-be Goldstone mode t absorbed by A_μ in a Higgs mechanism, let us look at

$$m_{\rm hh}^2 = \frac{1}{3}\delta^{ij} m_{ij}^2, \qquad m_{\rm vv}^2 = m_{z\bar{z}}^2, \qquad m_{\rm hv}^2 = \sqrt{\frac{1}{3}\delta^{ij} m_{iz}^2 m_{\bar{j}\bar{z}}^2}. \tag{4.40}$$

After a straightforward computation and using the constraints (4.39) imposed by the three-dimensional and two-dimensional Laplace equations satisfied by the functions f and l, one finds:

$$m_{\rm hh}^2 = \left[1 + \frac{4}{3}\tan^2\theta\right] m_A^2, \tag{4.41}$$

$$m_{\rm vv}^2 = \left[1 + 2\cot^2\theta\right] m_A^2, \tag{4.42}$$

$$m_{\rm hv}^2 = \left[\sqrt{\frac{2}{3}}(\tan\theta + \cot\theta)\right] m_A^2. \tag{4.43}$$

We see that as a result of the constraints imposed by $N = 2$ supersymmetry, and in particular (4.39), these average blocks are almost completely fixed, the only leftover parameter being the angle θ controlling the relative strength of the hyper- and vector multiplet sectors in the supersymmetry breaking process.

The first, qualitative information that we can extract from the knowledge of the above averaged blocks concerns the sign of the eigenvalues m_i^2. Some simple linear algebra shows that the full six-dimensional mass matrix m_{IJ}^2 can be positive definite only if the two-dimensional averaged mass matrix is also positive definite. This is the case if $m_{\rm hh}^2 > 0$, $m_{\rm vv}^2 > 0$ and $m_{\rm hh}^2 m_{\rm vv}^2 - m_{\rm hv}^4 > 0$. It is straightforward to check that all these three conditions are always satisfied by the expressions (4.41), (4.42) and (4.43), and this for any possible value of the angle θ. This suggests that it is a priori possible to adjust the parameters a_{ij} and $b_{z\bar{z}}$, b_{zz} subject to the constraints (4.39) in such a way to make all the eigenvalues m_i^2 positive.

The second, quantitative information that we can extract from the knowledge of the above averaged blocks concerns the size of the eigenvalues m_i^2. Since for a given θ all the averaged blocks are bounded, relative to the overall scale m_A^2, it is clear that the eigenvalues m_i^2 must also be bounded to lie in a certain interval, again relative to the overall scale m_A^2. More precisely, there must be an upper bound m_-^2 on how large the smallest m_i^2 can be, and also a lower bound m_+^2 on how small the largest m_i^2 can be. Through some simple linear algebra, one can show that these bounds $m_\pm^2$ are in fact simply the two eigenvalues of the two-dimensional matrix formed by the averaged mass blocks $m_{\rm hh}^2$, $m_{\rm vv}^2$ and $m_{\rm hv}^2$, and are thus given by:

$$m_\pm^2 = \frac{1}{2}\left(m_{\rm hh}^2 + m_{\rm vv}^2\right) \pm \sqrt{\frac{1}{4}\left(m_{\rm hh}^2 - m_{\rm vv}^2\right)^2 + m_{\rm hv}^4}. \tag{4.44}$$

Using the fact that $m_{\rm hh}^2 > 0$, $m_{\rm vv}^2 > 0$ and $m_{\rm hh}^2 m_{\rm vv}^2 - m_{\rm hv}^4 > 0$, one can then infer the following bounds, which can be derived by studying the necessary conditions for the matrix $m_{IJ}^2 - m_\pm^2 \delta_{IJ}$ to be negative or positive definite obtained after averaging and reducing to a two-dimensional matrix:

$$\min\{m_i^2\} \leqslant m_-^2 \leqslant \min\{m_{\rm hh}^2, m_{\rm vv}^2\}, \tag{4.45}$$

$$\max\{m_i^2\} \geqslant m_+^2 \geqslant \max\{m_{\rm hh}^2, m_{\rm vv}^2\}. \tag{4.46}$$

