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# Novel $NN$ interaction and the spectroscopy of light nuclei

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**Abstract**

Nucleon–nucleon ($NN$) phase shifts and the spectroscopy of $A \leqslant 6$ nuclei are successfully described by an inverse scattering potential that is separable with oscillator form factors.



Nucleon–nucleon ($NN$) potentials that describe available two-body data have a long and multi-faceted history. High precision fits have improved with time even as more precise experimental data have become available. Three-nucleon ($NNN$) potentials have a shorter history but are intensively investigated at the present time. Disparate foundations for these potentials, both $NN$ and $NNN$, have emerged. On the one hand, one sees the predominant meson-exchange potentials sometimes supplemented with phenomenological terms to achieve high accuracy in fitting $NN$ data (Bonn [1], Nijmegen [2], Argonne [3], Idaho [4], IS [5]) and $NNN$ data (Urbana [6,7], Illinois [8], Tucson–Melbourne [9,10]). On the other hand, one sees the emergence of potentials with ties to QCD which are either meson-free [11], or intertwined with meson-exchange theory [4,12].

All these potentials are being used, with unprecedented success, to explain a vast amount of data on light nuclei in quantum Monte Carlo approaches [7] and ab initio no-core shell model (NCSM) [13,14]. The overwhelming success of these efforts have led some to characterize these approaches as leading to a 'Standard Model' of non-relativistic nuclear physics.

Chief among the outstanding challenges is the computational intensity of using these $NN + NNN$ potentials within the presently available many-body methods. For this reason, most ab initio investigations have been limited to $A \leqslant 12$. The situation would be dramatically simpler if either the $NN$ potential alone would be sufficient or the potentials would couple less strongly between the low momentum and the high momentum degrees of freedom. If both simplifications

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Table 1
Non-zero matrix elements in $\hbar\omega = 40$ MeV units of the JISP6 matrices in the uncoupled partial waves

| $n$ | $V^l_{nn}$ | $V^l_{n,n+1} = V^l_{n+1,n}$ | $V^l_{n,n+2} = V^l_{n+2,n}$ | $n$ | $V^l_{nn}$ | $V^l_{n,n+1} = V^l_{n+1,n}$ | $V^l_{n,n+2} = V^l_{n+2,n}$ |
|---|---|---|---|---|---|---|---|
| $^1s_0$ partial wave | | | | $^3p_0$ partial wave | | | |
| 0 | −0.3708298354 | 0.1326630532 | | 0 | −0.1431645486 | 0.0207550691 | |
| 1 | −0.1488264739 | 0.0064481044 | | 1 | 0.0829881736 | −0.1200945062 | |
| 2 | 0.1528350732 | −0.1201935383 | | 2 | 0.3104470795 | −0.1161020719 | |
| 3 | 0.1871385321 | −0.0295044038 | | 3 | 0.0650449849 | 0.0136092039 | |
| 4 | −0.0055841242 | | | 4 | −0.0265550440 | | |
| $^1p_1$ partial wave | | | | $^3p_1$ partial wave | | | |
| 0 | 0.6310815765 | −0.2513829369 | 0.4133192379 | 0 | 0.2496797849 | −0.1647613526 | 0.1576028692 |
| 1 | −0.2933902473 | −0.1185398245 | | 1 | 0.0443279227 | −0.1766154808 | |
| 2 | 0.4541336329 | −0.2301860135 | | 2 | 0.5140992483 | −0.2757339299 | |
| 3 | 0.3480358376 | −0.0900432270 | | 3 | 0.4233249414 | −0.1082234804 | |
| 4 | 0.0492211782 | | | 4 | 0.0553972681 | | |
| $^1d_2$ partial wave | | | | $^3d_2$ partial wave | | | |
| 0 | −0.0406993900 | 0.0375316853 | | 0 | −0.6621132357 | 0.7597322690 | −0.5718515839 |
| 1 | −0.1117617458 | 0.0697916085 | | 1 | 0.1754482325 | 0.2736603208 | |
| 2 | −0.1349966182 | 0.0452650206 | | 2 | −0.2638801567 | 0.0860291227 | |
| 3 | −0.0312706313 | | | 3 | −0.0632310928 | | |
| $^1f_3$ partial wave | | | | $^3f_3$ partial wave | | | |
| 0 | 0.0194689323 | −0.0186312854 | | 0 | 0.0263262069 | −0.0142857575 | |
| 1 | 0.0835695955 | −0.0554882979 | | 1 | 0.0356744294 | −0.0167975664 | |
| 2 | 0.1068218756 | −0.0322733269 | | 2 | 0.0285435921 | −0.0082905860 | |
| 3 | 0.0210638602 | | | 3 | 0.0060369466 | | |

are obtained, the future for applications is far more promising.

