Journal of the Association of Arab Universities for Basic and Applied Sciences (2011) **10**, 28–32

University of Bahrain

Journal of the Association of Arab Universities for Basic and Applied Sciences

www.elsevier.com/locate/jaaubas
www.sciencedirect.com

ORIGINAL ARTICLE

Spatial dependence of moving atoms with a two-photon process

E.W. Ahmed [a,*], **A.M. Ahmed** [b]

[a] *BTC, Bahrain Teachers College, University of Bahrain, P.O. Box 32038, Bahrain*
[b] *Nuclear Research Center, Atomic Energy Authority, 13759 Cairo, Egypt*

Available online 23 July 2011

KEYWORDS

Moving atoms;
Two-photon process

Abstract We propose an alternative treatment to describe the spatial-dependence of the non-resonant two-photon mazer. Previous work presented a treatment for studying the cold atom micromaser (mazer) but under certain restrictive conditions. We now extend those results to a general case taking into account the spatial dependence, off-resonant interaction and Stark-shift. In a mesa mode profile, we obtain an exactly analytic solution of the model, by means of which we analyze the analytical form of the emission probability. We demonstrate that, when the spatial dependence effects are taken into consideration, the feature of the emission probability is influenced significantly.

1. Introduction

Cold and ultracold atoms introduce new regimes in atomic physics often not considered in the past (Wieman et al., 1999). Many new concepts and new phenomena, that involve ultracold atoms, are proposed or observed. Among them are atom optics (Adams et al., 1994), Bose Einstein condensation (Griffin et al., 1995), atom lasers (Bloch et al., 1999), nonlinear atom optics (Lenz et al., 1994), nonlinear optics of matter waves (Goldstein et al., 2000) and mazer action (Scully et al., 1996; Meyer et al., 1997; Loffler et al., 1997; Schröder et al., 1997; Zhang et al., 1999; Aran et al., 2000). In these phenomena, the interaction between ultracold atoms and quantum radiation fields plays an important role (Haroche et al., 1991; Englert et al., 1991) and the quantization of the center-of-mass (c.m.) motion of atoms must be taken into account in studies on this kind of interaction. Studies in Haroche et al. (1991) and Englert et al. (1991) have been extended by including the dissipation of the photon energy into the description in Battocletti and Englert (1994). In all these previous studies, the mazer properties were always presented in the resonant case where the cavity mode frequency is equal to the atomic transition frequency. In Obada and Abdel-Aty (2000) and Abdel-Aty and Obada (2002a,b) those restrictions have been removed and a general theory of the mazer has been established. Further, the research on quantum treatment of the atomic motion has been extended to study the off-resonant case (Bastin and Martin, 2003).

On the other hand, during the last decade many theoretical and experimental efforts have been done in order to study two-photon processes involving atoms inside a cavity, stimulated by the experimental realization of a two-photon micromaser.

* Corresponding author.
E-mail address: ewefky@btc.uob.bh (E.W. Ahmed).

Peer review under responsibility of University of Bahrain.
doi:10.1016/j.jaubas.2011.06.005

Production and hosting by Elsevier

ELSEVIER

In view of continuing technological improvements in micro-cavities even at optical frequencies recent work (Concannon et al., 1997) has motivated the examination of certain aspects of the two-photon mazer theory that are fundamental to the process. These aspects have their counterpart in the usual single-photon mazer, but rather different behavior is to be expected in the two-photon case, owing to the essential nonlinearity of the process. We have here in mind a degenerate two-photon mazer with the upper state connected to the lower one by a two-photon process.

We discuss the emission probability of the two-photon system, taking into account the spatio-temporal dependence. This is most conveniently accomplished in a quantum theory of the mazer formalism in terms of the dressed state approach (Scully et al., 1996; Meyer et al., 1997; Loffler et al., 1997; Schröder et al., 1997; Si-de et al., 1998, Zhang and He, 1998 and Zhang et al., 2002). To make the two-photon processes closer to the experimental realization, we include the effect of the dynamic Stark shift in the evolution of the emission probability, which is necessary and interesting. Related treatment discussing the quantum theory of the two-photon maser without the spatial dependence (i.e. in front and after leaving the cavity), have been presented in the literature (Zhang et al., 1999). However in this problem three regions are distinguished: in front of the cavity described by $\mu(z)$ where incident and reflected waves are present, inside the cavity represented by $\mu(z)$, $\mu(z, L)$ where transient regime occurs; and after leaving the cavity described by $\mu(z, L)$ where transmitted waves are present with $\mu(z)$ the step-function. Other extensions are made namely the off-resonance case and the Stark shift effect are considered. Contrary to what is claimed in Bastin and Martin, 2003 we find that the problem, in the mesa mode case, reduces to an elementary scattering problem over a potential barrier and a potential well defined by the cavity even in the presence of detuning and Stark shift.

