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Mathematical Modeling of Dynamics of Multi-Lever Linkages

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Abstract

This paper addresses both modeling and dynamics identification of planar multi-lever linkages with changeable close loop whose output link moves vertically. The equation of motion is derived in a symbolic form by means of the application of coordinate partitioning method on the basis of the Lagrangian equation of the second kind. The equation of motion of the mechanical system obtained through the use of Lagrange-Euler notations takes into account Coriolis, centrifugal and gravitational forces. Numerical experiment has confirmed the validity and correctness of the theoretical results and made it possible to determinate the oscillation amplitude and to evaluate the stiffness characteristics of the mechanical system.

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Keywords: Linkages; dynamics; Lagrange – Euler equation; Denavita-Hartenberg matrix transformations; motion equation.

1. Introduction

Over the past few years, linkages have received great attention in both theoretical researches and practical applications. Compared to classical four-bar mechanisms, they provide a number of advantages including the possibility of implementing complex laws of motion as well as the provision of optimal terms for force transmission.

The problems of the structural synthesis and the kinematics of the planar linkage (see fig. 1) are solved in ref. [10] and [13]. The discussed mechanism has one degree of freedom and consists of eleven rotational and one translational kinematic pairs of fifth class. Due to the changeable close loop DFECB in its design, the mechanism has a compact structure and a large operational space. The input link AD is a hydraulic cylinder. The movement of the rod leads to plane-parallel displacement of the output link PQL.

The suggested linkage can be used as the actuator of a lift machine. Due to its special design of the mechanism provides an increased the lateral and longitudinal stiffness and carries out the lifting action with only one hydraulic drive, and as a result, its control system will be less complex [10].

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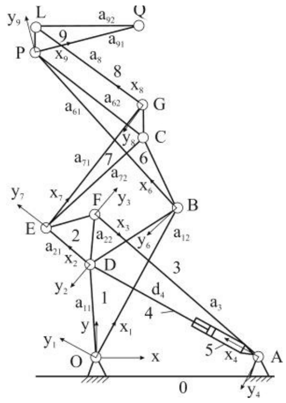

Fig. 1. Design of the multi-lever linkage.

Most modern machineries and mechanisms perform under the intensive conditions (e.g. accelerated to high velocities and operating under considerable loads), which requires application of exclusive standards to their strength, durability and precision operation [1]. Estimation of dynamic phenomena in machineries allows for the reduction in metal and energy consumption, improves controllability and as a result increases the quality of machinery operation. It is well known that the dynamics of a linkage is highly depended on its structure, geometric parameters, mechanical properties (e.g. inertia, elasticity and others) of its components and units as well as the type and characteristics of the drive, and, therefore, on its dynamic parameters. To achieve high performance in the operational space as well as to provide high accuracy during the fast motions, the following two conditions have to be satisfied:

- The dynamics model must be designed in such a way so as to insure the solution of the inverse dynamics problem in a real-time mode,
- The dynamic parameters of the mechanism including masses and their individual centers, moments of inertia and friction coefficients have to be estimated as accurately as possible in order to reduce errors in control algorithm developed on the bases of the model with feedforward or feedback.

The mechanism under consideration belongs to the holonomic mechanical systems with retaining stationary links. In general, the following assumptions are taking into account when the dynamics of such system is described:

- all components of the mechanical system are rigid bodies with distributed masses;
- kinematical pairs and transmission gears supposed to be ideal (the friction is negligible).

Such type of models reflects the properties of many executive mechanisms with reasonable accuracy and are widely used in analytical mechanics [2]. Different methods can be applied to dynamic modelling of the mechanism in the hypothesis of rigid links. Traditionally it is used to obtain the dynamic equations the follows: the Newton-Euler method [3, 4], the Lagrange method [5, 15], and the variation principle (the principle of virtual work [6, 7] as well as the principle of virtual power [8]). The equation of motion of the holonomic mechanical system under the action of potential forces can be formulated in other ways, for instance by making use of Hamilton method in which the unknown variable equals the sum of kinetic and potential energies [9, 14].

For simplification reasons, the values of some parameters, e.g. solid body parameters, can be obtained from manufacturers or derived with the help of computer-aided design software (CAD). Since the technical literature presents only particular cases of modelling applications of a mechanical system, specific to certain problem. However, in the case of complex linkage including many mechanical components, it is difficult or even impossible to evaluate the above parameters.