In the present work, we derive and apply a new class of potentials that have a very limited connection with the two well-established lines of endeavor. We develop $J$-matrix inverse scattering potentials (JISP) that describe $NN$ data to high accuracy and, with the off-shell freedom that remains, we obtain excellent fits to the bound and resonance states of light nuclei up to $A = 6$. Our $NN$ off-shell freedom is sufficient to describe these limited data without the need for $NNN$ potentials. As an important side benefit, we find that these potentials lead to rapid convergence in the ab initio NCSM evaluations presented here. We hope that these potentials will open a fruitful path for evaluating heavier systems and spur the development of extensions to scattering problems.

Our $NN$ potentials have the same symmetries as the conventional $NN$ potentials mentioned above (without charge symmetry breaking at present), but are not constrained by meson exchange theory, by QCD or by locality. This does not mean our $NN$ potentials are inconsistent with those constraints, however.

By means of the $J$-matrix inverse scattering approach [15] we construct $NN$ potentials as matrices in an oscillator basis with $\hbar\omega = 40$ MeV using the Nijmegen $np$ phase shifts [16]. Following Ref. [15], we obtain inverse scattering tridiagonal potentials (ISTP) that are tridiagonal (quasi-tridiagonal) in uncoupled (coupled) partial waves. The dimension of the potential matrix is specified by the maximum value of $N = 2n + l$ and is referred to as an $N\hbar\omega$ potential. In order to improve the description of the phase shifts, we develop a $9\hbar\omega$-ISTP in odd waves instead of the $7\hbar\omega$-ISTP of Ref. [15]. We retain an $8\hbar\omega$-ISTP in the even partial waves. To generate a high quality description of the two-body data, we find these low values of $N$ require a $\hbar\omega$ around 40 MeV.

Next we perform various phase equivalent transformations (PETs) of the obtained ISTP. In the coupled $sd$ waves, we perform the same PET as in Ref. [15] but with different rotation angle $\vartheta = 11.3°$ to improve

Table 2
Non-zero matrix elements in $\hbar\omega = 40$ MeV units of the JISP6 matrix in the coupled waves

**$sd$ coupled waves**

$V^{ss}_{nn'}$ matrix elements

| $n$ | $V^{ss}_{nn}$ | $V^{ss}_{n,n+1} = V^{ss}_{n+1,n}$ |
|---|---|---|
| 0 | −0.5082740408 | 0.2141564466 |
| 1 | −0.2761680295 | 0.0809077357 |
| 2 | −0.0094738037 | −0.0518814431 |
| 3 | 0.1528737343 | −0.0551935898 |
| 4 | 0.0375479299 | |

$V^{dd}_{nn'}$ matrix elements

| $n$ | $V^{dd}_{nn}$ | $V^{dd}_{n,n+1} = V^{dd}_{n+1,n}$ |
|---|---|---|
| 0 | 0.0508783491 | −0.0941736495 |
| 1 | 0.3221264718 | −0.1788087936 |
| 2 | 0.3085166731 | −0.0930126048 |
| 3 | 0.0612000372 | |

$V^{sd}_{nn'}$ matrix elements

| $n$ | $V^{sd}_{n,n-1} = V^{ds}_{n-1,n}$ | $V^{sd}_{nn} = V^{ds}_{nn}$ | $V^{sd}_{n,n+1} = V^{ds}_{n+1,n}$ |
|---|---|---|---|
| 0 | | −0.4117713554 | 0.2057319827 |
| 1 | −0.0485163427 | −0.0604765857 | |
| 2 | 0.0680444970 | −0.0801871065 | |
| 3 | 0.0494005788 | −0.0202056462 | |
| 4 | −0.0015039981 | | |

