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Synergistic Damage Mechanic Model for Stiffness Properties of Early Fatigue Damage in Composite Laminates

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Abstract

In the initial period of the life in the composite laminates, the principal types of damage are diffused ones, such as matrix crack, diffused fiber breaking and local delamination. On account of these diffused damages, a synergistic damage mechanic model was proposed for the stiffness properties. The model included the microcosmic responses of the physical damage and macroscopic performance of the material's stiffness. In micro-level, mesoscopic RVE(representative volume element) model was established to obtain crack opening displacement and crack sliding displacement, which were used to define the damage tensor. In macro-level, through homogenizing the material strain and the surface displacement of the damage, the relationship of the stiffness matrix of unidirectional laminate or laminates in damage statue and damage tense was set up. Due to restriction of NDT (non-destructive testing) technology development, only the constitutive relations of matrix cracks were constructed. The influences of the transverse matrix cracks on the stiffness properties of the laminates $[0/\pm 45]_s$ was analyzed with the present model and showed that it is capable to predict the reduction of the stiffness properties resulted from the fatigue diffused damage in the laminates.

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1. Introduction

Advanced composites have been widely used in mechanical and aerospace field for its specific strength and specific stiffness. For predicting the fatigue life of the composite structure, we need to investigate the mechanism of the composite damage and failure. The common damage modes in composites include matrix cracking, interface debonding, delamination, fiber breaking and so on. The residual stiffness of the composite reflects the damage status inside the material in the macro-level. Therefore, it's meaningful to construct the relations between each damage mode and the stiffness property of the composite laminates.

In the life of the composite, damage evolution goes through three periods, shown as Fig 1^[1]. In the prior period, matrix cracking distributes and dominates in the laminates. As the loading process, more and more cracks appear, until the crack density saturates. This configuration is termed “characteristic damage state” (CDS). Subsequently, the existing cracks can cause interfacial debonding, few fiber breakings and local delamination. Until this time, damages all present diffused characteristics. At last, large scale delamination and extensive fiber breakages emerge, which are highly localized and lead to lose the integrity of the laminates.

Composite damage has two characterizations: (1) in the prior and middle period, diffused multiple cracks dominate in the composite. While in the isotropic material, one isolated crack controls the damage evolution. This paper studies the influence of these fatigue diffused damage to the stiffness properties of the composites. (2) Damages evolve along the specific directions in the laminates, like matrix cracks always extend in the fiber direction. Hence, different diffused damages are defined according to the different directions.

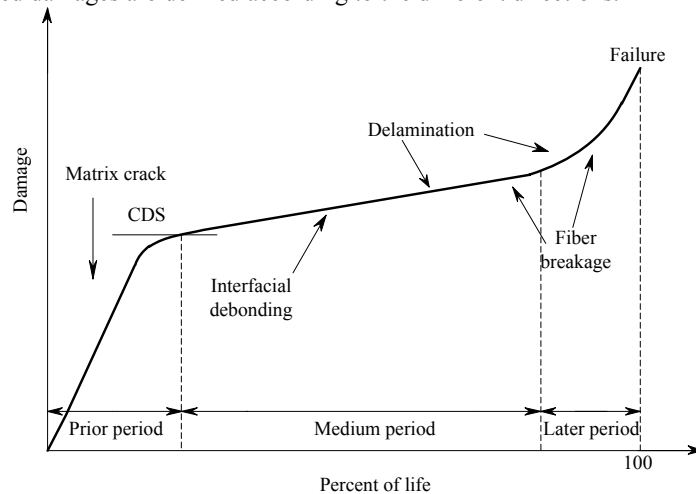


Fig.1. Damage evolution of the composites

Till now, scholars have analyzed each damage mode in detail. The study of matrix crack is more mature, and two different perspectives have been summarized^[2]. One directly concerns at the micro-scale, and hence can be called “micro-damage mechanics” (MIDM), which contains the well-known Shear-lag model^[3], Self-consistent scheme^[4], Variational methods^[5], Equivalent constrain model (ECM)^[6], Generalized plain strain analysis^[7], FEM^[8] and so on. These methods, through solving the stress and strain field in the vicinity of crack, can obtain the stiffness properties under a certain cracking status. However, the stiffness properties with all the damage modes in arbitrary lay-ups could not be predicted by some of them. This limitation can be alleviated by incorporating computational methods, so-called computational micromechanics. The other approach looks at macro or structure scale by defining internal variables to describe damages, e.g. continuum damage mechanics (CDM) proposed by Talreja^[9]. Combining thermodynamic equation, material constitutive with damage can be obtained. Whereas, traditional CDM needs a great quantity of experiments for determination of material coefficients, the engineering application is restricted. Synergistic damage mechanics (SDM) also proposed by Talreja^[10], makes up the deficiency of CDM, through

