Propagation of plane elastic waves at a plane interface between two electro-microelastic solid half-spaces

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Abstract

This work is concerned with the wave propagation and their reflection and transmission from a plane interface between two different electro-microelastic solid half-spaces in perfect contact. It is found that there exist five basic waves in an infinite electro-microelastic solid, namely an independent longitudinal micro-rotational wave, two sets of coupled longitudinal waves influenced by the electric effect, and two sets of coupled transverse waves. The existence of the two sets of coupled longitudinal waves is new. In the absence of microstretch and electric effects, these two coupled longitudinal waves reduce to a longitudinal displacement wave of micropolar elasticity. Amplitude and energy ratios of various reflected and transmitted waves are presented when (i) a set of coupled longitudinal wave is made incident and (ii) a set of coupled transverse wave is made incident. Numerical computations have been performed for a particular model and the variations of amplitude and energy ratios are obtained against the angle of incidence. The results obtained are depicted graphically. It has been verified that the sum of energy ratios is equal to unity at the interface and the amplitude ratios of reflected and transmitted waves depend upon the angle of incidence, frequency and elastic properties of the media. Results of some earlier workers have also been reduced from the present formulation.

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1. Introduction

The linear theory of micropolar elasticity developed by Eringen (1966) is well known and does not need much introduction. In this theory, the motion in a body is described by six degrees of freedom, three of translation and three of rotation. The interactions across a surface element between two parts of a micropolar body is transmitted not only by a force vector but also by a couple resulting in asymmetric force stress tensor and couple stress tensor. Different problems of waves and vibrations in micropolar elastic media have been investigated by various researchers. Some notable are Parfitt and Eringen (1969), Nowacki and Nowacki (1969), Ariman (1972), Tomar and Gogna (1995a,b) and Tomar et al. (1998) among several others.
Eringen (1971, 1990) has also developed a linear theory of thermo-microstretch elastic solids, which is a generalization of the theory of micropolar elasticity. Microstretch elastic solids are the solids in which the material particles can undergo expansion and contraction (stretches), in addition to the translation and rotation. Thus, the motion in a microstretch elastic body is characterized by seven degrees of freedom. Recently, Tomar and Garg (2005) investigated the wave propagation and reflection/transmission of plane waves at a plane interface between two different microstretch elastic solid half-spaces.

Eringen (2004) has further extended his theory of thermo-microstretch elastic solids (Eringen, 1990) to include the electromagnetic interactions and termed it as electromagnetic theory of microstretch elasticity. These materials include animal bones and nano-materials. Books on microcontinuum and electrodynamic theories (Eringen, 1999; Eringen and Maugin, 1990) are excellent monographs on this pertinent area of research.

In the present work, we have studied the possibility of wave propagation in an infinite electro-microelastic solid. The reflection/transmission phenomena at a plane interface between two different microstretch elastic solid half-spaces in perfect contact are also investigated when (i) a set of coupled longitudinal wave is incident and (ii) a set of coupled transverse wave is incident. It is found that there exist five basic waves in an infinite electro-microelastic solid, namely an independent longitudinal micro-rotational wave, two sets of coupled longitudinal waves having the electric effect, and two sets of coupled transverse waves. The amplitude ratios and energy ratios of various reflected and refracted waves are computed numerically for a specific model and their variations with angle of incidence are presented graphically. The variations of various amplitude ratios with frequency of the incident wave striking at a particular angle of incidence are also depicted graphically. The problems of Tomar and Gogna (1995a, b) and Tomar and Garg (2005) have been reduced as limiting cases of the present formulation.

2. Basic relations and equations

Eringen (2004) has introduced electromagentic fields in the continuum theory of microstretch elasticity. He presented constitutive relations and equations of motion for isotropic thermo-microstretch elastic solids subjected to electro-magnetic fields. In the absence of magnetic flux vector and thermal effect, the microstretch continuum will be subjected only to electric field. We shall call such continuum material as electro-microelastic solids. The reflection/transmission relations and equations of motion for isotropic thermo-microstretch elastic solids subjected to electro-magnetic fields. The microstretch continuum will be subjected only to electric field. We shall call such continuum material as electro-microelastic solids.

\[ t_{kl} = (\dot{\lambda}_0 \psi + \dot{\lambda}_u u_r) \delta_{kl} + \mu (u_{k,l} + u_{l,k}) + K (u_{l,k} - \epsilon_{klm} \Phi_r), \]
\[ m_{kl} = E_{r,l} \delta_{kl} + \beta \Phi_{k,1} + \gamma \Phi_{l,1} + b_0 \epsilon_{klm} \Phi_m, \]
\[ m_k = \alpha_0 \psi_{,k} + \lambda_2 E_k - b_0 \epsilon_{klm} \Phi_{l,m}, \]
\[ D_k = (1 + \gamma^E) E_k + \lambda_3 \epsilon_{lmk} \Phi_{l,m} + \lambda_2 \psi_{,k}, \]

where \( t_{kl} \) is the force stress tensor, \( m_{kl} \) is the couple stress tensor, \( m_k \) is the microstretch vector and \( D_k \) is the dielectric displacement vector; \( \dot{\lambda} \) and \( \mu \) are Lame’s constants; \( K, \alpha, \beta \) and \( \gamma \) are micropolar constants; \( b_0, \lambda_0 \) and \( \alpha_0 \) are microstretch constants; \( \gamma^E \) is the dielectric susceptibility, \( \lambda_3 \) and \( \lambda_2 \) are the coupling constants for micro-rotation-dielectric and microstretch-dielectric effects respectively; \( u_k \) and \( \Phi_k \) are the displacement and microstretch vectors respectively; \( \psi \) is a scalar microstretch and \( E_k \) is the electric field vector.