In this paper the dynamics model of the mechanical system is based on the formalism of Euler and Lagrange. This method can also be used for systems containing deformable elements (for example string, elastic rods and so on) if inertia can be neglected. The main subject of the article is the dynamic analysis of the multi-level planar linkage with changeable close loop while the major emphases is made on the study of the type and amplitude of oscillations and the estimation of the dynamic parameters of the mechanism. These parameters play the major role in both, the accuracy of the dynamic simulation and in the design of the advanced model-based control algorithm.

2. Dynamical model

The kinematic characteristics of the mechanism in question represent the initial data for the dynamics and are found by using Denavita-Hartenberg matrix transformations and are described in the articles [11].

The original form of Euler-Lagrange equation is as follows:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}=Q_i \quad , \tag{1}$$

In formula (1) i =1, … , 9 is a number of a joint. L stands for Lagrange function, which equals the difference between the kinematical K and potential P energies of the mechanical system. q_i are generalized coordinates; $\dot{q}_i$ is the first derivative of the generalized coordinates by time. Q_i are generalized forces (or moments) created in i^{th} joint during the given motion of i^{th} link.

2.1. Kinetic Energy

It is well known that the kinematic energy of a mechanism depends on the masses and the accelerations of its joints, the latter are defined as the first derivative of the velocity of a joint by time or the second derivative of the position of a point by time. Any point on the plane can be described with homogeneous coordinates of the radius vector r^i_i in the coordinate system of i^{th} link:

$$r_i^i=(x_i, y_i, z_i, 1)^T \quad . \tag{2}$$

Symbol r_i denotes a radius vector of a point in the reference coordinate system connected with the base of the linkage under consideration. Using the matrix of homogeneous transformation in Denavita-Hartenberg notation $A_i^0 = A_1^0 \cdot A_2^1 \cdot \ldots \cdot A_i^{i-1}$ the position in the fixed coordinate systems is describes with the expression:

$$r_i = A_i^0 \cdot r_i^i \quad , \tag{3}$$

Assume that the links of the mechanism are absolutely solid bodies, and any point with coordinates r_i^i has zero velocity in i^{th} local coordinate system, the velocity of the point in the basic coordinate system xOy is determined with the following formula:

$$v_i^0 = dr_i^0/dt = d\left(A_i^0 \cdot r_i^i\right)/dt \tag{4}$$

Note that r_i^i equals zero because the centers of the mechanism joints coincided with origins of the local coordinate systems.

The derivative of the homogeneous transformation matrix A_i^0 by generalized coordinate q_i (for rotational and translational kinematic pairs) is calculated with help of the matrix H_i that depends on the type of a joint:

$$H_i^{rat} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad H_i^{tran} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Thus, the result of the differentiation is as follows:

$$\partial A_i^0/\partial q_i = \begin{cases} A_1^0 A_2^1 \ldots A_{j-1}^{j-2} H_j A_j^{j-1} \ldots A_i^{i-1}, \; for\; j \le i, \\ 0, \; for\; j > i. \end{cases} \tag{5}$$

Expression (5) describes the displacement of the points of i[th] link caused by the relative movement in j[th] joint. If the partial derivative $\partial A_i^{i-1}/\partial q_i$ is denoted as U_{ij} the equation (5) can be rearranged as follows:

$$U_{ij} = \begin{cases} A_{j-1}^0 H_j . A_j^{i-1}, \; npu \; j \le i, \\ 0, \; npu \;\; j > i. \end{cases} \tag{6}$$

Matrix U_{ij} characterizes the velocity of i[th] link in the basic coordinate system and does not depend on the mass distribution in this link. Analytical dependence (4) can be transformed with the help of notation (6):

$$v_i^0 = \left[\sum U_{ij} \cdot \dot{q}_i \right] \cdot r_i^i \tag{7}$$

In accordance with the kinematic and kinetostatic parameters described in ref. [11, 13] and analytical expression (6) for the first rotational joint – rod OBD, the velocity of joint D in inertial coordinate system xOy, caused by the displacement of hydraulic cylinder rod AD will be equal to $U_{10} = \partial A_1^0 / \partial q_1 = H_1 A_1^0$.