obtaining the responses of damages in micro-level by constructing micro-level models. Singh^[11] presented nonlinear synergistic damage mechanics, which can get more accurate stiffness reduction with large matrix crack density by using high order damage tensor. In addition, Duan etc.^[12] adopted fracture mechanics to predict multi-directional stiffness reduction precisely. Gudmundson^[13] and Lundmark^[14] considered that crack surface displacements lead to the increasing strain, which results in decreasing stiffness. Nairn^[15], Talreja^[16] etc. have reviewed on this subject comprehensively.

On account of local delamination, scholars^[17, 18] tried to predict the decreasing stiffness with MIDM methods. Stress transfer mechanism of Fiber breakage was discussed by Piggott^[19] systematically. It was influenced by interface debonding, sliding friction and residual stress. Jiang etc.^[20] studied the fiber breakage with FEM and took the influence of interface debonding into account. Yao etc.^[21] established a meso-mechanics model with series-parallel method and predicted multi-directional stiffness decline with the increasing percentage of the fiber breakage.

Composite laminates with damage are considered as consisting of stationary entities (e.g. fibers and plies) and evolving entities (e.g. cracks and voids)^[22]. In CDM, the homogenization is illustrated in Fig. 2 as a two-step process. First step, after homogenizing stationary microstructures, composite laminates turn into a homogenized entity with diffused damages, which is the micro-level analysis object. Second step, homogenizing evolving microstructures is the process of the macro-level analysis. A meso-mechanics model is constructed in micro-level. It can not only reflect the damage micro responses, but also characterize the stiffness properties of the overall composite.

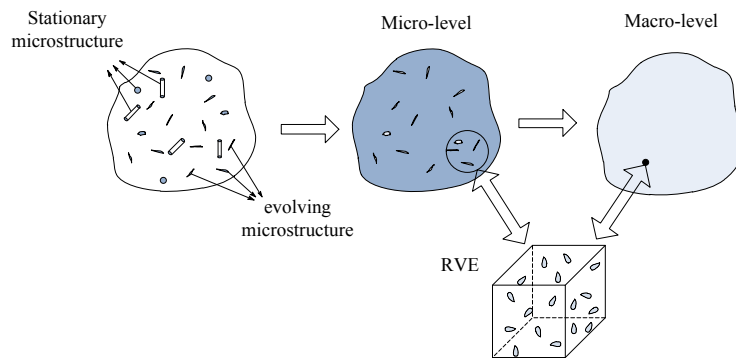


Fig.2. Analytical scales of continue damage mechanics in composites

In the present paper, a damage tensor was defined in micro-level, and damage micro responses were obtained by establishing meso-mechanics model. In macro-level, based on the similar idea with Gudmundson^[13] and Lundmark^[14], the surface displacements of the crack were normalized with the average stress field in the ply rather than the far field stress field. With homogenization of the material strain and the micro responses of the diffused damage, the damage stiffness matrix of unidirectional composite was deduced from the initial stiffness matrix and the damage tensor. Adopting this model, although, the constitutive relations of all the diffused damages could be constructed respectively, not all these diffused damages could be detected experimentally with regret. And then, the stiffness properties of the damage composite laminates can be characterized by classic laminate theory (CLT). At last, the influences of the transverse matrix cracks on the stiffness properties of the laminates $[0/\pm 45]_s$ was analyzed with the present model and showed that it is capable to predict the reduction of the stiffness properties resulted from the fatigue diffused damage in the laminates.

2. Damage definition

Crack is the basic component of the damage. A plane micro-crack element in space dS is shown in Fig. 3.

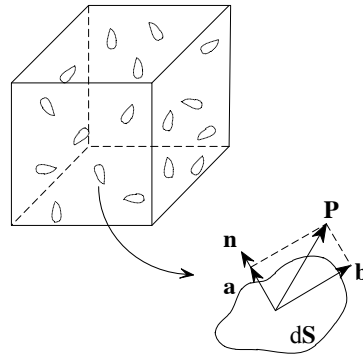


Fig.3. Schematic drawing of micro-crack

$$d\mathbf{S} = \mathbf{n} \cdot dS \quad (1)$$

Where \mathbf{n} is the normal vector on the micro-crack element. A spatial vector \mathbf{P} denotes the effect of the micro-crack on the material.