In the absence of body force density, body couple density, body microstretch force density and ignoring the current vector \( J \) and the volume charge density \( q_{e0} \), the field equations under Section 7 of Eringen (2004) for an isotropic and homogeneous electro-microelastic solid medium are given by

\[ (c_1^2 + c_2^2) \nabla \cdot \mathbf{u} - (c_1^2 + c_2^2) \nabla \times \nabla \times \mathbf{u} + c_3^2 \nabla \cdot \mathbf{F} + \tilde{\lambda}_0 \nabla \psi = \mathbf{\tilde{u}}, \]
\[ (c_1^2 + c_2^2) \nabla \cdot \mathbf{F} - c_3^2 \nabla \times \nabla \times \mathbf{u} + \omega_0^2 \nabla \cdot \mathbf{u} - 2 \omega_0^2 \mathbf{F} = \mathbf{\tilde{F}}, \]
\[ c_0^2 \nabla^2 \psi - c_1^2 \psi - c_3^2 \nabla \cdot \mathbf{u} + c_0^2 \nabla \cdot \mathbf{E} = \psi, \]
\[ \nabla \cdot \mathbf{D} = 0, \]
\[ \nabla \times \mathbf{E} = 0, \]
where \( c_1^2 = (\lambda + 2\mu)/\rho, \ c_2^2 = \mu/\rho, \ c_3^2 = K/\rho, \ c_4^2 = \gamma/\rho j, \ c_5^2 = (x + \beta)/\rho j, \ c_6^2 = 2\alpha_0/\rho j_0, \ c_7^2 = 2\lambda_1/3\rho j_0, \ c_8^2 = 2\lambda_2/3\rho j_0, \ c_9^2 = 2\lambda_3/\rho j_0, \omega_0^2 = c_3^2/j, \ Z_0 = \lambda_0/\rho, \lambda_1 \) is the microstretch constant, \( \rho \) is the density of the medium, \( j \) and \( j_0 \) are constants. The superposed dots on the right-hand sides of Eqs. (5)–(7) represent the double temporal derivative. It can be seen from Eqs. (5)–(9) that the coefficients \( b_0 \) and \( \lambda_3 \) do not contribute to the field equations. Therefore, we shall drop the terms containing these coefficients from the expressions of \( m_{kl}, \ k_b \) and \( D_k \) in the subsequent analysis. The reason of dropping the coefficient \( b_0 \) can be seen in Eringen (1999).

The inequalities among the material moduli which are necessary and sufficient conditions to make the strain energy density non-negative are (Eringen, 1999)

\[
3\lambda + 2\mu + K \geq \frac{3\lambda_0^2}{\lambda_1}, \quad 2\mu + K \geq 0, \quad K \geq 0, \quad 3\alpha + \beta + \gamma \geq 0,
\]

\( \gamma \geq \beta \geq -\gamma, \ \alpha_0 \geq 0, \ \lambda_1 \geq 0, \ \chi^2 \geq 0. \)

Introducing the scalar potentials \( q, \xi \) and \( \epsilon \); the vector potentials \( U \) and \( \Pi \), as follows:

\[
u \equiv \nabla q + \nabla \times U, \quad \Phi \equiv \nabla \xi + \nabla \times \Pi, \quad E \equiv -\nabla \epsilon, \quad \nabla \cdot U = \nabla \cdot \Pi = 0,
\]

and inserting them into Eqs. (5)–(9), we obtain

\[
(c_1^2 + c_3^2)\nabla^2 q + \overline{Z}_0 \psi = \psi,
\]

\[
(c_6^2 - c_{10}^2)\nabla^2 \psi - c_7^2 \psi - c_8^2 \nabla^2 q = \dot{\psi},
\]

\[
(c_2^2 + c_4^2)\nabla^2 U + c_5^2 \nabla \times \Pi = \dot{U},
\]

\[
c_3^2 \nabla^2 \Pi - 2\omega_0^2 \Pi + \omega_0^2 \nabla \times U = \dot{\Pi},
\]

\[
(c_2^2 + c_4^2)\nabla^2 \xi - 2\omega_0^2 \xi = \dot{\xi},
\]

\[
\nabla^2 \epsilon = \frac{\lambda_1}{1 + \chi^2} \nabla^2 \psi,
\]

where \( c_{10}^2 = \frac{2\lambda_2}{\rho_0(1+\chi^2)} \) and other symbols are defined earlier. It can be noticed that

(a) Eqs. (11) and (12) are coupled in scalar potentials \( q \) and \( \psi \),
(b) Eqs. (13) and (14) are coupled in vector potentials \( U \) and \( \Pi \),
(c) Eq. (15) is uncoupled in scalar potential \( \xi \), and
(d) Eq. (16) is coupled in scalar potentials \( \epsilon \) and \( \psi \).

3. Wave propagation

We consider the following form of plane waves traveling in the positive direction of a unit vector \( n \) as

\[
\{q, \psi, U, \Pi\} = \{a_1, b_1, A_0, B_0\} \exp\{ik(n \cdot r - \Omega t)\},
\]

where \( a_1 \) and \( b_1 \) are complex constants; \( A_0, B_0 \) may be complex constant vectors, \( V \) is the phase velocity, \( \mathbf{r} \) is the position vector and \( k \) is the wavenumber having the definition that \( \omega = kV; \ \omega \) being the circular frequency. Inserting the expressions of \( q \) and \( \psi \) from (17) into Eqs. (11) and (12) and then eliminating \( a_1 \) and \( b_1 \), we obtain

\[
AV^4 - BV^2 + C = 0,
\]

where \( A = 1 - \frac{\alpha_0^2}{3\mu} \left( \frac{\epsilon}{\rho} \right), \ B = \left( c_1^2 + c_3^2 - \frac{2\omega_0}{\lambda_1} \right) A + c_6^2 - c_{10}^2 + \frac{2\omega_0}{\lambda_1}, \ C = (c_1^2 + c_3^2)(c_6^2 - c_{10}^2), \) and \( \Omega = \frac{2\omega_0}{\rho^2} \). Eq. (18) is quadratic in \( V^2 \) and its roots are given by

\[
V_{1,2}^2 = \frac{1}{2A} \left[ B \pm \sqrt{B^2 - 4AC} \right],
\]

where the ‘+’ sign in the expression of Eq. (19) corresponds to \( V_1^2 \) and ‘-’ sign corresponds to \( V_2^2 \).
From Eqs. (11) and (17), it can be seen that the constants \( a_1 \) and \( b_1 \) are related to each other through the relation

\[
b_1 = \zeta a_1,
\]

where \( \zeta = \frac{\omega^2}{\lambda_0} \left[ c_1^2 + c_3^2 \right] - 1 \) is the coupling parameter between \( q \) and \( \psi \).

