Displacements of column-supported embankments over soft clay after widening considering soil consolidation and column layout:
Numerical analysis

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Abstract

The common challenges for constructing embankments on soft clay include low bearing capacity, large total and differential settlements, and slope instability. Different techniques have been adopted to improve soft clay, such as the use of foundation columns including stone columns, deep mixed columns, and vibro-concrete columns, etc. Due to increased traffic volume, column-supported embankments may be widened to accommodate the traffic capacity need. Adding a new embankment to an existing embankment generates additional stresses and deformations under not only the widened portion but also the existing embankment. Differential settlements between and within the existing embankment and the widened portion may cause pavement distresses. Limited research has been conducted so far to investigate widening of column-supported embankments. In this study, a two-dimensional finite difference numerical method was adopted. This numerical method was first verified against field data and then used for the analysis of widened column-supported embankments over soft clay. The modified Cam-Clay model was used to model the soil under the existing embankment and the widened portion. Mechanically and hydraulically coupled numerical models were created to consider the consolidation of the foundation soil under the existing embankment and the widened portion. Different layouts of foundation columns under the existing embankment and the widened portion were investigated. The numerical results presented in this paper include the vertical and horizontal displacements, the maximum settlements, the transverse gradient changes, and the stress concentration ratios, which depended on column spacing. The columns installed under the connection side slope were most effective in reducing the total and differential settlements, horizontal displacement, and transverse gradient change of the widened embankment.

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1. Introduction

Widening of embankments has been increasingly adopted in practice to increase highway capacities due to demand for higher traffic volume than previously designed. The 1989 Government's White Paper “Roads for Prosperity” (The Highway Agency, 1991) indicated that “about 60% of the motorway network in England as well as some truck roads will need to be widened by the provision of additional lanes”.

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The process of adding a new embankment adjacent to the existing embankment generates additional stresses and deformations underneath both the existing embankment and the widened portion. Ling et al. (2003) confirmed that a number of roadway pavements were exposed to overstresses due to widening of existing roads in China (Han et al., 2006). Ling et al. (2003) proposed some design criteria for widening of embankments.

The common challenges for constructing embankments on soft clay including embankment widening are low bearing capacity, large total and differential settlements, and slope instability. Different technologies have been used in practice to avoid, minimize or remedy roadway distresses due to the widening of embankments, such as the use of lightweight backfill, geosynthetic reinforcement, over-excavation and replacement, installation of piles or foundation columns, preloading, and a combination of the above alternatives. Foundation columns may include sand columns, stone columns, deep mixed (DM) columns, vibro-concrete columns, etc. DM columns were selected in this study as an example. Column-supported embankments have been increasingly used in soft soils in the past few years. A large number of studies have been conducted on this topic, for example, Han and Gabr (2002), Collin et al. (2005), Huang et al. (2005), (2009), Chen et al. (2008), Zheng et al. (2011), Filz et al. (2012), Khabbazian et al. (2012). Several factors influence the performance of column-supported embankments. The effect of the column stiffness on the displacement behavior is an obvious one, which has been investigated by the second author and his co-authors (Han and Gabr, 2002; Huang et al., 2009; Huang and Han, 2010).

Even though widening of embankments has been increasingly adopted in practice, so far very limited guidance for design is available for widening projects, especially for widening of column-supported embankments. Forsman and Uotinen (1999) investigated the effect of geosynthetic reinforcement on the settlements and horizontal displacements of embankments after widening. Geosynthetic reinforcement may not be needed if the spacing of columns is close and/or the height of the embankment is large. Han et al. (2007) investigated stresses and deformations of the widened embankments over soft soil with or without foundation columns. In this study, Han et al. (2007) modeled the soft soil as a linearly elastic–perfectly plastic material with the criterion of Mohr–Coulomb failure. This model cannot consider property change of the soft soil due to the reduction in the soil volume during the consolidation process. To overcome this problem, Han et al. (2007) assumed the improved properties of the soil after the consolidation. Mirjalili et al. (2012) presented a two-dimensional numerical analysis of a constructed levee on a normal river embankment to a broad width over soft soils. Several factors in influence the performance of column-supported embankments. The effect of the column stiffness on the displacement behavior is an obvious one, which has been investigated by the second author and his co-authors (Han and Gabr, 2002; Huang et al., 2009; Huang and Han, 2010).

The modified Cam–Clay model was employed in this study to represent the behavior of the soft clay under the existing embankment and the widened portion (i.e., the influence of volume change due to consolidation on soil properties is taken into consideration). The modified Cam–Clay model is an incremental hardening/softening elastoplastic model. Its features include a particular form of nonlinear elasticity and a hardening/softening behavior governed by volumetric plastic strain. The failure envelopes as investigated by Wood (1990) and Roscoe and Burland (1968) correspond to ellipsoids of rotation about the mean stress axis in the principal stress space. The Modified Cam Clay was classified as an incremental hardening/softening elastoplastic model based on the FLAC manual (Itasca, 2002a). Based on Roscoe and Burland (1968), the Modified Cam Clay (MCC) can be defined that the yield surface of the MCC is described by an ellipse and therefore the plastic strain increment vector (which is perpendicular to the yield surface) for the largest value of the mean effective stress is horizontal, and hence no incremental deviatoric plastic strain takes place for a change in mean effective stress (for purely hydrostatic states of stress). This is very convenient for constitutive modeling in numerical analysis. Based on Schofield and Worth (1968), Hardening phenomenon can be described as follows. Due to the positive volumetric strain increment, soil in a stressed state will compact to a smaller total volume at which it has a new larger yield curve. The soil becomes denser and a permanent load-increment can be added before bringing it to the verge of yielding as a point on the larger yield curve. At this stage, the soil is slightly deformed to be stronger or harder. In contrast, due to negative volumetric strain for a soil in another stressed state, the sample must expand to a larger volume and in this condition the new ‘looser’ soil is just in equilibrium governed by a smaller yield curve. The effect leads to the specimen being weaker or softer; this process is called softening. Therefore, the MCC model can consider strain hardening and softening. However, strain softening happens for highly over-consolidated soils when the applied stress is higher than the initial yield stress. The plastic flow rule is associated and no resistance to tensile mean stress is allowed.