Vakulenko and Kachanov^[23] put forward while studying plane crack: the damage of a crack can be described with two vectors ($d\mathbf{S}$ and \mathbf{P}). The influence vector \mathbf{P} denotes the possible displacements on the crack surface. Hence, the damage entity tensor is defined, that is a dyadic product of these two vectors integrated over the surface S .

$$\mathbf{d} = \int_S \mathbf{P} \otimes d\mathbf{S} \quad (2)$$

Under usual circumstances, damage tensor is asymmetric. Therefore, the influence vector \mathbf{P} should be decomposed along the normal and tangential directions on the damage surface:

$$\mathbf{P} = \mathbf{a} + \mathbf{b} \quad (3)$$

Where \mathbf{a} and \mathbf{b} are respectively the normal and tangential projects of vector \mathbf{P} . This paper proposes that \mathbf{a} and \mathbf{b} separately indicate normalized normal direction and tangential direction displacements on the crack surface, crack opening displacement (COD) and crack sliding displacement (CSD) under unit load. They are the micro responses of the damage. And hence:

$$\mathbf{d} = \mathbf{d}^{(1)} + \mathbf{d}^{(2)} = \int_S \mathbf{a} \otimes d\mathbf{S} + \int_S \mathbf{b} \otimes d\mathbf{S} \quad (4)$$

Aiming at the diffused damage in prior and middle periods, different damage modes in every ply can be homogenized respectively, according to the directional characterization of the composite damage. Denoting damage mode by $\alpha = 1, 2, \dots, n$, each damage mode tensor can be defined as

$$\mathbf{D}^{(\alpha)} = \frac{1}{V} \sum_{n_\alpha} \mathbf{d}_{n_\alpha} \quad (5)$$

Where V is the volume of the ply and n_α is the number of α damage mode in each ply. This tensor indicates the average damage of α damage mode in the ply.

3. Stiffness properties of damaged ply

3.1. Macro responses of damage

The constitutive of damaged ply can be expressed as

$$\{\sigma\} = [Q]_d \{\varepsilon\} \tag{6}$$

Where $\{\sigma\}$ and $\{\varepsilon\}$ are the stress and strain on the ply respectively, and $[Q]_d$ is the stiffness matrix of damaged ply.

Damaged material contains cracks and undamaged material inside. And homogenizing the stress and strain of the inner material, we get,

$$\sigma_{ij}^a = \frac{1}{V} \int_V \sigma_{ij} dV \quad \varepsilon_{ij}^a = \frac{1}{V} \int_V \varepsilon_{ij} dV \tag{7}$$

The superscript ‘‘a’’ means the average value in the volume V . We make assumption that the inside material satisfies the constitutive relation of undamaged state,

$$\{\sigma\}^a = [Q]_0 \{\varepsilon\}^a \tag{8}$$

Where is the initial stiffness matrix of the intact ply.

Allen and Yoon^[24] pointed out that the average strain of the inner material equals the strain applied to the material subtract the boundary averaged strain of the crack surface.

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \tau_{12} \end{Bmatrix}^a = \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} - \begin{Bmatrix} \omega_{11} \\ \omega_{22} \\ 2\omega_{12} \end{Bmatrix} \tag{9}$$

Where ω is theVakulenko-Kachanov tensor, which denotes the averaged strain of the crack surface and is expressed by

$$\omega = \frac{1}{V} \sum_n \int_{S_c} (\mathbf{a}^* + \mathbf{b}^*) \otimes dS_c \tag{10}$$

Where \mathbf{a}^* and \mathbf{b}^* are average relative COD and CSD in the volume V respectively, whose components are

$$a_i^* = \frac{1}{t} \int_{-\frac{t}{2}}^{\frac{t}{2}} \Delta u_n dx \quad b_i^* = \frac{1}{t} \int_{-\frac{t}{2}}^{\frac{t}{2}} \Delta u_t dx \tag{11}$$

Here Δu_n and Δu_t are relative COD and CSD of every crack, and t is the characteristic length of crack.