**$pf$ coupled waves**

$V^{pp}_{nn'}$ matrix elements

| $n$ | $V^{pp}_{nn}$ | $V^{pp}_{n,n+1} = V^{pp}_{n+1,n}$ | $V^{pp}_{n,n+2} = V^{pp}_{n+2,n}$ |
|---|---|---|---|
| 0 | −0.2570527690 | 0.2152699222 | −0.1713320974 |
| 1 | 0.0359505315 | 0.1037844371 | |
| 2 | −0.2092212263 | 0.1032216796 | |
| 3 | −0.1515463440 | 0.0373299671 | |
| 4 | −0.0158782011 | | |

$V^{ff}_{nn'}$ matrix elements

| $n$ | $V^{ff}_{nn}$ | $V^{ff}_{n,n+1} = V^{ff}_{n+1,n}$ |
|---|---|---|
| 0 | −0.0198361174 | 0.0082926722 |
| 1 | −0.0100583238 | 0.0006286653 |
| 2 | 0.0016462025 | −0.0009737977 |
| 3 | 0.0003885043 | |

$V^{pf}_{nn'}$ matrix elements

| $n$ | $V^{pf}_{n,n-1} = V^{fp}_{n-1,n}$ | $V^{pf}_{nn} = V^{fp}_{nn}$ | $V^{pf}_{n,n+1} = V^{fp}_{n+1,n}$ |
|---|---|---|---|
| 0 | | 0.0181636995 | 0.0032996664 |
| 1 | −0.0261868986 | 0.0234783463 | |
| 2 | −0.0247575890 | 0.0237074386 | |
| 3 | −0.0147089068 | 0.0062712798 | |
| 4 | 0.0000246531 | | |

the description of the deuteron quadrupole moment $Q$. We then find improvement in $^3$H and $^4$He binding energies. We also perform similar PETs mixing lowest oscillator basis states in the $^3p_2$, $^3p_1$, $^3d_2$ and $^1p_1$ waves with the rotation angles of $\vartheta = +8^\circ, -6^\circ, +25^\circ$ and $-16^\circ$ respectively to improve the description of the $^6$Li spectrum. The obtained interaction fitted to the spectrum of $A = 6$ nuclei, is referred to as JISP6. The non-zero matrix elements of the JISP6 interaction are presented in Tables 1 and 2 (in $\hbar\omega = 40$ MeV units). We use the same conventions for the oscillator wave functions as in Ref. [15].

The deuteron properties provided by JISP6 are compared with those of other potentials in Table 3.

We perform calculations of light nuclei in the NCSM with JISP6 plus the Coulomb interaction between protons. To improve the convergence, we perform the Lee–Suzuki transformation to obtain a two-body effective interaction as is discussed in Ref. [14]. We obtain the effective interaction in a new basis ($\hbar\omega = 15$ MeV) within an $N_{\max}\hbar\omega$ model space where $N_{\max}$ signifies the many-body oscillator basis cutoff. The results of our NCSM calculations for binding energies of $^3$H, $^3$He (in the $14\hbar\omega$ model space), $^4$He (in the $12\hbar\omega$ model space), $^6$He and $^6$Li (in the $10\hbar\omega$ model space) nuclei are compared in Table 4 with the calculations in various approaches (Faddeev, Green's-function Monte Carlo (GFMC), NCSM) with realistic $NN$ (CD-Bonn, Nijmegen-I (NijmI), Nijmegen-II (NijmII), and Argonne (AV18 and AV8')) and $NNN$ (Urbana (UIX), Tucson–Melbourne (TM and TM'), and Illinois (IL2)) potentials. To give an estimate of the convergence of our calculations, we present the difference between the given result and the result obtained in the next smaller model space in parenthesis after our JISP6 results. It is seen that the convergence of our calculations is adequate.

