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Generalized phantom energy

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Abstract

We examine cosmological models with generalized phantom energy (GPE). Generalized phantom energy satisfies the supernegative equation of state, but its evolution with the scale factor is generally independent, i.e., not determined by its equation of state. The requirement of general covariance makes the gravitational constant time-dependent. It is found that a large class of distinct GPE models with different evolution of generalized phantom energy density and gravitational constant, but the same equation of state of GPE have the same evolution of the scale factor of the universe in the distant future. The time dependence of the equation of state parameter determines whether the universe will end in a de Sitter-like phase or diverge in finite time with the accompanying “big rip” effect on the bound structures.

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Results of recent cosmological observations, such as distant supernovae of type Ia (SNIa) [1] and cosmic microwave background radiation (CMBR) [2], have dramatically altered our perception of the dynamics and composition of the universe and reshaped the landscape of standard cosmology [3]. The universe seems to be in the phase of accelerated expansion, which started at a relatively small redshift, $z \sim 1$. This acceleration is attributed to a new form of matter, usually referred to as *dark energy*, the nature of which is still not definitely established. Observations indicate that the energy density of the universe is very close to its critical density where dark energy presently accounts for approximately 2/3 of the total energy density, while the remaining 1/3 comes predominantly from *dark matter*, another unidentified component of

the universe. The most prominent and studied candidates for the title of dark energy are the cosmological constant [4–6] (together with its dynamical variants, such as renormalization group running cosmological constant [7–9]), quintessence [10] and the Chaplygin gas [11].

The majority of dark energy models share a common constraint on their equation of state (p_d and ρ_d represent pressure and energy density of dark energy, respectively)

$$p_d = w\rho_d, \quad (1)$$

where $w \geq -1$. Such a constraint is, however, not justified by the unbiased fits to the data of cosmological observations. Moreover, the allowed interval for the parameter of the equation of state extends significantly into the region with $w < -1$. The use of observational data on CMBR, large scale structure (LSS), SNIa and Hubble parameter measurements from the

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Hubble Space Telescope (HST) under the assumption of the redshift independent parameter w give the restriction $-1.38 < w < -0.82$ at the 95% confidence level [12]. Therefore, a possible supernegative equation of state of dark energy deserves due attention.

A new type of dark energy with the equation of state characterized by $w < -1$ was proposed in [13] and named *phantom energy*. Phantom energy is considered to be separate from other components of the universe and its energy–momentum tensor is conserved separately. In such a setting, the equation of state of dark energy determines its evolution with the scale factor a . The supernegative nature of the equation of state of the phantom energy leads to the growing energy density of phantom energy $\rho_d \sim a^{-3(1+w)}$, for a constant parameter w . The cosmological dynamics of the universe with such a phantom energy component possesses many interesting features [14]. The growth of the energy density of phantom energy drives the scale factor of the universe to infinity in finite time. The increasing negative pressure of phantom energy leads to the unbounding of all bound structures in the universe. This dramatic and picturesque scenario of the cosmic doomsday was appropriately named “big rip”. The formulation of microscopic models for phantom energy [15] relies on the machinery developed in quintessence models, namely the evolution of the scalar field in a suitably chosen potential. However, the description of phantom energy may require an introduction of some non-standard alterations, e.g., the negative kinetic term of the scalar field. Detailed considerations of the Lagrangians describing phantom energy show that in some cases the universe with phantom energy ends in a “big rip”, while in others it asymptotically approaches the de Sitter expansion.

In this Letter, we consider models with generalized phantom energy (GPE). First, we set up a more general model of the evolution of the universe with phantom energy. We assume that there are two components of the universe: the dark energy component (which will have the phantom energy characteristics), and the “ordinary” matter component with the respective energy densities ρ_d and ρ_m . The “ordinary” matter is taken to satisfy the equation of state

$$p_m = \gamma \rho_m, \quad (2)$$

where $\gamma \geq 0$. Furthermore, we assume that the energy–momentum tensor of the “ordinary” matter is conserved

$$T_{m;\nu}^{\mu\nu} = 0. \quad (3)$$

The equation given above ensures that the parameter of the equation of state governs the evolution of the “ordinary” matter energy density, i.e.,

$$\rho_m = \rho_{m,0} \left(\frac{a}{a_0} \right)^{-3(1+\gamma)}. \quad (4)$$

Dark energy has the equation of state

$$p_d = w \rho_d, \quad (5)$$

where w generally depends on time explicitly or implicitly, via explicit dependence on some other time-dependent quantity, such as the scale factor a . In the case of dark energy, we allow the possibility of non-conservation of the energy–momentum tensor, i.e.,

$$T_{d;\nu}^{\mu\nu} \neq 0. \quad (6)$$

Thus, the evolution of the dark energy density is not determined by the parameter from its equation of state.