Normalizing the average displacements with the average stress field in the ply gives:

$$a_i = a_i^* \frac{E_i}{\sigma_i^a} \quad b_i = b_i^* \frac{G}{\tau^a} \tag{12}$$

Where E and G are in-plane tensile modulus and shear modulus respectively. And then the relations between the components of the Vakulenko-Kachanov tensor ω and damage tensor D are obtained.

$$\begin{aligned}\omega_{11} &= \frac{1}{V} \sum_n \int_{S_1} a_1^* dS_1 = \frac{1}{V} \sum_n \int_{S_1} a_1 \frac{\sigma_1^a}{E_1} dS_1 = D_{11} \frac{\sigma_1^a}{E_1} \\ \omega_{22} &= \frac{1}{V} \sum_n \int_{S_2} a_2^* dS_2 = \frac{1}{V} \sum_n \int_{S_2} a_2 \frac{\sigma_2^a}{E_2} dS_2 = D_{22} \frac{\sigma_2^a}{E_2} \\ \omega_{12} &= \frac{1}{V} \sum_n \left(\int_{S_1} b_2^* dS_1 + \int_{S_2} b_1^* dS_2 \right) = \frac{1}{V} \sum_n \left(\int_{S_1} b_2 dS_1 + \int_{S_2} b_1 dS_2 \right) \frac{\tau_{12}^a}{G_{12}} = D_{12} \frac{\tau_{12}^a}{G_{12}}\end{aligned}\quad (13)$$

Defining the damaged matrix $[D^*]$:

$$[D^*] = \begin{bmatrix} D_{11} & 0 & 0 \\ 0 & D_{22} \frac{E_1}{E_2} & 0 \\ 0 & 0 & 2D_{12} \frac{E_1}{G_{12}} \end{bmatrix}\quad (14)$$

Then we have:

$$\{\omega\} = \frac{1}{E_1} [D^*] \{\sigma\}^a\quad (15)$$

Substituting Equation (9) in Equation (8) gives:

$$\{\sigma\}^a = [Q_0] (\{\varepsilon\} - \{\omega\})\quad (16)$$

And then substituting Equation (15) in Equation (16) shows:

$$\{\sigma\}^a = [Q_0] \{\varepsilon\} - \frac{1}{E_1} [Q_0] [D^*] \{\sigma\}^a\quad (17)$$

The Equation (17) can be rewritten as

$$\{\sigma\}^a = \left([1] + \frac{1}{E_1} [Q_0] [D^*] \right)^{-1} [Q_0] \{\varepsilon\}\quad (18)$$

Due to the balance between the inside and outside of the material, the average stress inside equals to the stress applied to the ply:

$$\{\sigma\}^a = \{\sigma\}\quad (19)$$

Stiffness matrix of the damaged ply can be expressed as:

$$[Q_d] = \left([I] + \frac{1}{E_1} [Q_0] [D^*] \right)^{-1} [Q_0] \quad (20)$$

Further, the modulus and Poisson ratio of the single ply can be obtained.

$$\begin{aligned} E_1 &= \frac{Q_{11}^d Q_{22}^d - Q_{12}^{d2}}{Q_{22}^d} & \nu_{12} &= \frac{Q_{12}^d}{Q_{22}^d} \\ E_2 &= \frac{Q_{11}^d Q_{22}^d - Q_{12}^{d2}}{Q_{11}^d} & G_{12} &= Q_{66}^d \end{aligned} \quad (21)$$

3.2. Micro responses of damage

In the present paper, damage is defined in the micro-level with \mathbf{a} and \mathbf{b} , the normalized displacements of crack surface, which objectively characterize the influences of cracks to the material properties. Meanwhile, they reflect the constraint to cracks from the adjacent material and are independent of the loading state. Besides, they form the damage tensor and the damaged matrix, which are the bridges between the damage stiffness matrix and the initial one.

Experimental method and FEM are two methods to determine the normalized displacements of crack surface. Varna^[25] designed a Micro-Specimen Test to measure COD directly with a set of precision instruments. Or through measuring engineering elastic constants under certain damage state, we can derive the displacements according to the present model. However, considering these displacements relate to the material and lay-ups closely, a large amount of experiments are required to determine the micro responses of damage in arbitrary material system. FEM compensates for this disadvantage very well. The average displacements of crack surface and the average stresses, extracted from the RVE meso-model, can derive the micro responses of damage.