Similarly, the dependences for the velocity of other links of the mechanism are developed the following way:

- point E of triangular link DEF: $U_{20} = A_1^0 H_1 A_2^1$;
- point F of rocker FA: $U_{30} = A_1^0 A_2^1 H_1 A_3^2$;
- joint P of triangular link BPC: $U_{60} = A_{11}^0 H_1 A_6^1$;
- point G of triangular link ECG: $U_{70} = A_1^0 A_2^1 H_1 A_7^2$;
- joint L of link GL: $U_{80} = A_1^0 A_2^1 A_7^2 H_1 A_8^7$;
- point Q of triangular link PLQ: $U_{90} = A_{11}^0 A_6^1 H_1 A_9^6$.

The analog velocity of the i[th] point in the basic coordinate system is obtained by substituting these expressions in formula (7).

Kinetic energy dK_i of i[th] link of the mechanism with mass dm is determined as follows:

$$dK_i = 1/2\left(x_i^{\,2} + y_i^{\,2} + z_i^{\,2}\right)dm = 1/2 \cdot Tr\left(\dot{T}_i J_i \dot{T}_i^T\right)dm \, , \tag{8}$$

Where x_i, y_i, z_i are the coordinates of the center of mass of i[th] link in inertial coordinate system xOy. $\dot{T}_i$ is the first derivative of the resulting homogeneous transformation $T_i = A_1^0 \cdot A_2^1 \cdot \ldots \cdot A_i^{i-1}$ by the generalized coordinate. T is named the matrix transpose operation. Tr is the notation of trace of the matrix which are equaled to the sum of its diagonal elements. And the inertia tensor of i[th] link is the matrix of the form:

$$J_i = \begin{bmatrix} J_{XX}^{(i)} & J_{XY}^{(i)} & J_{XZ}^{(i)} & m_i x_i^* \\ J_{YX}^{(i)} & J_{YY}^{(i)} & J_{YZ}^{(i)} & m_i y_i^* \\ J_{ZX}^{(i)} & J_{ZY}^{(i)} & J_{ZZ}^{(i)} & m_i z_i^* \\ m_i x_i^* & m_i y_i^* & m_i z_i^* & m_i \end{bmatrix}$$

Here m_i is a mass of i[th] moveable joint; x^*_i, y^*_i, z^*_i are coordinates of the center of mass of i[th] link in the local coordinate system. $J^{(i)}_{xx}$, $J^{(i)}_{yy}$, $J^{(i)}_{zz}$ are the axial inertia moments of i^{th} link relate to their own axes and $J^{(i)}_{XY}$, $J^{(i)}_{YX}$, $J^{(i)}_{ZY}$, $J^{(i)}_{ZX}$, $J^{(i)}_{YZ}$, $J^{(i)}_{XZ}$ are the centrifugal moments of inertia.

Substituting the velocity expression v_i of the i[th] moveable joint in formula (8) and adding the obtained analytical dependences for all links of the linkage the equation of the kinetic energy of the mechanical system can be derived:

$$K = \sum K_i = 0.5 \cdot \sum \sum \sum \left[Tr\left(U_{ip} J_i U_{ir}^T \dot{q}_p \dot{q}_r \right) \right] \tag{9}$$

The inertia tensor depends on the mass distribution in i[th] link of the mechanism and does not depend on the position and velocity of links. In the study of dynamics of the mechanism, we used the following assumptions:

- axis *Oz* of the inertial reference system *xOy* coincides with one of the major axes of the inertia tensor;
- the unit vectors of other two axes *Ox* and *Oy* are collinear and co-directional with the unit vectors of the other two principal axes respectively.

In accordance with the analytic dependence (9), the kinetic energy of the kinematic link OBD is defined as follows:

$$K_1 = 0.5 \cdot \sum Tr\left(\dot{T}_1 J_1 \dot{T}_1^T\right)\dot{q}_1^2 = 0.5\left(2J_Z^5 + m_1 a_{10}^2\right)\dot{q}_1^2$$

where T_1 is a product of U_{11} and $\dot{q}_1$.