The convergence patterns are also illustrated by Fig. 1 where we present the $\hbar\omega$ dependence of the $^6$Li ground state energy in comparison with the results of Ref. [19] obtained in NCSM with CD-Bonn interaction. The $\hbar\omega$ dependence with the JISP6 interaction is weaker over a wide interval of $\hbar\omega$ values. This is a signal that convergence is improved relative to CD-

Table 3
JISP6 deuteron property predictions in comparison with the ones obtained with various realistic potentials

| Potential | $E_d$, MeV | $d$ state probability, % | rms radius, fm | $Q$, fm$^2$ | As. norm. const. $\mathscr{A}_s$, fm$^{-1/2}$ | $\eta = \frac{\mathscr{A}_d}{\mathscr{A}_s}$ |
|---|---|---|---|---|---|---|
| JISP6 | −2.224575 | 4.1360 | 1.9647 | 0.2915 | 0.8629 | 0.0252 |
| Nijmegen-II | −2.224575 | 5.635 | 1.968 | 0.2707 | 0.8845 | 0.0252 |
| AV18 | −2.224575 | 5.76 | 1.967 | 0.270 | 0.8850 | 0.0250 |
| CD–Bonn | −2.224575 | 4.85 | 1.966 | 0.270 | 0.8846 | 0.0256 |
| Nature | −2.224575(9) | – | 1.971(6) | 0.2859(3) | 0.8846(9) | 0.0256(4) |

Table 4
The binding energies of $^3$H,$^3$He, $^4$He, $^6$He and $^6$Li nuclei obtained with JISP6 in NCSM with $\hbar\omega = 15$ MeV in comparison with the results obtained with modern $NN + NNN$ interaction models in various approaches

| Potential and model | $^3$H | $^3$He | $^4$He | $^6$He | $^6$Li |
|---|---|---|---|---|---|
| JISP6, NCSM | 8.461(5) | 7.751(3) | 28.611(41) | 29.24(17) | 31.48(27) |
| CD-Bonn+TM, Faddeev [17] | 8.480 | 7.734 | 29.15 | | |
| AV18+TM, Faddeev [17] | 8.476 | 7.756 | 28.84 | | |
| AV18 + TM′, Faddeev [17] | 8.444 | 7.728 | 28.36 | | |
| NijmI+TM, Faddeev [17] | 8.392 | 7.720 | 28.60 | | |
| NijmII+TM, Faddeev [17] | 8.386 | 7.720 | 28.54 | | |
| AV18+UIX, Faddeev [17] | 8.478 | 7.760 | 28.50 | | |
| AV18+UIX, GFMC [8] | 8.46(1) | 7.71(1) | 28.33(2) | 28.1(1) | 31.1(1) |
| AV18+IL2, GFMC [8] | 8.43(1) | 7.67(1) | 28.37(3) | 29.4(1) | 32.3(1) |
| AV8'+TM', NCSM [18] | | | | 28.189 | 31.036 |
| Nature | 8.48 | 7.72 | 28.30 | 29.269 | 31.995 |

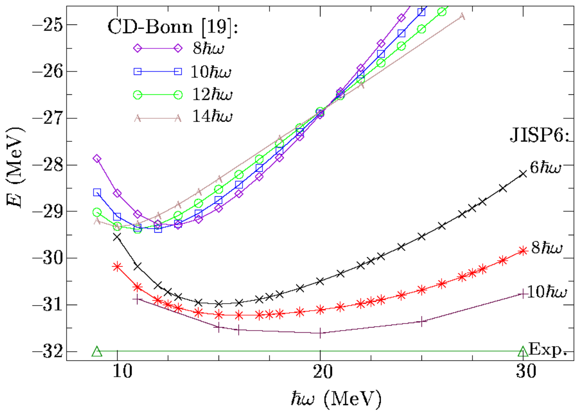


Fig. 1. The $\hbar\omega$ dependence of the $^6$Li ground state energy obtained with JISP6 interaction in comparison with the one obtained in NCSM with CD-Bonn potential [19].

Bonn. The variational principle cannot be applied to the NCSM calculations with effective interactions so the convergence may be either from above or below. However, we may surmise that the residual contributions of neglected three-body effective interactions is more significant in the CD-Bonn case.

The $\hbar\omega$ dependence of lighter nuclei is even weaker. That is why we present the results for all nuclei obtained with the same $\hbar\omega$ value. In this case the difference between ground state energies provide a consistent predictions for reaction $Q$ values.