The material of this paper is arranged as follows: in the section 'General scheme' we start with the theoretical description of the model. We obtain an exactly analytic solution of the model, by means of which we analyze the analytical form of the emission probability. Finally conclusions are presented in the last section.

2. General scheme

The concept of the mazer has been applied to the two-photon process in Zhang et al. (1999). They have studied the quantized-z-motion-induced emission and the photon statistics of the micromaser pumped by slow atoms after leaving the cavity thus they did not include terms which describe the incident and transient parts in the wave function, which when added alter the dynamics of the system. However in this problem we have three regions: one of them described by $\mu(z)$ which represents the wave function in front of the cavity, $\mu(z) - \mu(z, L)$ which represents inside the cavity of length L, and the last one with $\mu(z, L)$ which describes the wave function after leaving the cavity. But, if the ideas are to be contemplated for applications, the issue of propagation inside the cavity is crucial. Addressing this issue is the purpose of the present paper.

We consider a two-level atom moving along the z-direction in the way to a cavity of length L. The atom is coupled with a two-photon transition to a single-mode of the quantized field present in the cavity. The atom-field interaction is modulated by the cavity field mode function. The atomic center-of-mass motion is described quantum mechanically and the rotating-wave approximation is made. The Hamiltonian describing the system is given by

$$\begin{aligned}\widehat{H} &= \frac{\widehat{P}_z^2}{2m} + \hbar\omega(\hat{a}^\dagger\hat{a} + (|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|) + \hat{a}^\dagger\hat{a}(\beta_1|\uparrow\rangle \\ &\quad \times \langle\uparrow| + \beta_2|\downarrow\rangle\langle\downarrow|)) + \frac{\hbar}{2}\Delta(|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|) \\ &\quad + \hbar\lambda u(z)(|\downarrow\rangle\langle\uparrow|\hat{a}^{\dagger 2} + \hat{a}^2|\uparrow\rangle\langle\downarrow|), \end{aligned} \tag{1}$$

where P_z is the atomic center-of-mass momentum along z-axis. We denote by λ, the atom-field coupling strength for the interaction between the cavity field and the atom, $u(z)$ is the mode function of the cavity field and m is the atomic mass. β_1 and β_2 are parameters describing the intensity-dependent Stark shifts of the two levels that are due to the virtual transitions to the intermediate relay. The operator a (a^+) is the annihilation (creation) operator for the cavity field, ω is the field frequency and Δ the detuning parameter. When a cold atom is approaching the interaction region, it can be reflected or transmitted according to quantum scattering theory.

In the interaction picture, let us write Eq. (1) in the following form:

$$\begin{aligned}\widehat{H} &= \frac{\widehat{P}_z^2}{2m} + \widehat{V}, \\ \widehat{V} &= \beta_1(|\uparrow\rangle\langle\uparrow| + \beta_2\hat{a}^\dagger\hat{a}|\downarrow\rangle\langle\downarrow|) + \frac{\hbar}{2}\Delta(|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|) \\ &\quad + \hbar\lambda u(\hat{z})(|\downarrow\rangle\langle\uparrow|\hat{a}^{\dagger 2} + \hat{a}^2|\uparrow\rangle\langle\downarrow|).\end{aligned} \tag{2}$$