The objective of this paper was to investigate the effect of column layouts under existing and widened embankments on the displacement behavior using mechanically and hydraulically coupled numerical models considering consolidation. Particular attention was paid to the settlements, horizontal displacements, transverse gradients, and vertical stresses. The 2D numerical method was first verified against the measured field data available in the literature. Different layouts of columns under the existing embankment and the widened portion were investigated to evaluate their effectiveness.

2. Verification of numerical model

To ensure the reasonableness of the numerical model to be used for the parametric study, a field case study as described
below was selected for the verification. Due to unavailability of a well-documented roadway widening case study to the authors’ best knowledge and search, a field study with a new embankment over geosynthetic-reinforced deep mixed (DM) columns reported by Forsman et al. (1999) was adopted for this purpose. The details of this case study can be found in the literature (Forsman et al., 1999). The numerical verification based on a drained mechanical analysis with a simplified soil constitutive model (i.e., linearly elastic-perfectly plastic) can be found in the paper by Han et al. (2005) while the numerical verification based on a mechanically and hydraulically coupled analysis with the simplified soil constitutive model (i.e., linearly elastic-perfectly plastic) can be found in the paper by Huang et al. (2009). The limitation of the simplified soil constitutive model is that it cannot consider property change of soft soils due to consolidation, which is important for widening of embankments. Therefore, a mechanically and hydraulically coupled analysis with a Cam-Clay soil constitutive model was adopted in this study and a brief description of the verification of the numerical model based on this soil constitutive model is presented below.

2.1. Brief description of selected project

The selected project is a bridge embankment constructed on deep mixed (DM) columns beside the Sipoo River at Hertsby, Finland. Corrections and update on settlement measurements were made based on Forsman (2001). The soft foundation below the embankment consisted of a 1–1.5 m thick crust, 10–14 m thick soft clay, 0–6 m thick silt, and 1–5 m thick glacial till. The soft clay over the silt layer had an undrained shear strength of 10–15 kPa. The effective cohesion and friction angle were 8 kPa and 13°, respectively, determined from drained triaxial tests. The elastic moduli under drained and undrained conditions were 300–600 kPa and 3000–8000 kPa, respectively, also determined from triaxial tests. The determined Poisson’s ratio under drained conditions was 0.1 to 0.2. The embankment had a 0.05 m thick asphalt layer, 0.2 m thick crushed stone base coarse, 1.05 m thick gravel subbase, and 0.5 m thick sand working platform above the existing ground (i.e., at the base of the embankment). Based on Huang et al. (2009), the construction of the embankment was modeled in three stages. The duration and the lift thickness of each stage were based on the actual construction. Fig. 2 shows three stages of embankment construction. First stage built a 0.6 m thick platform fill immediately, which was maintained for two months. The embankment fill was constructed through two stages: 0.9 m thick fill placed immediately and maintained for eight months and the remaining fill placed immediately and maintained for 18 months. The last stage included the 0.3 m thick embankment fill with the asphalt layer as shown in Fig. 2. The traffic was simulated by applying an equivalent static, distributed pressure of 12 kPa on the crest of the embankment, which was assumed starting right after the placement of the asphalt layer in Stage 3 (i.e., the traffic loading was included in Stage 3). The soft foundation was treated with DM columns as shown in Fig. 1 to have enough bearing capacity and minimal compressibility. The columns had an average diameter of 0.8 m. Cement and by-product based binder were used as an admixture and the admixture content was 130 kg/m³. The top layer on the embankment shown in Fig. 1 was the pavement section (including asphalt, base course, and subbase course). The design shear strength of the columns was 150 kPa. One layer of woven geotextile and a 0.3 m sand layer were placed over the columns. The ultimate strength of this geotextile was 200 kN/m in both longitudinal and transverse directions. The secant stiffness of the geotextile layer was 1790 and 2120 kN/m at strains of 2% and 6%, respectively. The adjacent geotextile sheets were jointed together by seams. This constructed embankment was instrumented with horizontal hydrostatic profile gauges, settlement plates, and strain gauges on the geotextile sheet. After 5 years, the measured maximum settlements were approximately 120 mm (Forsman, 2001). The settlements had become relatively stable after two years since construction. The measured
strains in the geotextile in the longitudinal and transverse directions of the embankment were 0–0.2% and 0–1% respectively. The tensions in the geotextile corresponding to 0.2% and 1% strain are 3.6 kN/m and 18 kN/m, respectively.