With the properties of the components of the universe defined, we can specify the laws of its evolution. We start from the Einstein equation

$$G^{\mu\nu} = -8\pi G T^{\mu\nu}, \quad (7)$$

where $G^{\mu\nu}$ is the Einstein tensor and $T^{\mu\nu} = T_d^{\mu\nu} + T_m^{\mu\nu}$ is the total energy–momentum tensor. The reconciliation of the requirement of the general covariance of (7) and the non-conservation relation (6) is possible with the promotion of gravitational constant G into a space–time dependent quantity. This change can be interpreted as a modification of the dynamics of general relativity. This additional dynamics is effectively described by the introduction of space–time dependence of G . We consider the models where G is a function of time only, $G = G(t)$. Models with the time-dependent G were extensively studied in the framework of the time-dependent cosmological term $\Lambda(t)$ [16]. The covariant derivative of (7) then implies

$$(G(t)T^{\mu\nu})_{;\nu} = 0. \quad (8)$$

This equation can be rewritten in the form

$$d(G(\rho_m + \rho_d)a^3) = -G(p_m + p_d) da^3. \quad (9)$$

Combining the evolution laws (4) and (9) and introducing $w \equiv -1 + \kappa$ (where κ describes the deviation from the parameter of the equation of state inherent to the cosmological constant) we arrive at

$$\dot{G}(\rho_m + \rho_d) + G\dot{\rho}_d + 3\kappa HG\rho_d = 0. \quad (10)$$

Here $H = \dot{a}/a$ is the Hubble parameter, while dots denote time derivatives. Eq. (10) clearly shows the generality of the model. In the case of the constant G , we recover the standard equation of conservation of $T_d^{\mu\nu}$. Eq. (10) shows that the time evolution of G is the result of two competing effects. Namely, for dark energy with growing energy density, the second term in (10) causes the decrease of G , while for negative κ , the third term in (10) increases G with time.

Finally, Friedmann equations for the evolution of the scale factor complete the set of evolution equations (4) and (10)

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi}{3}G(\rho_m + \rho_d), \quad (11)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}G(\rho_m + \rho_d + 3p_m + 3p_d). \quad (12)$$

The set of Eqs. (4), (10) and (11) reveals that we have essentially two independent equations for three dynamical quantities G , ρ_d and a (assuming that κ is the function of these quantities and time). Without a more specific identification of the dynamics of G or ρ_d , it is not possible to solve the aforementioned set of equations. However, as we show below, with mild assumptions about the evolution of dark energy with the scale factor, it is possible to obtain information on the future evolution of the universe for general G and ρ_d satisfying the equations given above.

Next, we introduce the concept of generalized phantom energy (GPE). Generalized phantom energy is the form of dark energy satisfying the equation of state (1) with the non-conserved energy–momentum tensor (6) and the following two properties:

- (a) GPE energy density is a non-decreasing function of the scale factor,
- (b) GPE equation of state satisfies $\kappa \leq 0$.

We further examine the future evolution of the universe. In the sufficiently distant future we have $\rho_m \ll \rho_d$ and ρ_m can be neglected in the evolution

equations. Eqs. (10) and (11) thus become

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi}{3}G\rho_d \quad (13)$$

and

$$\frac{d}{dt}(G\rho_d) + 3\kappa HG\rho_d = 0. \quad (14)$$

Furthermore, from Eq. (14), we obtain

$$\frac{d(G\rho_d)}{G\rho_d} = -3\kappa \frac{da}{a}. \quad (15)$$

As the condition $-\kappa \geq 0$ is satisfied by assumption (b), we obtain

$$G\rho_d \geq (G\rho_d)_0. \quad (16)$$

Therefore, as $G\rho_d$ is a growing function in an expanding universe, for large a we can disregard the term k/a^2 in Eq. (13). For the flat universe, this approximation is exact, while for the closed or the open universe, this approximation is applicable in the sufficiently distant future.