Now, using the expressions of \( q \) and \( \psi \) from (17) into (10)\(_1\), the displacement vector \( \mathbf{u} \) can be obtained as

\[
\mathbf{u} = ika_1 \mathbf{n} \exp \{ i(k \mathbf{n} \cdot \mathbf{r} - \omega t) \}. \tag{21}
\]

This shows that the vector \( \mathbf{u} \) is parallel to the vector \( \mathbf{n} \). Thus, the displacement of the wave associated with \( q \) is in the direction of propagation of the wave, therefore this wave is longitudinal in nature and may be called as longitudinal displacement wave. By using the relations (20) and (21), we see that the waves associated with \( \psi \) are also longitudinal in nature and may be called as longitudinal microstretch wave. Since these waves are coupled, so we shall call them coupled longitudinal waves. Thus, there exist two coupled longitudinal waves consisting of a longitudinal displacement wave and a longitudinal microstretch wave. It can be seen that the velocities \( V_{1,2} \) depend on dielectric susceptibility \( \chi^E \) and the coupling constant for microstretch-dielectric effect \( \lambda_2 \). Therefore, both the coupled longitudinal waves propagating with velocities \( V_{1,2} \) are influenced by the electric effect.

When the constants corresponding to electric and microstretch effects vanish, then from Eq. (19) we obtain, \( V_{2}^2 = (\lambda + 2\mu + K)/\rho \) and \( V_{3}^2 = 0 \). This means that the waves corresponding to \( q \) and \( \psi \) are no longer coupled and the wave corresponding to microstretch would disappear in the absence of microstretch and electric effects. Thus, the longitudinal displacement wave of micropolar elasticity is recovered in this limiting case. Furthermore, it can be easily verified that in the absence of micropolar, microstretch and electric effects i.e. when \( K = \alpha_0 = \lambda_0 = \lambda_1 = \lambda_2 = \chi^E = 0 \), we obtain \( V^2 = (\lambda + 2\mu)/\rho \), as the only non-zero root of Eq. (18), which is the well-known velocity of longitudinal wave of classical elasticity.

Parfitt and Eringen (1969) have proved that equations Eqs. (13) and (14) represent two sets of coupled transverse waves propagating with velocities given by

\[
V_{3,4}^2 = \frac{1}{2(1 - \Omega)} \left\{ \epsilon \pm \sqrt{\epsilon^2 - 4c_1^2(1 - \Omega)(c_2^2 + c_3^2)} \right\}, \tag{22}
\]

where \( \epsilon = c_1^2 + c_2^2(1 - \Omega) + c_3^2(1 - \Omega/2) \). They have also proved that Eq. (15) represents a longitudinal micro-rotational wave propagating with velocity given by

\[
V_5^2 = c_4^2 + c_5^2 + \frac{2\alpha_0^2}{k^2}.
\]

Hence, it is seen that there exist five basic waves propagating in an infinite electro-microelastic solid medium. Two of them are coupled longitudinal waves that are influenced by the electric effect, two are coupled transverse waves, and the remaining wave is an independent longitudinal micro-rotational wave.

4. Reflection and transmission

4.1. Incident coupled longitudinal wave

Here, we shall discuss the reflection and transmission phenomena of coupled longitudinal wave at a plane interface between two different electro-microelastic solid half-spaces in perfect contact. Let the x-axis be taken along the interface and z-axis is taken to be along the direction pointing vertically downward. We take the lower half-space as medium \( M[Z > 0] \) and denote the elastic constants and density in this medium by \( \lambda, \mu, K, \gamma, \alpha_0, \lambda_0, \lambda_2, \chi^E \) and \( \rho \), while the upper half-space as medium \( M[Z < 0] \) and the corresponding elastic parameters therein are denoted by \( \lambda', \mu', K', \gamma', \alpha_0', \lambda_0', \lambda_2', \chi'^E \) and \( \rho' \). The complete geometry of the problem is shown in Fig. 1. The problem is two-dimensional in xz-plane, so we can take

\[
\mathbf{u} = (u_1, 0, u_3), \quad \Phi = (0, \phi_2, 0), \quad \frac{\partial}{\partial y} = 0. \tag{23}
\]
Using these into (10), we obtain

\[ u_1 = \frac{\partial q}{\partial x} - \frac{\partial U_2}{\partial z}, \quad u_3 = \frac{\partial q}{\partial z} + \frac{\partial U_2}{\partial x}, \quad \phi_2 = \frac{\partial \Pi_3}{\partial x} - \frac{\partial \Pi_1}{\partial z}, \]

where \( U_2 \) is the y-component of \( \mathbf{U} \), \( \Pi_1 \) and \( \Pi_3 \) are respectively the x- and z-components of \( \mathbf{\Pi} \).

Let a coupled longitudinal wave of amplitude \( A_0 \) propagating with velocity \( V_1 \) through the lower half-space \( M \) becomes incident at the interface making an angle \( \theta_0 \) with the z-axis. We postulate the following reflected and transmitted waves to satisfy the boundary conditions at the interface:

**Reflected waves:**
(i) Two sets of coupled longitudinal waves of amplitudes \( A_{1,2} \) propagating with velocities \( V_{1,2} \) in medium \( M \) and making angles \( \theta_{1,2} \) with the normal. The expressions of velocities \( V_{1,2} \) are defined earlier.

(ii) Two sets of coupled transverse waves of amplitudes \( A_{3,4} \) propagating with velocities \( V_{3,4} \) in medium \( M \) and making angles \( \theta_{3,4} \) with the normal. The expressions of velocities \( V_{3,4} \) are defined earlier.