2.2. Numerical modeling

A 2D finite difference method incorporated in the Fast Lagrangian Analysis Continua (FLAC) software Version 5 (Itasca, 2002a) was adopted in this study. The numerical model used for the verification against this case study is presented in Fig. 2, which modeled the cross section I–I across the DM walls without middle individual columns as shown in Fig. 1. Due to the symmetry of the problem, half of the section was used in the analysis. The DM columns with a wall pattern were modeled as two-dimensional soil-cement walls. The wall at the centerline had only half width.

In this study, the DM columns and the embankment fill were modeled as linearly elastic-perfectly plastic materials while the soft clay and the silt were modeled using the modified Cam-Clay model in which the modulus of elasticity increases as volumetric strain decreases. Based on Forseman (2001), the plasticity index (PI) of the clay ranged from 55 to 62% and 60% was selected for the calculation. As Budhu (2007) suggested, the slope of normal consolidation line $\lambda \approx 0.6/(PFI = 0.36$. Typically, the slope ratio of elastic swelling line to normal consolidation line $\kappa / \lambda$ is within the range of 1/10–1/5 (Budhu, 2007). Two cases were investigated in this study to evaluate the influence of the $\kappa / \lambda$ ratio: (1) $\kappa / \lambda = 1/7$ and (2) $\kappa / \lambda = 1/10$. Based on Huang et al. (2009), the effective friction angle of $25^\circ$ should be used; therefore, the frictional constant, $M = 6 \sin \phi' / (3 - \sin \phi') = 0.98$. The coefficient of lateral earth pressure was determined as $K_v = 1 - \sin \phi' = 0.577$. Based on Huang et al. (2009), the average moisture content of the soft clay was 70%; therefore, the initial void ratio of the clay was estimated as $e_o = w_G / S = 1.89$, where $w$ is the water content, $G_s$ is the specific gravity (assumed as 2.7), and $S$ is the degree of saturation (100% due to full saturation under groundwater table). Since, it was a normally consolidated clay; the vertical stresses $\sigma_v$ and $\sigma_h$ were estimated at the mid-depth of the soft clay with and without considering the clay crest of 1 m which was removed after installation of DM columns, respectively. The initial pressure was based on $p_o = (\sigma_v + 2\sigma_h) / 3$, where $\sigma_v$ and $\sigma_h$ are vertical and horizontal stresses at the mid layer respectively. Based on the FLAC manual (Itasca, 2002a), the maximum principal stresses $p$ and $q$ can be expressed as follows: $p_{max} = (\sigma_{v, max} + 2\sigma_{h, max}) / 3$ and $q_{max} = \sigma_{v, max} + \sigma_{h, max}$, where $\sigma_{v, max}$ and $\sigma_{h, max}$ are the maximum vertical and horizontal stresses, respectively. Based on Forsman et al. (1999), Poisson’s ratio of the clay was 0.2.

The preconsolidation pressure was obtained based on $p_c = (p_{max} + q_{max}^2 / M p_{max})$ suggested by the FLAC manual (Itasca, 2002a, 2002b). When the reference pressure of 1 Pa was chosen, the specific volume at this reference pressure was calculated by $e_r = e_0 + (\lambda - k) \ln p_c / k + k \ln p_0$. Therefore, the maximum elastic bulk modulus was calculated as $K_{max} = \varepsilon e_r p_{max} / k$ (Itasca, 2002a, 2002b). Based on Forsman et al. (1999), the slope of normal consolidation line $\lambda$ and the slope of elastic swelling line $k$ were 0.12 and 0.0060 for the silt layer, respectively. Head (2006) suggested the effective friction angle of the dense silt ranges from 30° to 34°. The friction angle of the silt of 30° was used in this study. Based on Huang et al. (2009), the average moisture content of the silt was 50%. Similar procedures for determining the parameters of the clay soil above were used to determine all the remained Cam-Clay parameters of the silt soil. The soil layers and the DM columns were extended to the depth of the firm glacial soil. A Mohr–Coulomb failure envelope was used as the failure criterion for the embankment fill and the DM columns while the failure criteria for the clay and silt were determined by the Modified Cam-Clay failure envelope. A cable element was used to simulate the geotextile layer, which was located 0.3 m above the top of DM columns. The properties of the soil, the DM columns, and the geosynthetic layer are provided in Tables 1 and 2. Interaction coefficient between geotextile and sand can be defined as the ratio of the friction coefficient between soil and geotextile to the friction coefficient for soil sliding on soil.

The elastic modulus of the DM columns was estimated based on a typical relationship of $E = 100q_u$ (e.g., Porbaha et al., 2000; Bruce, 2001), where $q_u$=unconfined compressive strength of the column (300 kPa). The effective thickness of the DM wall was estimated to be 0.7 m. The effective thickness was estimated based on the equivalency of the actual area of a series of DM columns to the area of the wall modeled in the analysis. The crust near the ground surface was not considered in the numerical analysis since the crust was removed after the installation of DM columns. The construction sequence was simulated by adding the embankment fill in three layers as Huang et al. (2009). A surcharge of 12 kPa was used to simulate the traffic loading. The problem was analyzed in a sequence of undrained and drained conditions at different stages of embankment filling. At the moment of filling, the problem was analyzed under an undrained condition and then it was turn into a drained condition. Under a drained condition,
an iterative procedure of the mechanically and hydraulically coupled numerical modeling was used, which consisted of mechanical and hydraulic loops. In each mechanical loop, the stresses and associated deformations were computed while in each hydraulic loop, the consolidation process was modeled by dissipation of excess pore water pressure through the soil. Both loops were coupled through the quasi-static Biot theory (Biot, 1956) as follows: (1) starting with the hydraulic loop, pore water pressure and specific discharge were computed based on Darcy’s law, the fluid mass balance law, and the computability law; (2) the resulted pore water pressures were transferred to a mechanical loop to compute effective stresses; (3) the effective stresses were used to check the failure characteristics and to compute the volumetric strain based on the selected constitutive model; and (4) the updated volumetric strain was passed back to the new hydraulic loop to compute a new change in the pore water pressure in an iterative procedure based on the linear quasi-static Biot theory (Itasca, 2002b). After many cycles of alternative loops, the force balance (within a tolerance) was reached and both pore water pressure and volumetric strain were updated.