Finally, we end up with the following two equations for the dynamics of the universe in the distant future:

$$H^2 = \frac{8\pi}{3}(G\rho_d), \quad (17)$$

$$\frac{d}{dt}(G\rho_d) + 3\kappa H(G\rho_d) = 0. \quad (18)$$

By combining Eqs. (17) and (18), we obtain an equation for the evolution of the Hubble parameter H with time

$$\frac{dH}{dt} + \frac{3}{2}\kappa H^2 = 0, \quad (19)$$

with the solution

$$H(t) = \frac{H(t_0)}{1 + (3/2)H(t_0) \int_{t_0}^t \kappa(t') dt'}. \quad (20)$$

Once we have found the expression for the evolution of the Hubble parameter, it is easy to obtain an expression for the evolution of the scale factor a

$$a(t) = a(t_0) \exp\left(\int_{t_0}^t dt' \frac{H(t_0)}{1 + (3/2)H(t_0) \int_{t_0}^{t'} \kappa(t'') dt''}\right). \quad (21)$$

General solutions (20) and (21) exhibit some interesting features. The evolution of the universe in the

sufficiently distant future is governed only by the parameter of the equation of state of dark energy. The precise form of the growth of ρ_d with the scale factor a is irrelevant in this limit. This implies that the entire class of models with different functional forms of ρ_d and G , obeying the same equation of state, show the same behaviour in the sufficiently distant future. Therefore, we can divide all GPE models with the characteristics specified above into classes with the same equation of state.

An important question regarding the fate of the universe is whether, for a particular class of generalized phantom energy models, a and H diverge in finite time or reach infinite values only in infinite time. For the Hubble parameter H , the answer is straightforward. There will be no divergence of H in finite time if the denominator of the expression on the right-hand side of (20) remains positive for all times. This leads to the condition

$$\int_{t_0}^{\infty} (-\kappa(t')) dt' < \frac{2}{3H(t_0)}. \quad (22)$$

As in this case there is no singularity in $H(t)$ in finite time, the scale factor $a(t)$ also does not diverge in finite time. In order to have the convergence of the integral $\int_{t_0}^{\infty} (-\kappa(t')) dt'$ required in (22), the function $\kappa(t)$ has to tend to zero at asymptotically large times. Therefore, for generalized phantom energy which exhibits no divergence of H or a in finite time, the parameter of the equation of state approaches -1 , i.e., generalized phantom energy approaches the time-dependent cosmological term.

In the case when the condition (22) is not satisfied, the Hubble parameter H diverges in finite time t . From Friedmann equations we have

$$\dot{a} = Ha, \quad (23)$$

$$\ddot{a} = \left(1 - \frac{3}{2}\kappa\right)H^2a. \quad (24)$$

These expressions indicate that, when H diverges in finite time t , both \dot{a} and \ddot{a} diverge as well, so the scale factor a cannot remain finite, but diverges in finite time t as well.

From the general expressions (20) and (21), we can obtain evolution laws for the conceptually simple, but

important case [13]

$$\kappa(t) = -\kappa_0. \quad (25)$$

With such a choice for the parameter of the equation of state of generalized phantom energy, we have the following evolution laws:

$$H(t) = \frac{H(t_0)}{1 - (3/2)H(t_0)\kappa_0(t - t_0)}, \quad (26)$$

$$a(t) = a(t_0) \left(1 - \frac{3}{2}H(t_0)\kappa_0(t - t_0)\right)^{-2/(3\kappa_0)}. \quad (27)$$

These solutions clearly show the onset of the divergence in H and a . The universe with generalized phantom energy with the constant parameter of the equation of state evolves to infinity in finite time.

Comparison with the case of the “standard” phantom energy [13,14] shows that, for the same parameter of the equation of state $\kappa(t)$, the scale factor follows the same evolution law. Given the fact that the parameter of the equation of state does not determine the scaling with a , and that G is variable in the framework of generalized phantom energy, it is by no means obvious that coincidence of this sort should exist. However, from Eq. (10), we readily see that for the case of constant G , we recover the equation of evolution for the “standard” phantom energy. As far as the evolution in the sufficiently distant future is concerned, the “standard” phantom energy model is just one instance of the class of generalized phantom energy models with the same function $\kappa(t)$.