The global Hilbert space of the system is given by $H = H_z \otimes H_A \otimes H_R$ with H_z the space of the wave functions describing the one-dimensional atomic center-of-mass motion, H_A the space describing the atomic internal degree of freedom, and H_R the space of the cavity single mode radiation. It is expedient to expand the atom-field state in terms of the states

$$\begin{aligned}|\Phi_0\rangle &= |0, \downarrow\rangle, \\ |\Phi_1\rangle &= |1, \downarrow\rangle, \\ |\Phi_n^+\rangle &= \sin\left(\frac{\varrho_n}{2}\right)|n, \uparrow\rangle + \cos\left(\frac{\varrho_n}{2}\right)|n+2, \downarrow\rangle, \\ |\Phi_n^-\rangle &= \cos\left(\frac{\varrho_n}{2}\right)|n, \uparrow\rangle - \sin\left(\frac{\varrho_n}{2}\right)|n+2, \downarrow\rangle,\end{aligned} \tag{3}$$

where

$$\begin{aligned}\varrho_n &= 2 \times \tan^{-1}\left(\frac{\lambda u(z)\sqrt{(n+1)(n+2)}}{\mu_n - \left(\frac{\Delta}{2} + \frac{1}{2}[n\beta_2 - (n+2)\beta_1]\right)}\right), \\ \mu_n &= \sqrt{\left(\frac{\Delta}{2} + \frac{1}{2}[n\beta_2 - (n+2)\beta_1]\right)^2 + \lambda^2 u^2(z)(n+1)(n+2)}.\end{aligned} \tag{4}$$

The states $|\Phi_n^\pm\rangle$ are z-dependent through the trigonometric functions, they satisfy

$$\begin{aligned}\frac{\partial}{\partial z}|\Phi_n^\pm\rangle &= \pm|\Phi_n^\mp\rangle\frac{d\varrho_n}{dz}, \\ \frac{\partial^2}{\partial z^2}|\Phi_n^\pm\rangle &= \pm|\Phi_n^\mp\rangle\frac{d^2\varrho_n}{dz^2} - |\Phi_n^\pm\rangle\left(\frac{d\varrho_n}{dz}\right)^2.\end{aligned} \tag{5}$$

Then, $\langle z|\Psi(t)\rangle$ can be expanded in the form $\langle z|\Psi(t)\rangle = \sum_n c_n^\pm(z,t)|\Phi_n^\pm\rangle$ and it satisfies the Schrödinger equation

$$i\frac{\partial}{\partial t}|\langle z|\Psi(t)\rangle = \widehat{H}\langle z|\Psi(t)\rangle.$$

Hence the coefficients $C_n^{\pm}(z,t)$ satisfy the coupled equation

$$\frac{\partial C_n^+(z,t)}{\partial t} = \left(-\frac{1}{2m}\frac{\partial^2}{\partial z^2} + V_n^+ - \left(\frac{d\varrho_n}{dz}\right)^2\right)C_n^+(z,t) - \left(2\frac{C_n^-(z,t)}{\partial z}\left(\frac{d\varrho_n}{dz}\right) + C_n^-(z,t)\left(\frac{d\varrho_n}{dz}\right)^2\right), \tag{6}$$

$$\frac{\partial C_n^-(z,t)}{\partial t} = \left(-\frac{1}{2m}\frac{\partial^2}{\partial z^2} + V_n^- - \left(\frac{d\varrho_n}{dz}\right)^2\right)C_n^-(z,t) + \left(2\frac{C_n^+(z,t)}{\partial z}\left(\frac{d\varrho_n}{dz}\right) + C_n^+(z,t)\left(\frac{d\varrho_n}{dz}\right)^2\right), \tag{7}$$

where

$$V_n^{\pm}(z) = \frac{\hbar}{2}[n\beta_2 + (n+2)\beta_1] \pm \hbar\sqrt{\left(\frac{\Delta}{2} + \frac{1}{2}[n\beta_2 - (n+2)\beta_1]\right)^2 + \lambda^2 u^2(\hat{z})(n+1)(n+2)}, \tag{8}$$

Eqs. (6) and (7) mean that, we get for each n two coupled partial differential equations. But once $u(z)$ is taken to be constant, then $\frac{d\rho_n}{dz}$ will vanish and these equations are decoupled over the entire z-axis and the problem reduces to an elementary scattering problem over a potential barrier and a potential well defined by the cavity even in the presence of detuning and Stark shift. This can be contrasted with what has been claimed earlier (Bastin and Martin, 2003).