Similarly, we have deduced the equations for the third, sixth, seventh, eighth and ninth movable joints of the linkage:

$$K_2 = 0.5\left(2J_Z^5 + m_1 a_{12}^2\right)\dot{q}_1^2,\ K_3 = 0.5\left(2J_Z^3 + m_3 a_3^2\right)\dot{q}_1^2,\ K_6 = 0.5\left(2J_Z^6 + m_6 a_6^2\right)\dot{q}_1^2,\ K_7 = 0.5\left(2J_Z^7 + m_7 a_7^2\right)\dot{q}_1^2,$$

$$K_8 = 0.5\left(2J_Z^8 + m_8 a_8^2\right)\dot{q}_1^2 \text{ and } K_9 = 0.5\left(2J_Z^9 + m_9 a_9^2\right)\dot{q}_1^2,$$

where lengths a_3, a_6, a_7, a_8 and a_9 are calculated in the following way: $a_3 = a_{12} + a_{22}$, $a_6 = a_{11} + a_{61}$, $a_7 = a_{10} + a_{21} + a_{71}$, $a_8 = a_{10} + a_{21} + a_{71} + a_8$, $a_9 = a_{11} + a_{61} + a_{91}$.

The expression for the calculation of the kinetic energy of the mechanism is founded as the algebraic sum of the kinetic energies of the separate links $K_1 - K_9$.

2.2. Potential energy

The potential energy of the mechanism in a force field of the Earth is worked out as:

$$\Pi_M = \sum \Pi_i = -\sum m_i \cdot G^T \cdot T_i \cdot R_i^* . \tag{10}$$

The potential energy Π_i of i^{th} link is calculated with the help of the formula:

$$\Pi_i = P_i \cdot y_O^* , \tag{11}$$

The weight P_i and the ordinate y_O^* of the center of mass of i^{th} link in inertial coordinate system xOy are calculated through the given information about mass and results of kinematical analysis.

The matrix form of the expression (11) can be transformed as follows:

$$\Pi_i = -m_i \cdot G^T \cdot T_i \cdot R_i^* , \tag{12}$$

In column matrix R_i^*, first three elements are Cartesian coordinates of the center of gravity of i^{th} link connected with their own local coordinate system. The row matrix $G^T = \left(g_x, g_y, g_z, 0\right)$ describes the gravitational acceleration in the basic coordinate system (g is the constant which equal approximately 9,8062 m/s^2).

According to formula (12) the expression of the potential energy of movable links Π_1, Π_2, Π_3, … Π_9 of the mechanism is derived as:

$$\Pi_1 = -m_1 \cdot G^T \cdot T_1 \cdot R_1^* = P_1 \cdot \left(y_1^* S_{10} - a_{10} C_{10}\right);\ \Pi_2 = -m_2 \cdot G^T \cdot T_2 \cdot R_2^* = P_2\left(y_2^* S_{21+10} - a_{21} C_{21+10}\right) \text{ and so on.}$$

The total potential energy (10) of the linkage with changeable close loop is equal to the algebraic sum of energy of its separate links:

$$\Pi_M = \left(\Pi_1 + \Pi_2 + \Pi_3 + \Pi_6 + \Pi_7 + \Pi_8 + \Pi_9\right) \cdot \dot{q}_1 . \tag{13}$$

2.3. Motion equation

Substitute the obtained dependence (21) for the kinetic energy in Lagrange equation (1):

$$\frac{d}{dt}\frac{\partial K}{\partial \dot{q}_i} - \frac{\partial K}{\partial q_i} = \sum_{i=1}^{n}\sum_{j=1}^{j}\sum_{k=1}^{i} Tr\left(U_{ijk} J_i U_{ij}^{\mathrm{T}}\right)\dot{q}_j \dot{q}_k + \sum_{i=j}^{n}\sum_{j=1}^{i} Tr\left(U_{ij} J_i U_{ij}^{\mathrm{T}}\right)\ddot{q}_j . \tag{14}$$

The first partial derivative of the potential energy of i^th joint (12) by the generalized coordinate is calculated as follows:

$$\frac{\partial \Pi}{\partial q_i} = -\sum_{i=1}^{n} m_i G^{\mathrm{T}} \frac{\partial T_i}{\partial q_i} R_i^* = -\sum_{i=1}^{n} m_i G^{\mathrm{T}} U_{ij} R_i^* . \tag{15}$$

The resultant formula for calculation the first and second derivative of Lagrange function is deduced as:

$$\sum_{i=1}^{n}\sum_{j=1}^{i} Tr\left(U_{ikl} \cdot J_i \cdot U_{ij}^T\right)\ddot{q}_j + \sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j} Tr\left(U_{ikl} \cdot J_i \cdot U_{ij}^T\right)\dot{q}_j \dot{q}_k - \sum_{i=1}^{n} m_i G_j^T U_{ij}^i R_{ij}^* = Q_i .$$