4. Stiffness properties of damaged laminates

On the basis of the model of the stiffness properties of damaged ply, the stiffness matrix of the damaged ply is obtained under the local coordinate system. Through coordinate transformation, the stiffness matrix under the local coordinate system turns into the one under the global coordinate system of the laminates, as follows:

$$[\bar{Q}_d]_i = [T]^{-1} [Q_d]_i [T]^{-T} \quad (22)$$

Where, $[T]$ is the coordinate transformation matrix, i donates the i_{th} ply.

Thereby assembling the damaged stiffness matrix of the plies gives that of the laminates:

$$[\bar{Q}_d]_{LAM} = \frac{1}{h} \sum_{i=1}^n t_i [\bar{Q}_d]_i \quad (23)$$

Here h is the thickness of the laminates, and t_i is the thickness of the i_{th} ply.

5. Analysis of example

Due to the restriction of NDT (non-destructive testing) technology development, experiment data of the diffused damages limits to the measurement of transvers crack. Generally, we get transvers crack density or the distance between cracks by the edge replica, X-ray deduction and so on. But the local delamination and fiber breaking cannot

be detected quantitatively. At the same time, the engineering elastic modulus is obtained, and then the one-to-one relationship between micro damage and macro stiffness is constructed.

5.1. Analysis object

The method for stiffness properties was tested by the experiment in the paper^[26]. Glass-fiber reinforced composite material made from 3M Scotchply 1003 was chosen, and the elastic properties are displayed in Table 1. The plate specimen has a size of 203mm×25.4mm with $[0/\pm 45]_s$ lay-up, and the single ply thickness is 0.203mm.

Table 1 Glass fiber reinforced polymer properties from 3M Scotchply 1003^[26]

E_1 (MPa)	E_2 (MPa)	G_{12} (MPa)	ν_{12}
41 700	13 000	3400	0.300

In the experiment, the specimens carried tension-tension fatigue load, which had 172MPa maximum stress level and stress ratio $R=0.1$. The extensometer with 50.8mm gauge length was adopted to measure the longitudinal elastic modulus. At 1, 5000, 10000, 50000 cycle respectively, damage states at the edge of the specimen were replicated and recorded as shown in Fig. 4.

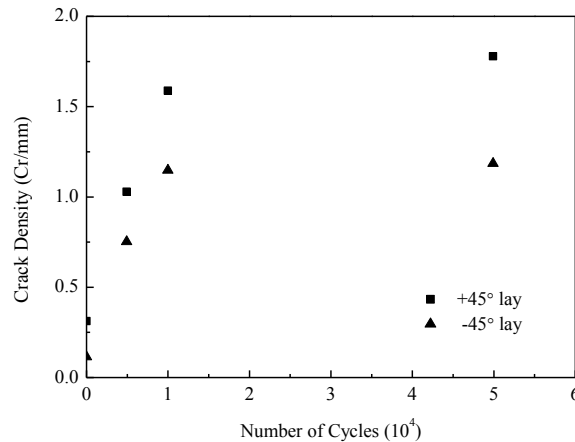


Fig.4. Matrix crack density in $\pm 45^\circ$ lays under fatigue load

5.2. Calculating normalized crack surface displacements

Loading in the longitudinal direction, transvers cracks are taken place in both 45° and -45° ply. Because load direction is not perpendicular to the crack surface, COD and CSD of the transvers crack should be considered at the same time. Assuming transvers matrix crack penetrates the specimen in the width direction, finite element model was constructed to get normalized COD and CSD.

Adopting ABAQUS FEM commercial software, 3D finite element model of RVE (representative volume element) was established as shown in Fig. 5. We chose 1/8 of the RVE to build the model according to the symmetry of structure and load. Moreover, considering the cracks in 45° and -45° ply interact with each other, 5 cracks with average distance in each damaged ply were preset by releasing nodes in the crack surface. Between the plies, cohesive element ply was inserted to simulate the deformation of adhesive. The model contained three symmetrical surfaces, which were separately set a displacement constraint and two rotational constraints. And a longitudinal displacement was loaded in the other side of RVE.

According to the results of FEM, the normalized COD and CSD could be obtained from the average displacements in the crack surface divided by the average stress field in the damaged ply.