The bottom boundary was fixed in both horizontal and vertical directions and the two side boundaries were fixed in the horizontal direction but free in the vertical direction.

### 2.3. Results and comparisons

Han et al. (2005) and Huang et al. (2009) provided detailed comparisons of the numerical results including the vertical displacements, tension in reinforcement, and vertical stresses with the field data. Herein only the key results are compared. Fig. 3 shows the comparison of the settlement-time curves for the two cases of the \( k/\lambda \) ratios at 1/7 and 1/10 respectively. S1 and S2 are the locations of interest shown in Fig. 2. Case 1 (i.e., \( k/\lambda = 1/7 \)) over-predicted the maximum settlement between the DM column walls as compared with the measured. Case 2 (i.e., \( k/\lambda = 1/10 \)) yielded a close agreement of the maximum settlement between the DM columns with the measured. The numerical analysis yielded the maximum tension from 10 kN/m (\( k/\lambda = 1/10 \)) to 12.3 kN/m (\( k/\lambda = 1/7 \)) while the measured maximum tension ranged from 3.6 to 18 kN/m. They are in a reasonably good agreement. In addition, the numerical results from the present study matched those obtained by Huang et al. (2009) reasonably well.

### 3. Numerical study

#### 3.1. Numerical model

Fig. 4 shows the cross section, boundary conditions, and dimensions of the numerical model for a baseline case without

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**Table 2**

Properties of fill, DM walls, and geotextile.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Embankment fill (base course)</th>
<th>Platform fill (subbase course)</th>
<th>DM walls</th>
<th>Geotextile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil model</td>
<td></td>
<td>MC(^a)</td>
<td>MC(^a)</td>
<td>MC(^a)</td>
<td></td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>(\nu)</td>
<td>0.33</td>
<td>0.33</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Shear modulus (GPa)</td>
<td>(G)</td>
<td>15</td>
<td>75.2</td>
<td>11.5</td>
<td>25.0</td>
</tr>
<tr>
<td>Bulk modulus (GPa)</td>
<td>(K)</td>
<td>39.2</td>
<td>19.6</td>
<td>25.0</td>
<td></td>
</tr>
<tr>
<td>Friction angle (deg)</td>
<td>(\phi)</td>
<td>38</td>
<td>32</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Cohesion (kPa)</td>
<td>(c)</td>
<td>5</td>
<td>5</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>Total density (kN/m(^3))</td>
<td>(\gamma)</td>
<td>20,000</td>
<td>20,000</td>
<td>20,000</td>
<td></td>
</tr>
<tr>
<td>Dry density (kN/m(^3))</td>
<td>(\gamma_d)</td>
<td>20,000</td>
<td>20,000</td>
<td>20,000</td>
<td></td>
</tr>
<tr>
<td>Tensile stiffness of geotextile (kN/m)</td>
<td>(J)</td>
<td>1700</td>
<td>85,000</td>
<td></td>
<td>0.8</td>
</tr>
<tr>
<td>Interation coefficient between geotextile and sand</td>
<td>(c_i)</td>
<td>0.8</td>
<td>85,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interface shear stiffness between geotextile and sand (kN/m/m)</td>
<td>(k_s)</td>
<td>85,000</td>
<td>0.8</td>
<td>85,000</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\)Linearly elastic-perfectly plastic Mohr–Coulomb model.
any columns, which is the same as that in Han et al. (2007). For clarity, the existing embankment refers to ADGF, and the widened portion refers to DEHG. The foundation of this existing embankment and the widened portion was divided into five zones for easy analyses and comparisons. This baseline case had excessive deformations and was not stable under a typical construction speed. To reduce the deformations and ensure the stability of the embankment, columns were installed under the embankment as shown in Fig. 5. To investigate the effect of column layout, three possible layouts of columns were considered for existing column-supported embankments, which resulted in three cases: Cases a, b, and c, as shown in Fig. 5. Case a had column spacing of 2 m, Case b had column spacing of 1.5 m, and Case c had column spacing of 1.0 m in the central portion and 2.0 m under the slopes. To widen the existing embankment (Case a), two column layouts were considered and investigated as labeled as Case a1 and Case a2 as shown in Fig. 6. Two same column layouts (Case b1 and Case b2) were used to widen the existing embankment (Case b) as shown in Fig. 7. Three column layouts (Case c1, Case c2, and Case c3) were adopted to widen the existing embankment (Case c) as shown in Fig. 8. The installation of columns under the existing slope is challenging in field. Benches or platforms may be needed for the installation of the columns, which is beyond the scope of this study.

