Research problems


Correspondent: Aviezri S. Fraenkel,
Department of Applied Mathematics,
The Weizmann Institute of Science,
Rehovot 76100,
Israel.

A two-person game is impartial if for every position the set of followers for one player is identical to the set of followers for the other player. Otherwise the game is partizan. At octal game as defined in Problem 38 is impartial. It is transformed into a partizan octal game by assigning distinct octals to the two players. Various properties of these games were determined in [1] for the case where the octals are restricted to quaternary numbers.

Explore the main properties of general partizan octal games.

Reference


Problem 40. Posed by Aviezri S. Fraenkel.

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Israel.

A poset game is a two-person game defined on any partially ordered set \((S, \succeq)\) of heaps of tokens. The first player selects a heap \(s_1 \in S\) and removes a nonempty subset of tokens from among all heaps \(s \succeq s_1\). The second player picks some \(s_2\) from among the remaining heaps of \(S\), if any, and removes a nonempty subset of tokens from among all heaps \(s \succeq s_2\). Play now reverts back to the first player who selects some \(s_3\) from among the surviving elements, if any, and again removes a nonempty subset of tokens from among all heaps \(s \succeq s_3\). Play continues in the indicated manner until all elements have been removed. In last-player-win (LPW)
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mode, the player first unable to move loses, his opponent wins. In last-player-lose (LPL) mode, the outcome is reversed.

Special cases: nim (finite sum of totally ordered sets), Von Neumann’s Hackendot (directed forest) [5], Gale’s matrix game (product of two totally ordered sets) [1], all subsets of \( \{1, \ldots, n\} \) of cardinality at most \( k \) (where \( A \succeq B \) if \( A \supseteq B \)) [2], Schuh’s game of divisors [4]. Incidentally, in all these games the rule is to take all \( s \succeq s^* \), not just any nonempty subset.

Whenever \( S \) contains a largest element \( s_0 \) (that is, \( s_0 \succeq s \) for all \( s \in S \)), then the first player can win in both the LPW and LPL mode of any poset game. Because if he can win by selecting \( s_0 \), we are done. Otherwise selecting \( s_0 \) is a losing move, and so selecting some \( s \in S \) is a winning response. Thus the first move of selecting \( s \) is winning.

Not much more than this (nonconstructive!) assertion is known about general poset games. It may be helpful to transform a poset game into an equivalent two-person game, directed node kayles, played on an acyclic digraph \( G = (V, E) \), where \( V \) is the set of game positions and \( (a, b) \in E \) if and only if \( a \succeq b \). A move consists of labeling an as yet unlabeled node \( a \) provided none of its followers \( b \) was labeled; that is, \( (a, b) \in E \) implies \( b \) not yet labeled. A simple reduction from node kayles [3] (see Problem 38) shows that directed node kayles is P-space-hard on a general cyclic digraph. Úlehla’s result [5] shows that it is polynomial on special acyclic digraphs, namely transitive closures of forests. Poset games lie somewhere in the middle. Where precisely? (We can show that the problem of deciding whether a game of directed node kayles can be won in at most \( K \) moves is NP-complete even if the acyclic digraph on which it is played is planar and degree constrained.)

References