A simple computation shows that the quantities $m_\pm^2$ are given by

$$m_\pm^2 = \left[1 + \cot^2\theta + \frac{2}{3}\tan^2\theta \pm \sqrt{\frac{2}{3}\cot^2\theta + \cot^4\theta + \frac{2}{3}\tan^2\theta + \frac{4}{9}\tan^4\theta}\right] m_A^2. \tag{4.47}$$

One can easily verify that $m_+^2 > 0$ and $m_-^2 > 0$ for any value of θ, as already implied by the analysis of the previous paragraph. One can, however, also study more quantitatively what happens when θ is varied. m_-^2 starts from a local minimum for $\theta = 0$ with value $\frac{2}{3}m_A^2$, then goes through an absolute maximum for $\theta = \frac{\pi}{4}$ with value m_A^2 and finally reaches a local minimum for $\theta = \frac{\pi}{2}$ with value $\frac{1}{2}m_A^2$. m_+^2 starts from a maximum for $\theta = 0$ with infinite value, goes through a minimum at $\theta \simeq 0.83$ (close to $\theta = \frac{\pi}{4}$) with value $4.27 m_A^2$ (close to $\frac{13}{3}m_A^2$) and then reaches again a maximum at $\theta = \frac{\pi}{2}$ with infinite value. We then conclude that:

$$\min\{m_i^2\} \leqslant m_A^2, \qquad \max\{m_i^2\} \gtrsim 4.27 m_A^2. \tag{4.48}$$

This result suggests that it should not only be possible to make all the mass eigenvalues m_i^2 positive, but actually all greater than or equal to m_A^2. In other words, it should be possible to achieve a genuinely metastable supersymmetry breaking vacuum with sizable masses for scalar fluctuations by adjusting the parameters of the model.

Note that the cases of theories with just one hypermultiplet or just one vector multiplet can formally be obtained as special cases of the more general situation studied here, by taking the limits $l \to +\infty$, $z \to z_0$ and $f \to +\infty$, $\vec{x} \to \vec{x}_0$, respectively. In those two limits one thus gets $\theta \to 0$ and $\theta \to \frac{\pi}{2}$, respectively, but also $m_A \to 0$ and $\Lambda \to +\infty$ with $V \to$ finite, in both cases. One then correctly recovers the vanishing upper bound for the smallest mass that leads to a no-go theorem in those cases [7,8], as a consequence of the vanishing of the trace of the relevant mass matrix block.

5. Existence of metastable vacua

The final question that we need to address is whether the full five-dimensional non-trivial part of the mass matrix $m_{I\bar{J}}^2$ defined by Eqs. (3.34)–(3.37) can really be made positive definite by a suitable choice of the parameters a_{ij} and $b_{z\bar{z}}, b_{zz}$, subject to the constraints (4.39). We saw that the necessary conditions for this to be possible that come from the study of the two-dimensional matrix obtained by averaging over each of the hyper and vector subsectors are satisfied for any value of θ, so the question is more precisely whether for any given θ and v_i, φ it is possible or not to make all the m_i^2 positive through a suitable choice of the parameters a_{ij} and $b_{z\bar{z}}, b_{zz}$. The answer to this question is yes, and in fact it turns out that one can always saturate the bounds defined by m_+^2 or m_-^2 by suitably adjusting a_{ij} and $b_{z\bar{z}}, b_{zz}$. An intuitive argument for this is as follows. Due to the restriction that $\delta^{ij} a_{ij} = 0$ and $b_{z\bar{z}} = 0$, the average of the eigenvalues of the two diagonal blocks of the mass matrix are fixed and cannot be changed. Moreover, the off diagonal block is also fixed and independent of the above parameters. As a result, there is certain amount of level-repulsion between the two groups of eigenvalues that the diagonal blocks would have on their own, and the average value of all the eigenvalues of the full matrix is also fixed. It is then clear that changing a_{ij} and $b_{\alpha\beta}$ can only affect the spread of the eigenvalues around what is dictated by the two-dimensional matrix obtained by averaging over the directions defining each subsector, and as a consequence it is possible to choose a_{ij} and $b_{z\bar{z}}, b_{zz}$ in such a way as to saturate the bounds defined by m_+^2 or m_-^2.