In what follows we study the mesa-mode case of the field which means that $u(z)$ is constant inside the cavity and zero outside the cavity. In this case the states (3) become the dressed states of the system. We assume that, initially, the atomic center-of-mass motion is not correlated to the other degrees of freedom. We describe it by the wave packet

$$X(z) = \langle z|\phi(0)\rangle = \int dk G(k) e^{ikz}\theta(-z). \tag{9}$$

We denote by $\theta(-z)$ the Heaviside step function (indicating that the atoms are incident from the left of the cavity). The Fourier amplitudes $G(k)$ are adjusted such that the center of the wave packet enters the cavity at time $t = 0$. The initial state of the system can be written

$$\begin{aligned}|\psi(0)\rangle &= \sum_{n=0}^{\infty} q_n|n\rangle\left[\cos\left(\frac{\tau}{2}\right)|\uparrow\rangle + \sin\left(\frac{\tau}{2}\right)e^{-i\varphi}|\downarrow\rangle\right] \\ &= \sin\left(\frac{\tau}{2}\right)e^{-i\varphi}(q_0|0,\downarrow\rangle + q_1|1,\downarrow\rangle) \\ &\quad + \sum_{n=0}^{\infty}\left\{q_n\cos\left(\frac{\tau}{2}\right)|n,\uparrow\rangle + q_{n+2}\sin\left(\frac{\tau}{2}\right)e^{-i\varphi}|n+2,\downarrow\rangle\right\} \\ &= \sin\left(\frac{\tau}{2}\right)e^{-i\varphi}(q_0|\Phi_0\rangle + q_1|\Phi_1\rangle) + \sum_{n=0}^{\infty}(Y_{n-}|\Phi_n^-\rangle + Y_{n+}|\Phi_n^+\rangle),\end{aligned} \tag{10}$$

where

$$\begin{aligned}Y_{n+} &= q_n\sin\left(\frac{\varrho n}{2}\right)\cos\left(\frac{\tau}{2}\right) + q_{n+2}\cos\left(\frac{\varrho n}{2}\right)\sin\left(\frac{\tau}{2}\right)e^{-i\varphi}, \\ Y_{n-} &= q_n\cos\left(\frac{\varrho n}{2}\right)\cos\left(\frac{\tau}{2}\right) - q_{n+2}\sin\left(\frac{\varrho n}{2}\right)\sin\left(\frac{\tau}{2}\right)e^{-i\varphi}.\end{aligned} \tag{11}$$

In a mesa mode profile, the wave function of the atom-field interaction can be obtained using a straightforward calculation, in the following form

$$\begin{aligned}\langle z|\Psi(t)\rangle = \int dk G(k)\exp\left(-i\frac{\hbar k^2 t}{2M}\right)\sum_{\infty}^{n=0} q_n \\ \times\Big[\Big(\Big\{e^{ikz} + \Big(A_n^+\sin\left(\frac{\varrho n}{2}\right)\cos\left(\frac{\tau}{2}\right) \\ + A_n^-\cos\left(\frac{\varrho n}{2}\right)\sin\left(\frac{\tau}{2}\right)e^{-i\varphi}\Big)e^{-ikz}\Big\}\theta(-z) \\ + \Big\{B_n^+\sin\left(\frac{\varrho n}{2}\right)\cos\left(\frac{\tau}{2}\right) + B_n^-\cos\left(\frac{\varrho n}{2}\right)\sin\left(\frac{\tau}{2}\right)e^{-i\varphi}\Big\} \\ \times e^{ik(z-L)}\theta(z-L) \\ + \Big\{\left(\alpha_n^+ e^{ik_n^+ z} + \beta_n^+ \times e^{-ik_n^+ z}\right)\sin\left(\frac{\varrho n}{2}\right)\cos\left(\frac{\tau}{2}\right) \\ \times\left(\alpha_n^- e^{ik_n^- z} + \beta_n^- e^{-ik_n^- z}\right)\cos\left(\frac{\varrho n}{2}\right)\sin\left(\frac{\tau}{2}\right)\times e^{i\varphi}\Big\} \\ \times[\theta(z) - \theta(z-l)]\Big)|n,\uparrow\rangle \\ + \Big(\Big\{A_n^+\cos\left(\frac{\varrho n}{2}\right)\sin\left(\frac{\tau}{2}\right)e^{-i\varphi} - A_n^-\sin\left(\frac{\varrho n}{2}\right)\cos\left(\frac{\tau}{2}\right)\Big\} \\ \times e^{-ikz}\theta(-z) + \Big\{B_n^+\cos\left(\frac{\varrho n}{2}\right)\sin\left(\frac{\tau}{2}\right)e^{-i\varphi} - B_n^-\sin\left(\frac{\varrho n}{2}\right) \\ \times\cos\left(\frac{\tau}{2}\right)\Big\}e^{ik(z-L)}\theta(z-L) \\ + \Big\{\left(\alpha_n^+ e^{ik_n^+ z} + \beta_n^+ e^{-ik_n^+ z}\right)\cos\left(\frac{\varrho n}{2}\right)\sin\left(\frac{\tau}{2}\right) \\ \times e^{-i\varphi}\left(\alpha_n^- e^{ik_n^- z} - \beta_n^- e^{-ik_n^- z}\right)\times\sin\left(\frac{\varrho n}{2}\right)\cos\left(\frac{\tau}{2}\right)\Big\} \\ \times[\theta(z) - \theta(z-L)]\Big)|n+2,\downarrow\rangle\Big],\end{aligned} \tag{12}$$