**Transmitted waves:**
(i) Two sets of coupled longitudinal waves of amplitudes \( A_{1',2'} \) propagating with velocities \( V_{1',2'} \) in medium \( M' \) and making angles \( \theta_{1',2'} \) with the normal.

(ii) Two sets of coupled transverse waves of amplitudes \( A_{3',4'} \) propagating with velocities \( V_{3',4'} \) in medium \( M' \) and making angles \( \theta_{3',4'} \) with the normal.

The expressions of velocities \( V_{1',2'} \) and \( V_{3',4'} \) are similar to the expressions of velocities \( V_{1,2} \) and \( V_{3,4} \) respectively, with appropriate dashes.

The potentials of various reflected and transmitted waves in the half-spaces \( M \) and \( M' \) are given by

\[ q = A_0 \exp\{ik_1(\sin \theta_0 x - \cos \theta_0 z) - i\omega_1 t\} + \sum_{p=1,2} A_p \exp\{ik_p(\sin \theta_p x + \cos \theta_p z) - i\omega_p t\}, \]

\[ \psi = \zeta_1 A_0 \exp\{ik_1(\sin \theta_0 x - \cos \theta_0 z) - i\omega_1 t\} + \sum_{p=1,2} \zeta_p A_p \exp\{ik_p(\sin \theta_p x + \cos \theta_p z) - i\omega_p t\}, \]

\[ U_2 = \sum_{p=3,4} A_p \exp\{ik_p(\sin \theta_p x + \cos \theta_p z) - i\omega_p t\}, \]

\[ \phi_2 = \sum_{p=3,4} \eta_p A_p \exp\{ik_p(\sin \theta_p x + \cos \theta_p z) - i\omega_p t\}, \]
\[ q' = \sum_{p=1,2} A_p' \exp\{ik'_p(\sin \theta'_p x - \cos \theta'_p z) - i\omega'_p t}\],
\[ \psi' = \sum_{p=1,2} \psi'_p A'_p \exp\{ik'_p(\sin \theta'_p x - \cos \theta'_p z) - i\omega'_p t\}, \]
\[ U'_2 = \sum_{p=3,4} A'_p \exp\{ik'_p(\sin \theta'_p x - \cos \theta'_p z) - i\omega'_p t\}, \]
\[ \phi'_2 = \sum_{p=3,4} \eta'_p A'_p \exp\{ik'_p(\sin \theta'_p x - \cos \theta'_p z) - i\omega'_p t\}, \]

where \( \iota = (-1)^{\frac{1}{2}} \), \( \omega_p = k_p V_p \), and \( \omega'_p = k'_p V'_p \) are well-known relations, \( \zeta_{1,2} \) are coupling parameters between \( q \) and \( \psi \), and \( \eta_{3,4} \) are coupling parameters between \( U_2 \) and \( \phi_2 \). The expressions of \( \zeta_i \) are determined earlier through Eq. (20) can be rewritten as
\[ \zeta_{1,2} = \frac{\omega_2^2}{\kappa_0} \left[ \frac{c_1^2 + c_3^2}{V_{1,2}^2} - 1 \right], \]

and the expressions of \( \eta_i \) are determined by applying curl operator in Eq. (14) and then inserting Eqs. (26) and (27) appropriately. Their expressions are given by
\[ \eta_{3,4} = \omega_0^2 \left[ \frac{V_{3,4}^2}{2} - \frac{2\omega_0^2}{k_{3,4}^2} - c_3^2 \right]^{-1}. \]

The \( \zeta_{1,2} \) and \( \eta_{3,4} \) are the coupling parameters between \( q' \) and \( \psi' \) and between \( U'_2 \) and \( \phi'_2 \) respectively. The expressions of \( \zeta_{1,2}' \) and \( \eta_{3,4}' \) are similar to the expressions \( \zeta_{1,2} \) and \( \eta_{3,4} \) and can be written by putting appropriate dashes.

Using equations from (10) into Eqs. (1)–(4), the requisite components of displacement, microrotation, stress and microstrecth are given by
\[ t_{zz} = (3 + 2\mu + K)q_{zz} + (2\mu + K)U_{2,zz} + \lambda q_{zz} + \lambda_0 \psi, \]
\[ t_{zx} = (2\mu + K)q_{zx} - (\mu + K)U_{2,zz} + \mu U_{2,zz} - K \phi_2, \]
\[ m_{zy} = \gamma \phi_2, \quad m_z = \left( \kappa_0^2 \frac{\lambda_0^2}{1 + \chi^2} \right) \psi, \]
\[ u_1 = q_x - U_{2,x}, \quad u_3 = q_z + U_{2,z}. \]

Similar expressions can be written for \( t_{xx} \), \( t_{xx}' \), \( m_{yx} \), \( m_y \), \( u'_1 \) and \( u'_3 \) with appropriate dashes e.g. \( m_{yx}' = \gamma \phi_2' \) and \( u'_1 = q_x' - U_{2,x}' \), etc. The comma in the subscripts denotes the spatial derivative.

The appropriate mechanical boundary conditions at the interface between two different electro-microelastic solid half-spaces are: (i) continuity of stresses, (ii) continuity of microstretch, (iii) continuity of displacement and microrotation. Mathematically, these boundary conditions can be written as: At the interface \( z = 0 \)
\[ t_{zz} = t_{zz}', \quad t_{zx} = t_{zx}', \quad m_{zy} = m_{zy}', \quad m_z = m_z', \]
\[ u_1 = u'_1, \quad u_3 = u'_3, \quad \phi_2 = \phi_2', \quad \psi = \psi'. \]