Let us illustrate the above statement with an explicit example of metastable supersymmetry breaking vacuum. For simplicity, we choose the vacuum point to be defined by values of the fields in the maximally symmetric direction such that

$$\theta = \frac{\pi}{4}, \qquad v_i = \sqrt{\frac{1}{3}}, \qquad \varphi = 0. \tag{5.49}$$

The values of the functions f and l at such a point are arbitrary and are mapped to arbitrary values for the scales m_A and Λ. The first derivatives of the functions f and l at such a point are instead completely fixed by the requirement that the stationarity conditions should be satisfied. Finally, the second derivatives of the functions f and l at such a point are arbitrary and are mapped to arbitrary values for the dimensionless parameters a_{ij} and $b_{z\bar{z}}, b_{zz}$. For generic values of the latter, we then get the following structure for the three blocks of the mass matrix:

$$m_{ij}^2 = \begin{pmatrix} \frac{7}{3} - \frac{1}{2}a_{11} & \frac{4}{3} - \frac{1}{2}a_{12} & \frac{4}{3} - \frac{1}{2}a_{13} \\ \frac{4}{3} - \frac{1}{2}a_{12} & \frac{7}{3} - \frac{1}{2}a_{22} & \frac{4}{3} - \frac{1}{2}a_{23} \\ \frac{4}{3} - \frac{1}{2}a_{13} & \frac{4}{3} - \frac{1}{2}a_{23} & \frac{7}{3} - \frac{1}{2}a_{33} \end{pmatrix} m_A^2, \tag{5.50}$$

$$m_{\alpha\bar{\beta}}^2 = \begin{pmatrix} 3 - b_{z\bar{z}} & 2 - b_{zz} \\ 2 - b_{\bar{z}\bar{z}} & 3 - b_{z\bar{z}} \end{pmatrix} m_A^2, \tag{5.51}$$

$$m_{i\bar{\beta}}^2 = \begin{pmatrix} -\sqrt{\frac{8}{3}} & -\sqrt{\frac{8}{3}} \\ -\sqrt{\frac{8}{3}} & -\sqrt{\frac{8}{3}} \\ -\sqrt{\frac{8}{3}} & -\sqrt{\frac{8}{3}} \end{pmatrix} m_A^2. \tag{5.52}$$

Recalling the constraints $\delta^{ij} a_{ij} = 0$ and $b_{z\bar{z}} = 0$, in this case we have

$$m_{\rm hh}^2 = \frac{7}{3} m_A^2, \qquad m_{\rm vv}^2 = 3 m_A^2, \qquad m_{\rm hv}^2 = \sqrt{\frac{8}{3}} m_A^2, \tag{5.53}$$

and

$$m_-^2 = m_A^2, \qquad m_+^2 = \frac{13}{3} m_A^2. \tag{5.54}$$

We can finally make some definite choice for the parameters a_{ij} and $b_{z\bar{z}}, b_{zz}$ and compute the mass eigenvalues m_i^2 explicitly. As expected, by choosing appropriate values for these parameters it is possible to make all the m_i^2 positive, but at least one of these is always lighter that $m_-^2 = m_A^2$ and one is always heavier than $m_+^2 = \frac{13}{3} m_A^2$. A very simple working example of the above type is obtained by making the following choice of parameters:

$$a_{ij} = 0, \qquad b_{z\bar{z}}, b_{zz} = 0. \tag{5.55}$$

In this case, the five non-trivial eigenvalues of the full mass matrix can be computed analytically and are found to be:

$$m_i^2 = \{1, 1, 1, 1, 9\} m_A^2. \tag{5.56}$$

These are all positive, and the vacuum is thus metastable. We moreover see that in this simple example the upper bound on the lightest mass is saturated.