So the value of the generalized force acts on i^th joint can be estimated by using the differential equation:

$$Q_i = \sum_{k=1}^{n} D_{ik} \ddot{q}_k + \sum_{j=1}^{n}\sum_{k=1}^{n} h_{ijk} \dot{q}_j \dot{q}_k + c_i , \qquad (i=1, \ldots, 9). \tag{16}$$

In matrix form, expression (16) can be written down as:

$$Q(t) = D(q(t))\ddot{q}(t) + h\left(q(t), \dot{q}(t)\right) + c(q(t)), \tag{17}$$

Elements of n-dimensional column vector $Q(t) = (Q_1(t), Q_2(t), \ldots, Q_n(t))^T$ are the generalized forces in the joints, which are generated by an actuator. Vectors $q(t) = (q_1(t), q_2(t), \ldots, q_n(t))^T$, $\dot{q}(t) = \left(\dot{q}_1(t), \dot{q}_2(t), \ldots, \dot{q}_n(t)\right)^T$ and $\ddot{q}(t) = \left(\ddot{q}_1(t), \ddot{q}_2(t), \ldots, \ddot{q}_n(t)\right)^T$ are matrices of the generalized coordinates, velocities and accelerations respectively. The elements of the square symmetric matrix *D(q(t))* in expression (16) are calculated with formula:

$$D_{ik} = \sum_{j=\max(i,k)}^{9} Tr\left(U_{ij} J_i U_{ik}^T\right), \text{ where } (i=1, \ldots, 9). \tag{18}$$

Column matrix of the Coriolis and centrifugal forces $h\left(q(t),\dot{q}(t)\right)=(h_1,h_2,...,h_n)^T$ in formula (17) is obtained as follows:

$$h_i=\sum_{j=1}^{9}\sum_{k=1}^{9}h_{ijk}\dot{q}_j\dot{q}_k=\sum_{l=\max(i,j,k)}^{9}Tr\left(U_{ijk}J_iU_{ij}^T\right),\qquad i,j\ \ k=1,\ldots,9; \tag{19}$$

Matrix of the gravitational forces $c(q(t))=(c_1,c_2,...,c_n)^T$ in the dependence (16) is equal to:

$$c_i=\sum_{j=i}^{n}\left(-m_iG^TU_{ij}R_j^*\right). \tag{20}$$

Elements of matrices c_i, D_{ik}, h_{ijk}, are functions which depend on the generalized coordinates and dynamic parameters of the linkage under consideration. The physical meaning of them consists of:

- matrix c_i takes into account forces due to gravity;
- vector D_{ik} describes the link between forces (moments) and accelerations acting on joins.
- matrix h_{ijk} characterizes the dependence between the forces acting on joints and the velocities of the generalized coordinates changes.

The analytical expression for the element D_{11} of the inertial matrix $D(q)$ is obtained as follows:

$$D_{11}=U_{11}J_1U_{11}^T=2J_Z^5+m_1a_{12}^2.$$

Other elements of the matrix (18) are worked out the similar way. The elements of the column matrix $h\left(q(t),\dot{q}(t)\right)$ are obtained by making use of the expression:

$$h_i\left(q(t),\dot{q}(t)\right)=\dot{q}^T\cdot H_{i,v}\cdot\dot{q}\cdot$$

For example, the Coriolis and centrifugal forces acting on the first link are described as:

$$h_1=\sum_{j=1}^{9}\sum_{k=1}^{9}h_{1jk}\cdot\dot{q}_j\cdot\dot{q}_k.$$

Respectively the others elements of this matrix are calculated by using in much the same manner.

Take into account all founded dependences for the components (18-20) of the general force the equation of motion of the studied linkage with changeable close loop can be derived as follows:

$$\sum_{i=1}^{9}\sum_{j=1}^{i}Tr\left(U_{ij}\cdot J_i\cdot U_{ij}^T\right)\ddot{q}_j+\sum_{i=1}^{9}\sum_{j=1}^{i}\sum_{k=1}^{j}Tr\left(U_{ijk}\cdot J_i\cdot U_{ijk}^T\right)\dot{q}_j\,\dot{q}_k-\sum_{i=1}^{9}m_iG_j^TU_{ij}^iR_{ij}^*=Q_i. \tag{21}$$

The equation of motion (21) is an ordinary nonlinear differential equation of second order and it can be solved with the standard method.