Returning to the results presented in Tables 3 and 4, we see that the JISP6 interaction provides a realistic description of the ground states of light nuclei competitive with the quality of descriptions previously achieved with both $NN$ and $NNN$ forces.

This conclusion is supported by the spectra and ground state properties of $A = 6$ nuclei summarized in Table 5. We again present in parenthesis the difference between the given value and the result obtained in the next smaller model space. Note that the $^6$Li spectrum was found [18] to be sensitive to the presence of the $NNN$ force and a high-quality description of the $^6$Li spectrum seemed impossible without $NNN$ forces. It is seen that the $^6$Li spectrum is well-reproduced in our calculations and competitive with realistic $NN + NNN$ models. The most important difference with the experiment is the excitation energy of the $(1_2^+, 0)$ state. However $E_x(1_2^+, 0)$ goes down

Table 5
Excitation energies $E_x$ (in MeV) and ground state point-proton rms radii $r_p$ (in fm) and quadrupole moments $Q$ (in $e\,\mathrm{fm}^2$) of $A = 6$ nuclei

| Potential method | Nature | JISP6 NCSM, $10\hbar\omega$ | AV8'+TM' NCSM, $6\hbar\omega$ [18] | AV18+UIX GFMC [7,20] | AV18+IL2 GFMC [8,20] |
|---|---|---|---|---|---|
| $^6$Li | | | | | |
| $E_x(1_1^+, 0)$ | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| $r_p$ | 2.32(3) | 2.083(25) | 2.054 | 2.46(2) | 2.39(1) |
| $Q$ | −0.082(2) | −0.194(55) | −0.025 | −0.33(18) | −0.32(6) |
| $E_x(3^+, 0)$ | 2.186 | 2.102(4) | 2.471 | 2.8(1) | 2.2 |
| $E_x(0^+, 1)$ | 3.563 | 3.348(24) | 3.886 | 3.94(23) | 3.4 |
| $E_x(2^+, 0)$ | 4.312 | 4.642(2) | 5.010 | 4.0(1) | 4.2 |
| $E_x(2^+, 1)$ | 5.366 | 5.820(4) | 6.482 | | 5.5 |
| $E_x(1_2^+, 0)$ | 5.65 | 6.86(36) | 7.621 | 5.1(1) | 5.6 |
| $^6$He | | | | | |
| $E_x(0^+, 1)$ | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| $r_p$ | 1.912(18) | 1.694(3) | 1.707 | 1.95(1) | 1.91(1) |
| $E_x(2^+, 1)$ | 1.8 | 2.505(86) | 2.598 | 1.9(1) | 2.0 |

rapidly when the model space is increased and better results are anticipated in a larger model space.

The point–proton rms radius $r_p$ and the quadrupole moment $Q$ have a more prominent $\hbar\omega$ dependence than the binding energy. $\hbar\omega = 15$ MeV is not the optimal value for these observables in $A = 6$ nuclei and hence their convergence is not very good. The exponential extrapolation of the $^6$Li point–proton rms radius using the results obtained with different $\hbar\omega$ values results in the value of $r_p \approx 2.14$ fm. The $^6$Li quadrupole moment $Q$ is a recognized challenge due to a delicate cancellation between deuteron quadrupole moment and the $d$ wave component of the $\alpha$–$d$ relative wave function, various cluster model calculations cannot reproduce even the negative sign of $Q$. Our results for $Q$ are seen to be competitive with the ones obtained with $NN + NNN$ potentials.

We return to the underlying rationale for our approach and ask why it is conceivable that an $NN$ interaction alone may be competitive with the $NN+NNN$ potentials mentioned at the outset. That this is feasible may be appreciated from the theorem of Polyzou and Glöckle [21]. They have shown that changing the off-shell properties of two-body potentials is equivalent to adding many-body interactions. This theorem coupled with our limited results suggests that our inverse scattering $NN$ potential plus off-shell modifications is roughly equivalent, for the observables so far investigated, to the successful $NN + NNN$ potential models.

Clearly, more work will be needed to carry this to nuclei with $A \geqslant 7$ and see if the trend continues. Based on the results presented, the additional off-shell freedoms remaining may well serve to continue this line of fitting properties for some time. When it eventually breaks down, $NNN$ potentials may be needed.

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