Owing to Eqs. (24)–(31), the boundary conditions given above in Eq. (33) are identically satisfied if and only if \( k_i \sin \theta_i = k'_i \sin \theta'_i \) and \( \omega_i = \omega'_i \), we obtain
\[
\begin{align*}
\left[ \lambda + (2\mu + K) \cos^2 \theta_0 - \frac{\lambda_{0z}^r}{k_1^2} \right] k_1^2 A_0 + \sum_{p=1,2} \left[ \lambda + (2\mu + K) \cos^2 \theta_p - \frac{\lambda_{0z}^r}{k_p^2} \right] k_p^2 A_p \\
- \sum_{p=1,2} \left[ \lambda' + (2\mu' + K') \cos^2 \theta'_p - \frac{\lambda'_{0z}^r}{k_p^2} \right] k_p^2 A'_p + (2\mu + K) \sum_{p=3,4} \sin \theta_p \cos \theta_p k_p^2 A_p \\
+ (2\mu' + K') \sum_{p=3,4} \sin \theta'_p \cos \theta'_p k_p^2 A'_p = 0,
\end{align*}
\]
(34)

\[
(2\mu + K) \sin \theta_0 \cos \theta_0 k_1^2 A_0 - \sum_{p=1,2} \left[ (2\mu + K) \sin \theta_p \cos \theta_p k_p^2 A_p + (2\mu' + K') \sin \theta'_p \cos \theta'_p k_p^2 A'_p \right] \\
+ \sum_{p=3,4} \left[ \mu \cos 2\theta_p + K \cos^2 \theta_p - \frac{K\eta'_p}{k_p^2} \right] k_p^2 A_p - \left[ \mu' \cos 2\theta'_p + K' \cos^2 \theta'_p - \frac{K'\eta'_p}{k_p^2} \right] k_p^2 A'_p = 0,
\]
(35)

\[
\sum_{p=3,4} \left[ \eta_p \cos \theta_p k_p A_p + \eta'_p \cos \theta'_p k_p A'_p \right] = 0,
\]
(36)

\[
\left( x_0 - \frac{\lambda_0^2}{1 + K} \right) \left[ \zeta_1 \cos \theta_0 k_1 A_0 - \sum_{p=1,2} \zeta_p \cos \theta_p k_p A_p \right] - \left( x_0 - \frac{\lambda_0^2}{1 + K} \right) \sum_{p=1,2} \zeta'_p \cos \theta'_p k'_p A'_p = 0,
\]
(37)

\[
\sin \theta_0 k_1 A_0 + \sum_{p=1,2} \left[ \sin \theta_p k_p A_p - \sin \theta'_p k'_p A'_p \right] - \sum_{p=3,4} \cos \theta_p k_p A_p + \cos \theta'_p k'_p A'_p = 0,
\]
(38)

\[
\cos \theta_0 k_1 A_0 - \sum_{p=1,2} \left[ \cos \theta_p k_p A_p + \cos \theta'_p k'_p A'_p \right] - \sum_{p=3,4} \left[ \sin \theta_p k_p A_p - \sin \theta'_p k'_p A'_p \right] = 0,
\]
(39)

\[
\sum_{p=3,4} \left[ \eta_p A_p - \eta'_p A'_p \right] = 0,
\]
(40)

\[
\zeta_1 A_0 + \sum_{p=1,2} \left[ \zeta_p A_p - \zeta'_p A'_p \right] = 0.
\]
(41)

Eqs. (34)–(41) enable us to provide the amplitude ratios of various reflected and transmitted waves. These equations can be written in matrix form as

\[
[a_{ij}] [Z] = [M],
\]
(42)

where \([a_{ij}]\) is a 8 \times 8 matrix, \([Z] = [Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7, Z_8]^T\) is a column matrix, here ‘t’ in the superscript represents the transpose of the matrix and \(Z_r = \frac{\lambda_r}{V_1} (r = 1, 2, 3, 4)\) are the reflection coefficients, while \(Z_{r+4} = \frac{\lambda_r}{V_1} \) are the transmission coefficients for incident coupled longitudinal wave traveling with velocity \(V_1\). The non-zero entries of the matrix \([a_{ij}]\) together with the column matrix \([M]\) are given in Appendix A.

Next, we shall discuss the partitioning of incident energy between various reflected and transmitted waves at the interface \(z = 0\). The rate of energy transmission per unit area denoted by \(P^*\) is given by

\[
P^* = t_{zz} \dot{u}_3 + t_{z\dot{u}} + m_{zz} \dot{\phi}_2 + m_{z\dot{\phi}},
\]
(43)

where superposed dot represents the time derivative.

The expressions of energy ratios \(E_i (i = 1, 2, \ldots, 8)\) are given by
Each energy ratio \( E_t \) gives the rate of energy transmission at the interface for the respective reflected and transmitted wave to the rate of energy transmission for the incident coupled longitudinal wave propagating with velocity \( V_1 \).

### 4.2. Incident coupled transverse wave

We shall now study the reflection and transmission at an interface between two different electro-microelastic solid half-spaces in perfect contact when a coupled transverse wave propagating with velocity \( V_1 \) having amplitude \( A_0 \) is incident at an angle \( \theta_0 \) with the normal. The geometry of the problem and the set of reflected and transmitted waves will remain same as considered in the previous case of incident coupled longitudinal wave. Also, the boundary conditions will be the same as given in Eq. (33). In the present formulation, the potentials in lower half-space \( M \) are given by

\[
U_2 = A_0 \exp \{ ik_3 (\sin \theta_0 x - \cos \theta_0 z) - i \omega_3 t \} + \sum_{p=3,4} A_p \exp \{ ik_p (\sin \theta_p x + \cos \theta_p z) - i \omega_p t \},
\]

\[
\phi_2 = \eta_3 A_0 \exp \{ ik_3 (\sin \theta_0 x - \cos \theta_0 z) - i \omega_3 t \} + \sum_{p=3,4} \eta_p A_p \exp \{ ik_p (\sin \theta_p x + \cos \theta_p z) - i \omega_p t \},
\]

\[
q = \sum_{p=1,2} A_p \exp \{ ik_p (\sin \theta_p x + \cos \theta_p z) - i \omega_p t \},
\]

\[
\psi = \sum_{p=1,2} \zeta_p A_p \exp \{ ik_p (\sin \theta_p x + \cos \theta_p z) - i \omega_p t \},
\]

and the potentials in the upper half-space \( M' \) are given by:

\[
g' = \sum_{p=1,2} A'_p \exp \{ ik'_p (\sin \theta'_p x - \cos \theta'_p z) - i \omega'_p t \},
\]

\[
\psi' = \sum_{p=1,2} \zeta'_p A'_p \exp \{ ik'_p (\sin \theta'_p x - \cos \theta'_p z) - i \omega'_p t \},
\]

\[
U'_2 = \sum_{p=3,4} A'_p \exp \{ ik'_p (\sin \theta'_p x - \cos \theta'_p z) - i \omega'_p t \},
\]

\[
\phi'_2 = \sum_{p=3,4} \eta'_p A'_p \exp \{ ik'_p (\sin \theta'_p x - \cos \theta'_p z) - i \omega'_p t \}.
\]