In the numerical modeling, the ground was first formed by applying a gravity force under a small-strain mode. The small-strain mode does not update the mesh coordinates. Any deformation induced during this process was zeroed out. The properties of the soil corresponding to columns under the existing embankment were changed to those of the columns. In all the cases, the columns were modeled as walls in the plane strain condition. The thickness of the column wall used in this analysis was 0.5 m. All the columns started from the base of the embankment down to the bedrock. The construction sequence of the existing embankment was simulated by adding the embankment fill in five layers of equal thickness (i.e., 1.0 m) and the waiting period after each loading was one month. The numerical analysis was conducted under undrained and drained conditions. The mesh size for the foundation below the embankment in the horizontal direction was 0.5 m to accommodate the size of the column and that in the vertical direction was 0.5 m. The coupled analysis included mechanical and hydraulic loops. Each mechanical loop was cycled for 100 times. A completed mechanical loop was followed by a hydraulic loop which was cycled once.

During each loading, it was analyzed under an undrained condition (i.e., no drainage and pore water pressure dissipation). After the existing column-supported embankment was constructed, the model was analyzed under a traffic load of 12 kPa and under a drained condition (i.e., drainage and pore water pressure dissipation). The excess pore water pressure and the settlement along the centerline of the embankment were monitored during the analysis. The analysis was terminated at the time when the excess pore water pressure and the settlement were less than 1% their maximum values, which is considered as the end of the consolidation. This computation process was also under a small-strain mode. The small strain mode was selected to ensure no distortion of mesh before the widening of the existing embankment. At the end of consolidation of the existing embankment, all the displacements were zeroed out again because the focus of this study was to investigate the behavior of the embankment after widening. At this point, the columns under the widened portion were installed by changing the soil properties to column properties. The widened portion and the traffic loading on the widened portion were added and analyzed in a large-strain mode, which updated the mesh coordinates due to displacements. The widened portion was placed through five stages similar to the existing embankment and the mechanically and hydraulically coupled analysis was conducted up to the end of consolidation. Due to page limit, the paper is focused on the numerical results of the widened embankment at the end of consolidation.

In the numerical analysis, embankment fill and the bedrock were modeled as elastic materials whereas the bedrock had a high modulus and the compression of the bedrock was negligible. Columns were modeled as a linearly elastic–perfectly plastic material with the Mohr–Coulomb failure criterion while the soft soil was modeled using the Cam–Clay model. The material properties of the soft clay were selected based on the values of Case 2 listed in Table 1. According to the verification, \( k \lambda = 1/10 \) was used in this numerical study. The properties of the columns used in this analysis were the same as those in Table 2. To simplify the analysis, only one type of embankment fill was considered. The elastic moduli of the embankment fill and the bedrock were 20 and 100 MPa respectively and their Poisson’s ratios...
were 0.25 and 0.35 respectively. Different from the study conducted by Han et al. (2007) previously, this study considered the effect of soil consolidation on the soil property changes under the existing embankment and the widened portion. This model better simulates soil behavior in field.

3.2. Numerical results and discussion

3.2.1. Vertical displacement contour and maximum settlement

Vertical displacement (settlement) contours of three widened embankments at the end of consolidation are presented in Fig. 9. Please note different scales were used for better presentation. It is shown that the maximum settlements developed at the base of the embankment under the connection side slope in Case a2 (2.0 m column spacing) and Case b2 (1.5 m column spacing) while the maximum settlement developed under the base of the widened portion in Case c3 (varied column spacing). The additional columns installed under the connection side slope minimized the maximum settlement effectively. It is intuitively correct that columns with smaller spacing resulted in less maximum settlement. The change of the colors in contours at the same elevation indicates the differential settlement and displacement gradient. Fig. 9 shows that the maximum differential settlement and displacement gradient developed at the base of the embankment and decreased with an increase of the elevation due to arching action. On the crest of the embankment, the differential settlement and displacement gradient were relatively small.

These results are different from those obtained by Han et al. (2007). The differences resulted from different ways of modeling the loading process and the soil consolidation. In the Han et al. (2007) study, the widened portion was placed in one stage, which is more appropriate for rapid construction. In addition, Han et al. (2007) did not consider the soil property changes due to consolidation.

Table 3 provides the maximum settlements on the crest and at the base of all the existing embankment and the widened embankments. The maximum settlement on the crest was less than that at the base for all the cases. It is clearly shown that the decrease of the column spacing from 2.0 m (Case a) to 1.5 m (Case b) reduced the maximum settlements on the crest and at the base by more than half. The reduction of the column spacing in the central portion to 1.0 m (Case c) reduced the maximum settlements on the crest and at the base by more than 80% as compared with Case a and more than 55% as compared with Case b. The ratio of the settlement on the crest to that at the base ranged from 63% to 74% except Case c and Case c3. The closer spacing in the central portion of the existing embankment and under the connection side slope of the widened embankment made these two column layouts most effective and efficient in reducing the settlements on the crest of the existing embankment and the widened embankment, respectively.

Table 3 also shows that the existing embankment and the widened portion on largest column spacing (2 m) resulted in the largest settlements on the crest and at the base of the
Fig. 6. Widening of the existing embankment Case a. (a) Case a1 and (b) Case a2.