2.4. Analytical solution of the motion equation

Analysis of free oscillations in the studied linkage is performed with respect to the configuration when the drive is stopped completely, thus the generalized force is equal to zero i.e. Q=0. Dependence (21) is rearranged to obtain the homogeneous differential equation of the second order with the constant coefficients:

$$2A_1\ddot{q}_1-A_2\dot{q}_1=0, \tag{22}$$

In expression (22), the matrix A_1 contains the inertial coefficients and other matrix $A_2 = B \cdot \sin(q_1) + C \cdot \cos(q_1)$ includes the dissipative coefficients.

To simplify the dependence (29) the change the variables $\dot{q}_1 = z,\ \ddot{q}_1 = z\frac{dz}{dq}$ is carried out:

$$2A_1 z\frac{dz}{dq_1} - (B \cdot \sin(q_1) + C \cdot \cos(q_1))z = 0$$

Integrate the differential $dz = \frac{1}{2A_1}(B\sin(q_1) + C\cos(q_1))dq_1$, the formulas for calculation of parameters z and t are derived:

$$z = \frac{1}{2A_1}(-B\cos(q_1) + C\sin(q_1)) + C_1 \qquad t = \int\frac{dq_1}{z(qC_1)} = \int\frac{dq_1}{C_1 + C_2\cos(q_1) + C_3\sin(q_1)} \tag{23}$$

Coefficients C_1 and C_2 are determined as follows: $C_1 = -B/2A_1$; $C_2 = C/2A_1$. Due to the trigonometric substitution $tg(0.5q_1) = v$, the integrand t can be rearranged:

$$t = \int\frac{2dv}{C_1 + C_2 + 2C_3v + (C_1 - C_2)v^2} \tag{24}$$

In formula (24) the symbol a denotes the sum of C_1 and C_2. The letter b stands for function C_3. And the difference between C_1 and C_2 is named the lowercase letter c. Thus, the denominator of the integrand is converted into the quadratic equation $X = a + 2bv + cv^2$. Taking into account the possible value of the discriminant $\Delta = ac - b^2$ the result of integration of the function (24) is equal to:

$$t = \int\frac{dv}{X} = \begin{cases} \dfrac{2}{\sqrt{ac-b^2}}arctg\left(\dfrac{b + c\cdot tg\left(\frac{q_1}{2}\right)}{\sqrt{ac-b^2}}\right) + C_4, & when\ \Delta > 0; \\ \dfrac{1}{\sqrt{b^2-ac}}\ln\left|\dfrac{\sqrt{b^2-ac} - b - c\cdot tg\left(\frac{q_1}{2}\right)}{\sqrt{b^2-ac} + b + c\cdot tg\left(\frac{q_1}{2}\right)}\right| + C_4, & when\ \Delta < 0; \\ \dfrac{-2}{b + c\cdot tg\left(\frac{q_1}{2}\right)} + C_4, & when\ \Delta = 0. \end{cases} \tag{25}$$

According to the formula (25) the motion law of the output link of the suggested linkage with changeable close loop depends on the integration constants C_1, C_2, C_3, C_4 which are determined by using the given initial and boundary conditions.

The general solution of the differential equation (22) is characterized the free oscillations of the moveable joints and can be derived as:

$$q(t) = a \cdot b \cdot \sin(\omega t + \delta) \tag{26}$$

In the formula (26), the following denotes are used amplitude b, oscillation frequency ω and initial phase of oscillations δ. Parameters a and b are constant and are found through the initial conditions.

Angle of rotation q_1 and velocity $\dot{q}_1$ are estimated with the condition of the static equilibrium for the given scheme of the mechanism in the initial moment.

There are many numerical methods to solve the ordinary differential equation of the second order [17]. In this study it is used the approximation method Runge-Kutta [12]. The step of numerical integration was selected significantly less than the period of the free oscillations in the investigated linkage.

Substituting expression (25) into formula (22) we change the subject of the linear differential equation:

$$\left(A_2 - \omega^2 A_1\right)b = 0 \quad (27)$$

The characteristic equation for formula (27) will be follows:

$$\det\left(A_2 - \omega^2 A_1\right) = 0 \quad (28)$$

Due to positive of matrices A_1 and A_2, the roots of the equation (28) are real and positive too. Value ω is the natural oscillation frequency of the system. Amplitude vector *b* is found from the equation (28) by using the calculated value ω.