where, the coefficients $A_n^{\pm}$, of the reflected waves $B_n^{\pm}$, of the transmitted waves and $\alpha_n^{\pm}$ and $\beta_n^{\pm}$ of the transient regime are given by

$$\begin{aligned}A_n^{\pm}(k) &= \frac{i\Upsilon_n^{\pm}\sin(k_n^{\pm}L)}{\cos(k_n^{\pm}L) - i\delta_n^{\pm}\sin(k_n^{\pm}L)}, \quad B_n^{\pm}(k) = \frac{e^{-ikL}}{\cos(k_n^{\pm}L) - i\delta_n^{\pm}\sin(k_n^{\pm}L)}, \\ \alpha_n^{\pm}(k) &= \frac{\frac{1}{2}\left(1 + \frac{k}{k_n^{\pm}}\right)e^{-ik_n^{\pm}L}e^{-ikL}}{\cos(k_n^{\pm}L) - i\delta_n^{\pm}\sin(k_n^{\pm}L)}, \quad \beta_n^{\pm}(k) = \frac{\frac{1}{2}\left(1 - \frac{k}{k_n^{\pm}}\right)e^{ik_n^{\pm}L}e^{-ikL}}{\cos(k_n^{\pm}L) - i\delta_n^{\pm}\sin(k_n^{\pm}L)},\end{aligned} \tag{13}$$

where

$$\begin{aligned}\delta_n^{\pm} &= \frac{1}{2}\left(\frac{k_n^{\pm}}{k} + \frac{k}{k_n^{\pm}}\right), \quad \Upsilon_n^{\pm} = \frac{1}{2}\left(\frac{k_n^{\pm}}{k} - \frac{k}{k_n^{\pm}}\right), \\ k_0^2 &= \sqrt{k^2 + \frac{m\Delta}{\hbar}}, \quad k_1^2 = \sqrt{k^2 + \frac{m}{\hbar}(\Delta - 2\beta_2)}, \\ k_n^{\pm} &= \sqrt{k^2 - \frac{\gamma^2}{2}[n\beta_2 - (n+2)\beta_1] \mp \gamma^2\sqrt{\Delta^2/\lambda^2 + (n+1)(n+2)}}.\end{aligned} \tag{14}$$

It is to be noted that the vacuum coupling energy $\hbar\lambda = (\hbar\gamma)^2/2m$, and $\hbar k$ is the atomic center-of-mass momentum. To make a shortcut to the two-photon JC-model considered in the standard studies of the quantum optics we may write the time dependent exponent $\exp[it\hbar k^2/2m]$ in the term with $[\theta(z) - \theta(z-l)]$, i.e., inside the cavity in the following form $\exp\left(-i\left[\frac{\hbar k_n^{\pm}}{2m} \pm E_n\right]t\right)$, where $E_n = \frac{1}{2}[n\beta_2 + (n+2)\beta_1] + \hbar\sqrt{\Delta^2 + \lambda^2(n+1)(n+2)}$.

When the spatial dependence is not taken into consideration, the wave function goes automatically to the well known wave function for the standard two-photon JC-model. The solution (10) contains the regions inside and outside the cavity. The region inside the cavity, the contributions of the dynamic Stark effect and the off-resonant case have not been considered in earlier studies for the two-photon cases (Zhang et al., 1999).