Fig. 7. Widening of the existing embankment Case b. (a) Case b1 and (b) Case b2.
embankment. The settlement decreased when columns with closer spacing (1.5 m) used under the widened portion (i.e., Case a2) or under the existing embankment (i.e., Case b1). The reduction was more significant when the columns with closer spacing (1.5 m) were used under the existing embankment. The difference between Case a1 and Case c1 is the column spacing under the central portion of the existing embankment. It is clearly shown that the columns with closer spacing in the central portion of the existing embankment also reduced the maximum settlements on the crest and at the base of the embankment. However, the benefit of the settlement reduction on the crest by the columns with closer spacing in the central portion of the existing embankment (i.e., Case c1) was not that significant as that by the columns with closer spacing under the connection side slope (i.e., Case b1). As Cases a1, b1, and c1 are compared Cases a2, b2, and c2, respectively, the columns with closer spacing under the widened portion reduced the maximum settlements on the crest and at the base of the widened embankment. Table 3 shows the most significant reduction in the maximum settlements was Case c3, in which additional columns were installed between the existing columns under the connection side slope. As compared with Case c2, the percentages of settlement reduction on the crest and at the base of the embankment were approximately 91% and 78%, respectively.

Table 3 shows the location of the maximum base settlement from the left toe of the existing embankment, which was determined from the settlement profile at the base of the embankment as discussed in the next section. The distance of 15 m indicates the center of the existing embankment.
The increase of the distance implies the location of the maximum base settlement shifted from the center towards the widened portion. When the distance was greater than 30m, the location of the maximum base settlement was on the right to the right toe of the existing embankment. It is shown that all the existing embankments (Cases a, b, and c) had the location of the maximum base settlement at 15 m. The widening of the embankment shifted the location of the maximum base settlement towards the widened portion. The existing embankments having columns with smaller spacing in the central portion (Cases c1, c2, and c3) had larger distances of the maximum base settlements because the central portions were stiffer than the widened portions. The columns with smaller spacing under the widened portion pushed the location of the maximum base settlement towards the existing embankment (for example, Cases a2 and b2). The additional columns under the connection side slope (i.e., Case c3) made the location of the maximum base settlement shifted farthest right and pass the right toe of the existing embankment. The location of the maximum base settlement also represents the center of the soil movement horizontally, which will be discussed later.

3.2.2. Settlement profile

Fig. 10 presents the settlement profiles at the bases of the existing column-supported embankments at the end of consolidation before widening. It is clearly shown that the

Table 3
Maximum settlements at the base and on the crest of the widened embankment at the end of consolidation.

<table>
<thead>
<tr>
<th>Location</th>
<th>Case (unit: mm)</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>a1</th>
<th>a2</th>
<th>b1</th>
<th>b2</th>
<th>c1</th>
<th>c2</th>
<th>c3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crest</td>
<td></td>
<td>430</td>
<td>171</td>
<td>52</td>
<td>400</td>
<td>330</td>
<td>280</td>
<td>230</td>
<td>292</td>
<td>240</td>
<td>21</td>
</tr>
<tr>
<td>Base</td>
<td></td>
<td>630</td>
<td>270</td>
<td>118</td>
<td>550</td>
<td>480</td>
<td>380</td>
<td>320</td>
<td>420</td>
<td>350</td>
<td>78</td>
</tr>
<tr>
<td>Crest/base (%)</td>
<td></td>
<td>68.3</td>
<td>63.3</td>
<td>44.1</td>
<td>72.2</td>
<td>68.9</td>
<td>73.7</td>
<td>71.8</td>
<td>69.5</td>
<td>68.6</td>
<td>26.9</td>
</tr>
<tr>
<td>Location of maximum base settlement from left toe (m)</td>
<td></td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>25</td>
<td>24</td>
<td>25</td>
<td>24</td>
<td>27</td>
<td>26</td>
<td>31</td>
</tr>
</tbody>
</table>
decrease of the column spacing from 2.0 m (Case a) to 1.5 m (Case b) reduced the maximum and differential settlements by more than 50%. The reduction of the column spacing in the central portion to 1.0 m (Case c) reduced the maximum and differential settlements by more than 80% as compared with Case a and more than 55% as compared with Case b. There was almost no differential settlement on the crest of the embankment in Case c. It is interesting to point out that Case c had the same number of columns as Case b. Therefore, Case c should be used over Case b in practice for better performance.

Fig. 11 presents the settlement profiles at the bases of the widened embankments at the end of consolidation, which shows the effect of column layout on the performance of the widened embankment. The reduction of column spacing under the widened portion from Case a1 to Case a2, Case b1 to Case b2, or Case c1 to Case c2 reduced the maximum and differential settlements at the base of the embankment. The comparison between Case a2 (i.e., column spacing at 2 m under the existing embankment and 1.5 m under the widened portion) and Case b1 (i.e., column spacing at 1.5 m under the existing embankment and 2 m under the widened portion) shows that the effect of column spacing under the existing embankment was greater than that under the widened portion. The difference in their maximum settlements at the bases was more than 20%.

As discussed earlier, due to the influence of the soft soil under the existing embankment, the additional columns installed under the connecting side slope of the existing embankment significantly reduced the maximum settlement of the widened embankment. The location of the maximum base settlement in Case c3 shifted significantly towards the widened portion as discussed earlier. This shift is because the soft soil improved by the columns under the widened portion had lower composite strength and modulus than those under the connecting side slope of the existing embankment.