Inserting the above potentials into the boundary conditions given in Eq. (33), we see that the boundary conditions are satisfied if
\[(2\mu + K) \sin \theta_0 \cos \theta_0 k_0^2 A_0 - \sum_{p=1,2} \left[ \lambda + (2\mu + K) \cos^2 \theta_p - \frac{\lambda q_p}{k_p^2} \right] k_p^2 A_p \]

\[+ \sum_{p=1,2} \left[ \lambda' + (2\mu' + K') \cos^2 \theta_p' - \frac{\lambda q_p}{k_p^2} \right] k_p^2 A_p' - (2\mu + K) \sum_{p=3,4} \sin \theta_p \cos \theta_k k_p^2 A_p \]

\[- (2\mu' + K') \sum_{p=3,4} \sin \theta_p' \cos \theta_p' k_p^2 A_p' = 0, \quad (52)\]

\[\begin{bmatrix}
\mu \cos 2\theta_0 + K \cos^2 \theta_0 - \frac{K \eta_3}{k_3^2} \\
\mu \cos 2\theta_0 + K \cos^2 \theta_0 - \frac{K \eta_3}{k_3^2} \\
\end{bmatrix} k_3^2 A_0 - (2\mu + K) \sum_{p=1,2} \sin \theta_p \cos \theta_k k_p^2 A_p \]

\[- (2\mu' + K') \sum_{p=1,2} \sin \theta_p' \cos \theta_p' k_p^2 A_p' + \sum_{p=3,4} \left[ \mu \cos 2\theta_p + K \cos^2 \theta_p - \frac{K \eta_p}{k_p^2} \right] k_p^2 A_p \]

\[- \sum_{p=3,4} \left[ \mu' \cos 2\theta_p' + K' \cos^2 \theta_p' - \frac{K' \eta_p}{k_p^2} \right] k_p^2 A_p' = 0, \quad (53)\]

\[
\gamma \eta_3 \cos \theta_0 k_3 A_0 - \sum_{p=3,4} \left[ \gamma \eta_p \cos \theta_p k_p A_p + \gamma' \eta_p' \cos \theta_p' k_p' A_p' \right] = 0, \quad (54)\]

\[
\sum_{p=1,2} \left[ \left( \chi_0 - \frac{\lambda_0^2}{1 + \lambda_0^2} \right) \zeta_p \cos \theta_p k_p A_p + \left( \chi_0' - \frac{\lambda_0^2}{1 + \lambda_0^2} \right) \zeta_p' \cos \theta_p' k_p' A_p' \right] = 0, \quad (55)\]

\[
\cos \theta_0 k_3 A_0 + \sum_{p=1,2} \left[ \sin \theta_p k_p A_p - \sin \theta_p' k_p' A_p' \right] - \sum_{p=3,4} \left[ \cos \theta_p k_p A_p + \cos \theta_p' k_p' A_p' \right] = 0, \quad (56)\]

\[
\sin \theta_0 k_3 A_0 + \sum_{p=1,2} \left[ \cos \theta_p k_p A_p + \cos \theta_p' k_p' A_p' \right] + \sum_{p=3,4} \left[ \sin \theta_p k_p A_p - \sin \theta_p' k_p' A_p' \right] = 0, \quad (57)\]

\[
\eta_3 A_0 + \sum_{p=3,4} \left[ \eta_p A_p - \eta_p' A_p' \right] = 0, \quad (58)\]

\[
\sum_{p=1,2} \left[ \zeta_p A_p - \zeta_p' A_p' \right] = 0. \quad (59)\]

The system of Eqs. (52)–(59) can be written in matrix form as

\[\begin{bmatrix}
b_{ij} \end{bmatrix} [Z'] = [N], \quad (60)\]

where \([b_{ij}]\) is a 8×8 matrix, \([Z'] = [Z'_1, Z'_2, Z'_3, Z'_4, Z'_5, Z'_6, Z'_7, Z'_8]'\) is a column matrix, \(Z'_r = A_r/A_0\) and \(Z'_{(r+4)} = A'_r/A_0\) \((r = 1, 2, 3, 4)\) defined earlier are now the reflection and transmission coefficients respectively, when coupled transverse wave traveling with velocity \(V_3\) is made incident. The non-vanishing elements of coefficient matrix \([b_{ij}]\) and the column matrix \([N]\) are given in Appendix B. Thus, Eq. (60) will enable us to provide the expressions of reflection and transmission coefficients in the present case.

Expressions for energy ratios \(E_i \quad (i = 1, 2, 3, \ldots, 8)\) of various reflected and transmitted waves in this case, are given by
Here, each energy ratio $E_i$ gives the rate of energy transmitted at the interface for the respective reflected and transmitted wave to the rate of energy transmitted for the incident coupled transverse wave propagating with velocity $V_3$.