3. Emission spectra

If the cavity field is initially prepared in the coherent state, we have the following photon-number distribution

$$P(n) = \exp(\bar{n})\frac{\bar{n}^n}{n!} = q_n^2, \tag{15}$$

where $\bar{n}$ is the averaged photon number. With the wave function calculated, any property related to the atom or the field can be calculated. Let us denote by $\rho(t)$ the atom-field density matrix, that its elements ρ_{ij} are

$$\rho_{ij} = \sum_n \langle i, z|\psi(t)\rangle\langle\psi(t)|z, j\rangle. \tag{16}$$

With the wave function calculated, any property related to the atom or the field can be calculated.

$$\rho_{ij} = \sum_n |\langle i, z|\psi(t)\rangle|^2. \tag{17}$$

The probability of finding the atom in the upper state is given by $\rho_{ee} = \langle C|C\rangle$ and the probability of being in the ground state is given by $\rho_{gg} = \langle S|S\rangle$, where

$$\begin{aligned}|C\rangle = \int dk G(k)\exp\left(-i\frac{\hbar k^2 t}{2M}\right)\sum_n qn \\ \times \left(\left[e^{ikz} + \left(A_n^+ \sin\theta_n + A_n^- \cos\theta_n\right) \times e^{-ikz}\right]\theta(-z)\right. \\ +\left(B_n^+ \sin\theta_n + B_n^- \cos\theta_n\right)e^{ik(z-L)}\theta(z-L) \\ +\left\{\left(\alpha_n^+ \times e^{ik_n^+ z} + \beta_n^+ e^{-ik_n^+ z}\right)\sin\theta_n\right. \\ \left.+\left(\alpha_n^- e^{ik_n^- z} + \beta_n^- e^{-ik_n^- z}\right)\cos\theta_n\right\}[\theta(z) - \theta(z-L)]\Big)|n\rangle,\end{aligned} \tag{18}$$

$$\begin{aligned}|S\rangle = \int dk G(k)\exp\left(-i\frac{\hbar k^2 t}{2M}\right)\sum_n qn\Big(\left(A_n^+ \cos\theta_n - A_n^- \sin\theta_n\right)e^{-ikz} \\ \times\theta(-z) + \left(B_n^+ \cos\theta_n - B_n^- \sin\theta_n\right)e^{ik(z-L)}\theta(z-L) \\ +\left\{\left(\alpha_n^+ e^{ik_n^+ z} + \beta_n^+ e^{-ik_n^+ z}\right)\cos\theta_n - \left(\alpha_n^- e^{ik_n^- z} - \beta_n^- e^{-ik_n^- z}\right)\sin\theta_n\right\} \\ \times[\theta(z) - \theta(z-L)]\Big)|n+2\rangle.\end{aligned} \tag{19}$$

In what follow, we shall investigate the properties of the spatial dependence on the emission probability when we take the distribution function $G(k) = \delta(k - k_0)$. The emission probability is given by

$$\begin{aligned}P_s = \sum_n P(n)\left|\left\{A_n^+ \cos\theta_n - A_n^- \sin\theta_n\right\}\right|^2 \\ +\left|\left\{B_n^+ \cos\theta_n - B_n^- \sin\theta_n\right\}\right|^2 \\ +\left|\left\{\left(\alpha_n^+ e^{ik_n^+ z} + \beta_n^+ e^{-ik_n^+ z}\right)\cos\theta_n - \left(\alpha_n^- e^{ik_n^- z} - \beta_n^- e^{-ik_n^- z}\right)\sin\theta_n\right\}\right|^2.\end{aligned} \tag{20}$$

The spatial dependence of the cavity field shows that the emission probability depends not only on the statistics of the field but also on the momentum distribution of the atomic mass center. The first part in Eq. (20) is the contribution from the reflected waves, the second part is due to the transmitted waves, and the third part is the contribution of the transient regime.

This last part plays an essential role in the emission probability. It does not appear possible to express the sums in Eq. (20) in closed form, but for not too large mean photon number, direct numerical evolutions can be performed. Resorting to Eq. (20), in the following we investigate the response of the atom to the coherent cavity field when it experiences a transition from classical regime to the quantum regime.