### 3.2.3. Horizontal displacement

The numerical results of the horizontal displacements in three cases before widening at the end of consolidation are presented in Fig. 12 while those in seven cases after widening are presented in Figs. 13–15. The sections (AA’, BB’, CC’, and DD’) are shown in Fig. 4. Fig. 12 shows that the horizontal displacements before widening were symmetric and decreased as the column spacing decreased from Case a, Case b to Case c. Doubling the number of columns (i.e., reducing the spacing by twice) from Case a to Case c in the central portion of the existing embankment before widening reduced the maximum horizontal displacements by 80%.

Figs. 13–15 show the horizontal displacements of the soil under the widened embankment. The horizontal displacements along Sections AA’ and BB’ were almost same in all the cases except Case c3; therefore, the column layout under the existing embankment had a minor effect on the horizontal displacements at these two locations. The installation of the columns under the connection side slope significantly reduced the vertical deformation of the soft soil thus reducing the horizontal movement. It is shown that Section CC’ had smaller horizontal displacements in all the cases except Case c1 and Case c2. As shown in Table 3, Cases a1, a2, b1, and b2 had the locations of the maximum base settlements close to 25 m from the left toe of the existing embankment, which is close to the location of Section CC’. This result implies that Section CC’ was close to the axis of the center of the soil horizontal movement under the connection side slope. The reason for Case c3 to have small horizontal displacements is that the overall soil deformation and movement in this case were small. Due to the shift of the locations of the maximum base settlements in Cases c1 and c2, they had increased horizontal displacements towards the existing embankment. Section DD’ had the largest horizontal displacement in all the cases except Case c3. It is also shown that the widened embankment with larger column spacing under the existing embankment (Case a1 or a2) had the larger horizontal displacements at Sections DD’ and EE’ than that with smaller column spacing (b1, b2, c1, c2, or c3). The reduction of the column spacing under the widened portion (for example, Case a2 vs. Case a1, Case b2 vs. Case b1, or Case c2 vs. Case c1) reduced the horizontal displacements at Sections DD’ and EE’ as expected.

![Fig. 10. Settlement profile at the base of the existing embankment at the end of consolidation.](image)

![Fig. 11. Settlement profile at the base of the widened embankment at the end of consolidation.](image)
3.2.4. Transverse gradient change

Han et al. (2007) defined the transverse gradient change as the distortion (i.e., different settlement/distance) of the pavement in percent due to widening of the embankment. This distortion is also the slope change of the pavement in the transverse direction. In this numerical study, the pavement of the existing embankment was assumed to be horizontal initially (this condition was ensured by the small-strain mode); therefore, the slope change of the pavement from the initial position was equal to the current slope of the crest after widening.

Fig. 16 presents the settlement profiles of seven cases on the crest of the embankment at the end of consolidation. The transverse gradient changes for the widened embankment can be calculated by plotting the settlements on the crest of the existing embankment and the widened portion against the distances from the location of the maximum settlement to the shoulders of the widened embankment. In other words, the transverse gradient changes for the existing embankment and the widened portion were determined as the slopes of two straight lines as shown in Fig. 16. The results of the transverse gradient changes are summarized in Table 4. Fig. 16 and Table 4 both show that the transverse gradient change for the widened portion was always less than that for the existing embankment. This result is different from that presented by Han et al. (2007), in which the widened portion was constructed once. The settlement on the crest by staged construction in this study did not include the settlement occurring before the pavement was placed. The comparison between Case a1 and Case s2, Case b1 and Case b2, or Case c1 and Case c2 shows that the reduction of the column spacing under the widened portion increased the transverse gradient change under the widened portion but reduced that under the existing embankment. On the contrary, the reduction of column spacing under the existing embankment had more significant effect on the reduction of transverse gradients under both the existing embankment and the widened portion (for example, Case b1 vs. Case a2).

The gradient difference between the widened portion and the existing embankment, \( g_w - g_e \), is an important parameter to evaluate the performance of a widened embankment, where \( g_w \) and \( g_e \) are the transverse gradient changes for the widened portion and the existing embankment, respectively. Ling et al. (2003) suggested that \( 0.18 \leq g_w - g_e \leq 0.43 \). Table 4 shows that the reduction of the gradient difference under the widened portion and the existing embankment, \( g_w - g_e \), from Case a1 to Case a2,
Case b1 to Case b2, or Case c1 to Case c2 was not more than 5% while the reduction between Case a2 and Case b1 reached 29%. This result implies that the reduction of column spacing under the existing embankment was more effective in reducing the gradient difference than that under the widened portion. As compared with Case c2, the installation of additional columns under the connection side slope (i.e., Case c3) resulted in 86% reduction in the gradient difference. Table 4 shows that Case c3 had $g_{w} - g_{e} = 0.39$, which was considered satisfactory based on the requirement suggested by Ling et al. (2003).