6. Generalization to supergravity

The results that we have derived here in the context of rigid supersymmetry can be generalized to local supersymmetry. To do so, one needs to consider a generic supergravity theory with one hyper- and one vector multiplet. The hypermultiplet sector is described by a four-dimensional quaternionic-Kähler manifold with negative Ricci curvature set by the Planck scale, and this must again admit a triholomorphic isometry. Fortunately, the most general space with these properties is also known and goes under the name of the Przanowski–Tod space [29,30]. This is the Ricci-curved generalization of the Gibbons–Hawking space, and is based on a function of three variables satisfying the non-linear three-dimensional Toda equation, rather than the linear three-dimensional Laplace equation. The vector multiplet sector is instead described by a two-dimensional local-special-Kähler manifold. This can also be described in a completely general way. The new feature is again a deformation in the structure of the curvature by effects linked to the Planck scale.

A detailed analysis of the structure of the mass matrix, the bounds that can be put on its eigenvalues and the constraints on the possibility of achieving metastable de Sitter vacua in this kind of theories can be performed by using the technology described in [31] and will be presented elsewhere [32], along with some explicit examples. It is however clear that the existence of metastable supersymmetry-breaking vacua in the rigid limit directly implies also the existence of metastable supersymmetry breaking de Sitter vacua in supergravity. This shows that Fayet–Iliopoulos terms and non-Abelian gauge symmetries are not necessary ingredients to achieve metastable supersymmetry breaking even within supergravity, again contrary to the common lore in the literature and in particular the claim of [33]. The only subtle point concerns the values of the cosmological constant V and the gravitino mass $m_{3/2}$ that can be compatible with metastability. It is obvious that in the limit where $V \gg m_{3/2}^2 M_{\rm Pl}^2$, gravitational effects on supersymmetry breaking and on the masses are small and it must therefore be possible to achieve metastable de Sitter vacua exactly as in the rigid case. On the other hand, in the limit where $V \ll m_{3/2}^2 M_{\rm Pl}^2$, gravitational effects on supersymmetry breaking and on the masses are sizable and the possibility of achieving metastable de Sitter vacua must be carefully reinvestigated. The quantitative question that one then has to deal with consists in understanding for which range of values of the dimensionless ratio $V/(m_{3/2}^2 M_{Pl}^2)$ metastability can be achieved. This is a particularly relevant question, since small and large values of the above parameter are needed in applications to particle physics and inflation, respectively.

7. Conclusions

In this Letter, we have demonstrated that metastable spontaneous breaking of global $N = 2$ supersymmetry is possible even in very simple theories that do not involve Fayet–Iliopoulos terms or non-Abelian gaugings. We then argued that the same qualitative result also holds true in the presence of gravity, although the relative size of the cosmological constant and the gravitino mass allowing for metastable vacua might be constrained and remains to be analyzed.

To conclude, let us compare our findings with the general statement in [13] that $N = 2$ theories admitting an $SU(2)_R$ symmetry and a supercurrent conservation law based on a linear superconformal anomaly multiplet cannot spontaneously break supersymmetry. Our examples of $N = 2$ theories possessing metastable supersymmetry-breaking vacua have a priori no $SU(2)_R$ symmetry, since the generic Gibbons–Hawking manifolds we considered do not admit an isometry group that could contain this. We believe that this is the reason why they evade the result of [13]. As a consistency check of this interpretation, we verified that in the special models built on spaces with a larger isometry group, such as flat space and the Eguchi–Hanson manifold, there are in fact no supersymmetry-breaking vacua. Nevertheless, it would be interesting to understand whether or not our models admit a linear superconformal anomaly multiplet coping with the potential problems emphasized in [34,35].

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