4. Conclusions

In this paper we have presented the non-resonant two-photon mazer in the presence of Stark shift effect taking into account the spatial-dependence. The full solution is given and the case of the inter-cavity is considered in detail. The situation here is somewhat different from the cold atom scheme that has already been examined (Scully et al., 1996; Meyer et al., 1997; Loffler et al., 1997; Schröder et al., 1997). The emission probability of the system is also calculated and investigated with special emphasis on its spatial dependence. The results of this paper may be tested with micromaser-like experiments by using a high-Q micromaser pumped by cold atoms with very high principal quantum number (Walther et al., 2000; Varcoe et al., 2000; Weidinger et al., 1999).

References

Abdel-Aty, M., Obada, A.-S.F., 2002a. J. Phys. B: At. Mol. Opt. Phys. 35, 807–813.

Abdel-Aty, M., Obada, A.-S.F., 2002b. Mod. Phys. Lett. B 16, 117–125.

Adams, C., Sigel, M., Mlynek, J., 1994. Phys. Rep. 240, 143–192.

Aran, R., Agarwal, G.S., Scully, M.O., Walther, H., 2000. Phys. Rev. A 62, 023809–023821.

Bastin, T., Martin, J., 2003. J. Phys. Rev. A 67, 053804–053816.

Battocletti, M., Englert, B.-G., 1994. J. Phys. II France 4, 1939–1945.

Bloch, I., Hansch, T.W., Esslinger, T., 1999. Phys. Rev. Lett. 82, 3008–3011.

Concannon, H.M., Brown, W.J., Gardner, J.R., Gauthier, D.J., 1997. Phys. Rev. A 56, 1519–1523.

Englert, B.-G., Schwinger, J., Barut, A.O., Scully, M.O., 1991. Europhys. Lett. 14, 25–32.

Goldstein, E.V., Moore, M.G., Meystre, P., 2000. Nonlinear optics of matter waves. In: Letokhov, V.S., Meystre, P. (Eds.), Advances in Laser Physics. Harwood, Amsterdam, pp. 117–124.

Griffin, A., Snoke, D.W., Stringari, S., 1995. Bose Einstein Condensation. Cambridge University Press, Cambridge.

Haroche, S., Brune, M., Raimond, J.M., 1991. Europhys. Lett. 14, 19–27.

Lenz, G., Meystre, P., Wright, E.M., 1994. Phys. Rev. A 50, 1681–1696.

Loffler, M., Meyer, G.M., Schroder, M., Scully, M.O., Walther, H., 1997. Phys. Rev. A 56, 4153–4163.

Meyer, G.M., Scully, M.O., Walther, H., 1997. Phys. Rev. A 56, 4142–4152.

Obada, A.-S.F., Abdel-Aty, M., 2000. Acta Phys. Pol. 31, 589–601.

Schröder, M., Vogel, K., Schleich, W.P., Scully, M.O., Walther, H., 1997. Phys. Rev. A 56, 4164–4174.

Scully, M.O., Meyer, G.M., Walther, H., 1996. Phys. Rev. Lett. 76, 4144–4149.

Si-de, D., Lu-wei, Z., Zhi-zhan, X., Wen-qi, Z., Yuan, L., 1998. J. Phys. B: At. Mol. Opt. Phys. 31, 4895–4904.

Varcoe, B.T.H., Brattke, S., Weidinger, M., Walther, H., 2000. Nature 403, 743–751.

Walther, H., 2000. In: Xu, Z.Z., Xie, S.W., Zhu, S.Y., Scully, M.O. (Eds.), Frontiers Laser Physics and Quantum Optics. Springer, Berlin, p. 39.

Weidinger, M., Varcoe, B.T.H., Heerlein, R., Walther, H., 1999. Phys. Rev. Lett. 82, 3795–3799.

Wieman, C.E., Pritchard, D.E., Wineland, J., 1999. Rev. Mod. Phys. 71, S253–S258.

Zhang, Z.M., He, L.S., 1998. Opt. Commun. 157, 77–86.

Zhang, Z.-M., Lu, Z.-Y., He, L.-S., 1999. Phys. Rev. A 59, 808–816.

Zhang, Z.-M., Chi, S., Cheng, B.-C., 2002. J. Opt. B: Quantum Semiclass. Opt. 4, 30–46.