5. Limiting cases

(i) If we assume that both the half-spaces are free from the electric and microstretch effects, then we shall be left with the relevant problem in micropolar elastic solid half-spaces. In this case, the wave propagating with velocity $V_2$ would not appear in the medium $M$. Similarly, the wave propagating with velocity $V'_2$ would not appear in the medium $M'$. Therefore, $A_2 = A'_2 = 0$. Thus, we note that the boundary Eqs. (37) and (41) are identically satisfied and the remaining Eqs. (34)–(36), (38)–(40) reduce to

\[
E_{1,2} = Z_{1,2}^2 P_2 \left[ \lambda + 2\mu + K - \frac{k_{1,2}^2}{k_{1,2}^2} \frac{e_{21,2}^2}{k_{1,2}^2} + \frac{\lambda_{21,2}^2}{(1 + \chi^2)k_{1,2}^2} \right] k_{1,2}^3 \cos \theta_{1,2},
\]

\[
E_3 = Z_3^2,
\]

\[
E_4 = Z_4^2 P_2 \left[ \mu + K - \frac{e_4^2}{k_4^2} (\gamma e_4 + K) \right] k_4^3 \cos \theta_4,
\]

\[
E_{5,6} = Z_{5,6}^2 P_2 \left[ \lambda' + 2\mu' + K' - \frac{k_{1,2}^2}{k_{1,2}^2} \frac{e_{21,2}^2}{k_{1,2}^2} + \frac{\lambda_{21,2}^2}{(1 + \chi^2)k_{1,2}^2} \right] k_{1,2}^3 \cos \theta'_{1,2},
\]

\[
E_{7,8} = Z_{7,8}^2 P_2 \left[ \mu' + K' - \frac{e_4^2}{k_4^2} (\gamma e_4' + K') \right] k_4^3 \cos \theta'_{3,4},
\]

where

\[
P_2 = \left[ \left( \mu + K - \frac{e_4^2}{k_4^2} (\gamma e_4 + K) \right) k_4^3 \cos \theta_0 \right]^{-1}.
\]

After converting the angle of incidence to the angle of emergence, one can verify that the above equations exactly match with those obtained by Tomar and Gogna (1995b) for the case of coupled longitudinal wave incident at the interface between two micropolar elastic solid half-spaces.

Similarly, when coupled transverse wave propagating through the medium $M$ with velocity $V_3$ become incident at a plane interface between two electro-microelastic solid half-spaces, then in this limiting case, we see...
that Eqs. (55) and (59) are satisfied identically and the remaining boundary equations exactly match with those obtained by Tomar and Gogna (1995a) after converting the angle of incidence into the angle of emergence.

(ii) If we assume that both the half-spaces are free from electric effect, then we will be left with the relevant problem in linear homogeneous microstretch elastic solid half-spaces. In this limiting case, it can be seen that the matrix Eq. (42) reduce to

\[
\begin{align*}
\lambda + (2\mu + K) \cos^2 \theta_0 - \frac{\lambda_{11}^\prime}{k^2_1} k^2_1 A_0 + \sum_{p=1,2} \left[ \lambda + (2\mu + K) \cos^2 \theta_p - \frac{\lambda_{0p}^\prime}{k^2_p} \right] k^2_p A_p \\
- \sum_{p=1,2} \left[ \lambda + (2\mu + K') \cos^2 \theta_p' - \frac{\lambda_{0p}^\prime}{k^2_p} \right] k^2_p A_p' + (2\mu + K) \sum_{p=3,4} \sin \theta_p \cos \theta_p k^2_p A_p \\
+ (2\mu + K') \sum_{p=3,4} \sin \theta_p' \cos \theta_p k^2_p A_p' = 0,
\end{align*}
\]  

\[\text{(67)}\]

\[
(2\mu + K) \sin \theta_0 \cos \theta_0 k^2_1 A_0 - \sum_{p=1,2} \left[ (2\mu + K) \sin \theta_p \cos \theta_p k^2_p A_p + (2\mu + K') \sin \theta_p' \cos \theta_p' k^2_p A_p' \right] \\
+ \sum_{p=3,4} \left[ \mu \cos 2\theta_p + K \cos^2 \theta_p - \frac{K \eta_p}{k^2_p} \right] k^2_p A_p - \left[ \mu' \cos 2\theta_p' + K' \cos^2 \theta_p' - \frac{K' \eta_p'}{k^2_p} \right] k^2_p A_p' = 0,
\]

\[\text{(68)}\]

\[
\sum_{p=3,4} \left[ \eta_p \cos \theta_p k_p A_p + \eta_p' \cos \theta_p' k_p A_p' \right] = 0,
\]

\[\text{(69)}\]

\[
\alpha_0 \zeta_1 \cos \theta_0 k_1 A_0 - \sum_{p=1,2} \left[ \alpha_0 \zeta_p \cos \theta_p k_p A_p - \alpha_0 \zeta_p' \cos \theta_p' k_p A_p' \right] = 0,
\]

\[\text{(70)}\]

\[
\sin \theta_0 k_1 A_0 + \sum_{p=1,2} \left[ \sin \theta_p k_p A_p - \sin \theta_p' k_p A_p' \right] - \sum_{p=3,4} \left[ \cos \theta_p k_p A_p + \cos \theta_p' k_p A_p' \right] = 0,
\]

\[\text{(71)}\]

\[
\cos \theta_0 k_1 A_0 - \sum_{p=1,2} \left[ \cos \theta_p k_p A_p + \cos \theta_p' k_p A_p' \right] - \sum_{p=3,4} \left[ \sin \theta_p k_p A_p - \sin \theta_p' k_p A_p' \right] = 0,
\]

\[\text{(72)}\]

\[
\sum_{p=3,4} \left[ \eta_p A_p - \eta_p' A_p' \right] = 0,
\]

\[\text{(73)}\]

\[
\zeta_1 A_0 + \sum_{p=1,2} \left[ \zeta_p A_p - \zeta_p' A_p' \right] = 0.
\]

\[\text{(74)}\]

These equations exactly match with the boundary equations obtained by Tomar and Garg (2005) for the case of a coupled longitudinal wave incident at an interface between two different microstretch elastic solid half-spaces.

Similarly, when coupled transverse wave propagating with velocity \( V_1 \) through the medium \( M \) is incident at an interface, and then making the substitutions as considered above into the matrix Eq. (60), we see that the reduced equations exactly match with the boundary equations obtained by Tomar and Garg (2005) for the case of a coupled transverse wave incident at an interface between two different microstretch elastic solid half-spaces.