Given the same evolution properties of the broad class of GPE models with the same $\kappa(t)$, it is natural to look at the destiny of bound structures, another peculiarity of phantom energy models [14]. The relevant quantity with respect to the stability of the bound structures is the analogue of the gravitational potential proportional to the quantity $G(\rho_d + 3p_d) = (-2 + 3\kappa)G\rho_d$. Eq. (17) shows that $G\rho_d \sim H^2$ and $G\rho_d$ grows with time. If the condition (22) is not satisfied, H and $G\rho_d$ diverge in finite time. Furthermore, as ρ_d grows with the scale factor, $G\rho_d$ certainly increases compared to G . For gravitationally bound systems, the GPE contribution of the order $\sim G(\rho_d + 3p_d)R^3$ (where R denotes the characteristic spatial scale of the bound system) overwhelms the “mass” contribution $\sim GM$ (M denotes the mass of the bound system). Gravitationally bound systems fall apart in finite time.

For the systems bound by electromagnetic or strong forces, mere growth of $G\rho_d$ ensures their unbounding at some finite time before the time at which scale factor goes to infinity. Consequently, all bound structures are unbound in finite times. The scenario of the “big rip” is present in generalized phantom energy models as well.

Finally, let us make some comments on fundamental aspects of the GPE model. As the gravitational constant $G(t)$ is time-dependent, the description of the gravitational sector in the GPE model represents a declination from the Einsteinian gravity. One important aspect is whether the scale factor a really describes the growth of length scales. One can raise two arguments in favour of the standard interpretation of the scale factor a . The first is that no intervention in the geometrical structure or interpretation of the left-hand side of Eq. (7) has been made. The other, more physical one, is that the density of “non-relativistic” matter scales as $\rho_m \sim a^{-3}$ in our GPE model, Eq. (4) with $\gamma = 0$. Given that no interaction (production or annihilation) of the “ordinary” matter component with other components is assumed, this fact establishes a as a natural measure of the growth of length scales.

In some theories with the time-dependent effective gravitational constant, such as scalar-tensor or non-minimally coupled scalar field theories, one can construct many mathematically equivalent theories using conformal transformations. It turns out that all these theories are not physically equivalent, i.e., some formulations are more physically viable than others (the Einstein frame formulation is more viable than the Jordan frame formulation) [17]. Generally, it might be of interest to consider conformally related models of GPE obtained by the transformation of the type $\tilde{g}_{\mu\nu} = f(G(t))g_{\mu\nu}$, where f is a suitably chosen function. However, the time variation of $G(t)$ in our model can be very general and includes possibilities to which requirements on the choice of the conformal frame do not necessarily apply. Some examples of such a variation are the renormalization group running of G [7–9] or the time variation of G emanating from extra dimensions [18].

In conclusion, in this Letter we have considered cosmological models with the time-dependent gravitational constant G and dark energy with the supernegative equation of state (phantom energy). Phantom en-

ergy is generalized in the sense that its equation of state does not determine its evolution with the scale factor a , i.e., GPE density becomes an independent function of the scale factor. The requirement of general covariance in this setting imposes conditions on the gravitational constant G which acquires time dependence. Investigation of future dynamics of the generalized phantom energy models with growing generalized phantom energy density and the parameter of the equation of state less than -1 exhibits some general properties. A large class of models with different evolutions of ρ_d and G , but the same equation of state of GPE, have the common law of the evolution of the scale factor a in the sufficiently distant future. The time dependence of the GPE parameter of the equation of state determines whether the universe evolves infinitely in a de Sitter regime or diverges in finite time. One would expect that bounds on the variation of G in the past epochs of the evolution of the universe would produce the most stringent constraints on the parameters of the GPE model. Therefore, it is important to point out that our main results qualitatively do not depend on the size of the parameter $|\kappa|$ or on the intensity of growth of ρ_d (of course, within classes of these parameters that satisfy or do not satisfy the condition (22)). For smaller parameter values and slower varying functions ρ_d and G , the onset of the general evolution (dependent only on κ) will come later. For instance, for constant and negative κ , but very small $|\kappa|$, the entire class of GPE models leads to the “big rip” event, but at very late times.

Clearly, the present accelerating phase of the evolution of the universe carries the seed of the possibly very dramatic future of our cosmos. Therefore, more precise observations of the past variation of ρ_d and G with time (redshift) will be able to unravel the fate of the universe.

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