### 3.2.5. Vertical stress

The vertical stresses at the depth of 0.25 m below the base of the widened embankment were extracted from the numerical analysis, which shows stress concentration on columns. To evaluate the stress concentration, a stress concentration ratio, $n$, defined as the ratio of the average stress on the columns to average stress in the surrounding soil, was used in this study. Table 5 summarizes the calculated $n$ values in four zones under the widened embankment. $n_1$, $n_2$, $n_3$, and $n_4$ correspond to the column-improved ABB′A′, BCC′B′, CDD′C′, and DEE′D′ zones, respectively in Fig. 4. Table 5 shows that the stress concentration ratios under the widened embankment were not uniform. Cases a1 and b2 had nearly symmetric stress concentration ratios under the exiting embankment and the widened portion due to the uniform distribution of the columns, but those under the slopes were higher than those under the crest. It can be noted that the column spacing had a significant effect on the $n$ value. The stress concentration ratio increased as the column spacing decreased. Case b2 had more than 80% higher stress concentration ratios than Case a1 due to the reduction of column spacing from 2.0 to 1.5 under both the existing embankment and the widened portion. The reduction of the column spacing under the widened portion from 2.0 to 1.5 m in Cases a1, b1 and c1 as compared with Cases a2, b2, and c2 respectively led to an increase of the $n_4$ value by 44–64%. The addition of the columns under the connection side slope further increased the stress concentration ratio. The increase of the stress concentration ratio reduced the vertical stresses on the surrounding soil thus reducing the horizontal resistance to soil movement. As a result, Cases c1 and c2 had larger horizontal displacements along Section CC′ compared to other cases as discussed earlier.

### 4. Limitations

It should be pointed out that the numerical model used in this study has some limitations, for example:
A two-dimensional numerical model was used in the numerical analysis. In field, columns are often arranged in individual column, grid, wall, block patterns or a combination of these patterns. Under such conditions (except for the wall pattern), a three-dimensional model is more appropriate.

In the analysis, the modulus of the columns was assumed as constant. In reality, the modulus may be nonlinear and change with time and stress level.

In the numerical study, the soft soil was assumed being underlain by bedrock. This condition is not necessarily always true in real projects.

The geotextile was modeled as a linearly elastic material with a constant tensile stiffness. In field, a geotextile often has a nonlinear behavior including strain dependence and creep and/or stress relaxation with time.

5. Conclusions

Widening of existing column-supported embankments over soft soil presents an important geotechnical and pavement problem which has not been well addressed previously. Field observations showed that cracks, drop-off of roads, and instability of slopes occurred after widening. This paper presents a two-dimensional numerical study that first verified the numerical model against field data and then investigated the effect of column layout on the performance of widened embankments on columns in soft clay. This study adopted the Modified Cam-Clay constitutive model for the soft clay to consider its consolidation under existing embankments and during and after widening. The following conclusions can be made from this study:

1. The mechanically and hydraulically coupled two-dimensional numerical model with the Modified Cam-Clay model for the soft soils reasonably simulated the performance of deep-mixed columns-supported embankment in field.

2. The column layout and spacing had influences on the behavior of exiting and widened column-supported embankments over soft soil.

3. Considering the soil consolidation during the construction and service of the existing embankment and the widened portion, the maximum settlement occurred at the base of the widened embankment, which is different from the finding in the previous study without considering soil consolidation. The location of the maximum settlement varied due to the installation of columns in different layout.

4. The ratio of the maximum settlement on the crest to that at the base of the widened embankment was approximately 70% except the case with the closer spacing in the central

Fig. 15. Horizontal displacements below the widened embankment at the end of consolidation after widening Case c. (a) Case c1, (b) Case c2 and (c) Case c3.
portion of the existing embankment and the one under the connection side slope of the widened embankment.

5. The columns with smaller spacing in the central portion of the foundation under the existing embankment most effectively reduced the total and differential settlements as compared with those under the slopes.

6. The installation of columns under the connection side slope most effectively reduced the total and differential settlements and the horizontal displacements under the widened embankment.

7. The reduction of the column spacing under the widened portion reduced the horizontal displacements more effectively under the widened portion than that under the existing embankment.

8. The transverse gradient change for the widened portion was always smaller than that for the existing embankment. The reduction of column spacing under the existing embankment had more significant effect on the reduction of

![Graph](attachment:image.png)

Fig. 16. Transverse gradient change of the widened embankment at the end of consolidation. (a) Case a1 and Case a2, (b) Case b1 and Case b2, (c) Case c1, Case c2, and Case c3.

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Transverse gradient change (%)</th>
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<tbody>
<tr>
<td>Case</td>
<td>a1</td>
</tr>
<tr>
<td>Existing, $g_e$</td>
<td>-2.92</td>
</tr>
<tr>
<td>Widened, $g_{w_c}$</td>
<td>1.67</td>
</tr>
<tr>
<td>$\Delta g_{w_c}$</td>
<td>4.59</td>
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</table>

<table>
<thead>
<tr>
<th>Table 5</th>
<th>Stress concentration ratio $n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
<td>$n_1$</td>
</tr>
<tr>
<td>a1</td>
<td>10.92</td>
</tr>
<tr>
<td>a2</td>
<td>11.04</td>
</tr>
<tr>
<td>b1</td>
<td>19.55</td>
</tr>
<tr>
<td>b2</td>
<td>19.63</td>
</tr>
<tr>
<td>c1</td>
<td>25.21</td>
</tr>
<tr>
<td>c2</td>
<td>25.49</td>
</tr>
<tr>
<td>c3</td>
<td>26.27</td>
</tr>
</tbody>
</table>
transverse gradients and gradient difference under both the existing embankment and the widened portion.

9. The stress concentration ratios under the widened embankment were not uniform. The stress concentration ratio increased as the column spacing decreased.

References