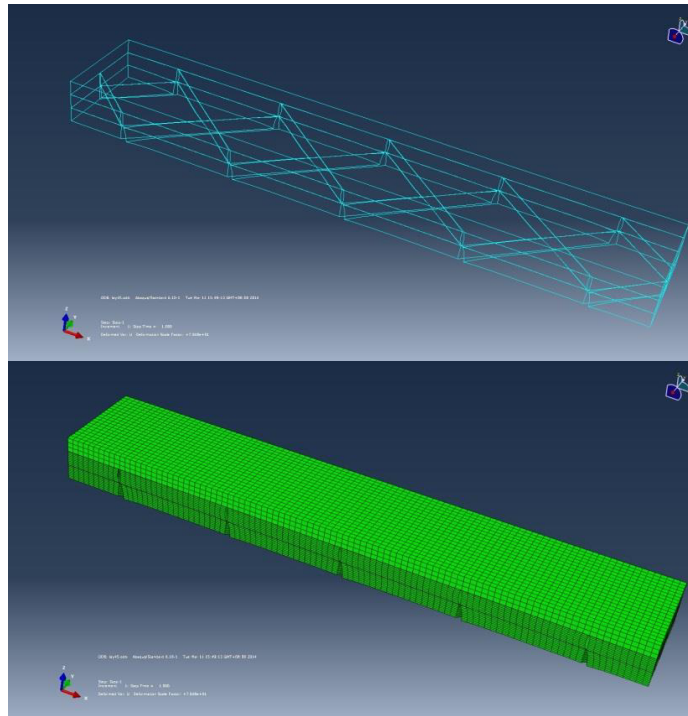


Fig.5. RVE finite element model of $[0/\pm 45]_s$ laminates

5.3. Analysis result

The trend of the longitudinal normalized stiffness of the laminates $[0/\pm 45]_s$ with the load cycles is shown in Fig. 6.

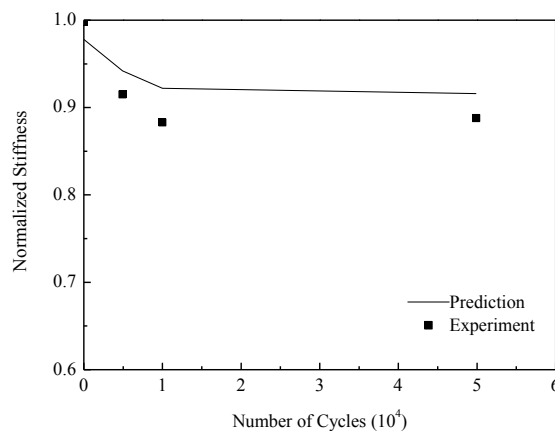


Fig.6. Normalized stiffness change with load cycles in $[0/\pm 45]_s$ laminates

From the figure, the predicted results fit the experiment ones well, with relative errors less than 5%. However, the prediction results are larger than the experiment ones, which are resulted from the existence of local delamination at the transvers crack tips. This damage mode was not taken into consideration, which leads to the errors.

6. Discussion and conclusion

The present paper characterized the deduction of the stiffness properties because of diffused damage, considering the average stain of the internal material is equivalent to the applied strain outside subtracted by the average strain of the crack surface (inner boundary), like the methods from Gudmundson^[13] and Lundmark^[14]. After comparing, this model is simpler than Gudmundson's one, and crack surface displacements are more accurate with FEM. Not like Lundmark's model, which obtained the stiffness properties directly in the laminate level and normalized COD and CSD with far field stress, this paper started in the ply level, and used the average stress inside of the ply to normalize COD and CSD. The definition of damage tensor was proposed with normalized crack surface displacements. This change made this model suitable to not only transvers matrix crack mode but also other diffused damage modes. Constrained by NDT technology development, only transvers cracks in fatigue load were characterized. Other modes are waiting for experimental testifying, and more accurate result of transvers crack will be obtained.

The synergistic damage mechanic model characterizes the deduction of stiffness properties in the composite laminates caused by diffused damages, which dominate the prior period of fatigue life.

- In micro level, damage tensor is defined by normalized COD and CSD, which can be obtained from RVE finite element model.
- Through detail analysis in multi-scale, the stiffness properties of composite are determined by the initial stiffness and the damage tensor.
- This paper tried to use a self-consistent model to characterize all the diffused damage, even though NDT technology cannot satisfy the experimental requirement.
- Through analysis of example, the result showed that the model can predict the reduction of the stiffness properties resulted from the diffused matrix cracks under fatigue load.

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