The simulation of the linkage is implemented by MathCad [16], which has a well-defined position among many other software products (Maple, Mathematica, MatLab, Adam ect.) being worldwide standard for technical computation. For numerical testing of the dynamic model of the mechanism in question, a given liner trajectory between two points in Cartesian space was used. To convert to dimensionless variables and parameters of the dynamics model and simulate the operating process of the mechanism as well as to obtain the numerical data for the analysis of free oscillations of the linkage was used the following assumptions that are not loss of generality:

- the mass of the levers are equal to one;
- the moment of inertia of levers are equal to 1/3.

The results of the simulation of the mechanism show that the lower (main) oscillation frequencies are close for all links, thus, the mechanism is moved mainly in the plane. The natural frequencies of links are calculated:

- conditional input link OBD $\omega_1 \approx 13\,Hz$,
- triangle link EDF $\omega_2 \approx 21\,Hz$;
- dyad FA $\omega_3 \approx 26\,Hz$;
- triangle links BCP and EGF $\omega_6 \approx 15\ Hz$ and $\omega_7 \approx 18\ Hz$ respectively;
- lever GL $\omega_8 \approx 24\,Hz$ and output link PLQ $\omega_9 \approx 13\ Hz$.

In figure 2, the diagram of the dependence between the oscillation amplitude of the link AF and the natural oscillation frequencies is shown.

Since the oscillations of the mechanical system components (links) are close to single frequency according to the analysis of the graphs of the resonance frequencies of free oscillations that were built for the individual links of the suggested mechanism during the simulation, it is necessary and sufficient to consider only its first form of oscillations.

At the beginning of motion the oscillation amplitude the output link *PLQ* of the linkage is the value about (2÷4) $10^{-3} \cdot l$, where *l* is the kinematic parameter of the link. The relatively high oscillation frequencies of some certain links, in particular *BCP* and *ECG*, are indicated on high strength of the mechanism and as a result the rapidly damped oscillation process.

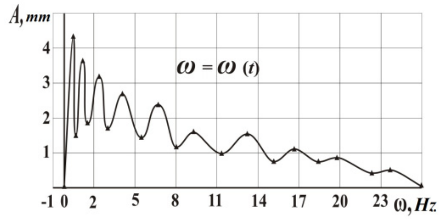

Fig. 2. Diagram of the oscillation amplitude.

Conclusion

Generally, the lifting machines are constructed with the help of scissors lift lever system called Nuremberg scissors and a lifting cargo platform. The key disadvantages of such systems are the following: limited lifting capacity; low stability at the highest position of the platform; low operation parameters and durability; structural complexity and high material capacity.

The proposed structure of the multi-lever planar mechanism with a changeable close loop have increased lateral and longitudinal stiffness and provides lifting action with aid of the only one hydraulic actuator. The results of kinematical and kinetostatical analysis in ref. [11, 13] show that the operating platform moves in the vertical plane by about 1,5 m. The output link of the mechanism in question has a negligible inclination, namely 2,51 degrees to the horizontal, but it does not influence the operational capacity of the equipment. The load capacity of the proposed mechanism is 200 kg. It can be used as an actuator of a lifting mechanism in a repair works as well as internal and external construction works.

The article describes a comprehensive and systematic method, which is used to estimate the dynamics parameters of multi-lever linkages. The proposed method is based on the Euler and Lagrangian formalism. It is important to note that the suggested method can be applied to both planar and spatial linkages in order to solve the dynamic problem. Especial attention was paid to description of the dynamics of the planar linkage with changeable close loop. In this paper, the Langrange method is used to derive the dynamic model of the planar linkage with changeable close loop, dynamic modelling in the hypothesis of rigid links.

The numerical result of simulation of the mechanism are conducted by the mathematical software application MathCad. The presented numerical results of the simulation prove the feasibility of the proposed approach. Relatively high oscillation frequency and rapidly damped oscillation of some levers indicate high strength of the mechanism. The amplitude of oscillations of the loaded mechanism is reduced to $(0,5 \div 2)\cdot 10^{-3}\cdot l$.

The study of kinematic and dynamic characteristics of the multi-lever mechanism with changeable closed loop makes it possible to deduce the equation of motion and evaluate some dynamic parameters, which influence productivity, reliability and durability of machinery. Further researches will be devoted to the development on the improved dynamics model, which will take into account friction effect to reduce model uncertainties and, therefore, reduce the position error of the mechanism.

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