(iii) To discuss the problem of reflection of coupled longitudinal wave from free plane surface of an electromicroelastic solid half-space, we shall assume that the medium \( M' \) is absent. Here, the boundary conditions will be corresponding to vanishing of loads at the free boundary. Thus, putting the quantities having dashes equal to zero into Eqs. (34)–(37), we obtain the following equations:

\[
\begin{align*}
\lambda + (2\mu + K) \cos^2 \theta_0 - \frac{\lambda_{11}^\prime}{k^2_1} k^2_1 A_0 + \sum_{p=1,2} \left[ \lambda + (2\mu + K) \cos^2 \theta_p - \frac{\lambda_{0p}^\prime}{k^2_p} \right] k^2_p A_p \\
+ (2\mu + K) \sum_{p=3,4} \sin \theta_p \cos \theta_p k^2_p A_p = 0,
\end{align*}
\]

\[\text{(75)}\]
\[ (2\mu + K) \sin \theta_0 \cos \theta_0 k_1^2 A_0 - (2\mu + K) \sum_{p=1,2} \sin \theta_p \cos \theta_p k_p^2 A_p \]
\[ + \sum_{p=3,4} \left[ \mu \cos 2\theta_p + K \cos^2 \theta_p - \frac{K \eta_p}{k_p^2} \right] k_p^2 A_p = 0, \]  
\[ (z_0 - \frac{j_2^2}{1 + \chi^2}) \zeta_1 \cos \theta_0 k_1 A_0 - \left( z_0 - \frac{j_2^2}{1 + \chi^2} \right) \sum_{p=1,2} \zeta_p \cos \theta_p k_p A_p = 0. \]  

Similarly, when a coupled transverse wave propagating with velocity \( V_1 \) is incident at the free plane surface of an electro-microelastic solid half-space, then making the corresponding substitutions into the Eqs. (52)–(55), we obtain the following equations:

\[ (2\mu + K) \sin \theta_0 \cos \theta_0 k_3^2 A_0 - (2\mu + K) \sum_{p=1,2} \left[ \lambda + (2\mu + K) \cos^2 \theta_p - \frac{\lambda_0 \zeta_p}{k_p^2} \right] k_p^2 A_p \]
\[ - (2\mu + K) \sum_{p=3,4} \sin \theta_p \cos \theta_p k_p^2 A_p = 0, \]
\[ \left[ \mu \cos 2\theta_0 + K \cos^2 \theta_0 - \frac{K \eta_3}{k_3^2} \right] k_3^2 A_0 - (2\mu + K) \sum_{p=1,2} \sin \theta_p \cos \theta_p k_p^2 A_p \]
\[ + \sum_{p=3,4} \left[ \mu \cos 2\theta_p + K \cos^2 \theta_p - \frac{K \eta_2}{k_p^2} \right] k_p^2 A_p = 0, \]
\[ \gamma \eta_3 \cos \theta_0 k_3 A_0 - \gamma \sum_{p=3,4} \eta_p \cos \theta_p k_p A_p = 0, \]
\[ \left( z_0 - \frac{j_2^2}{1 + \chi^2} \right) \sum_{p=1,2} \zeta_p \cos \theta_p k_p A_p = 0. \]  

The formulae (75)–(78) and (79)–(82) yield the reflection coefficients at a free plane boundary in the respective case.

6. Numerical results and discussions

In order to examine this study in greater detail, we have computed the amplitude ratios and energy ratios of various reflected and refracted waves for a particular model. Keeping in view the restrictions on constitutive constants given earlier, we have taken the following values of the relevant parameters in medium \( M \)

\[ \lambda = 7.85 \times 10^{11} \text{ dyne/cm}^2, \quad \mu = 6.46 \times 10^{11} \text{ dyne/cm}^2, \quad K = 0.0139 \times 10^{11} \text{ dyne/cm}^2, \]
\[ \lambda_0 = 0.085 \times 10^{11} \text{ dyne}, \quad \lambda_0 = 0.038 \times 10^{11} \text{ dyne/cm}^2, \quad \lambda_1 = 0.030 \times 10^{11} \text{ dyne/cm}^2, \]
\[ j_2^2 = 0.3364 \times 10^{11} \text{ dyne}, \quad j = j_0 = 0.0212 \text{ cm}^2, \quad \rho = 1.9 \text{ g/cm}^3, \quad \gamma = 0.365 \times 10^{11} \text{ dyne}, \]
\[ \chi^E = 318, \text{ and the relevant values of the parameters in medium } M' \]
\[ \lambda' = 7.59 \times 10^{11} \text{ dyne/cm}^2, \quad \mu' = 1.89 \times 10^{11} \text{ dyne/cm}^2, \quad K' = 0.0149 \times 10^{11} \text{ dyne/cm}^2, \]
\[ j' = j'_0 = 0.0196 \text{ cm}^2, \quad \rho' = 2.2 \text{ g/cm}^3, \quad \gamma' = 0.345 \times 10^{11} \text{ dyne}, \quad \chi'^E = 298, \text{ and } \omega/\omega_0 = 10. \]

In Fig. 2, we have plotted the modulus values of reflection coefficients as a function of angle of incidence, when coupled longitudinal wave becomes incident obliquely at the interface between the half-spaces \( M \) and \( M' \). The reflection coefficient \( Z_1 \) begins with the value 0.1132 near the normal incidence, it decreases till \( \theta_0 = 35^\circ \), and afterwards it increases very slowly till \( \theta_0 = 50^\circ \). Beyond \( \theta_0 = 50^\circ \), the reflection coefficient \( Z_1 \) decreases very smoothly till \( \theta_0 = 61^\circ \) and thereafter it increases frequently and attains the maximum value
i.e. unity at the grazing incidence. The reflection coefficient $Z_2$ first decreases with increase of $\theta_0$ in the range $1^\circ \leq \theta_0 \leq 28^\circ$, then it increases sharply to attain its maximum value at $\theta_0 = 64^\circ$, afterwards its value decreases and approaches to zero at $\theta_0 = 90^\circ$. The value of reflection coefficient $Z_3$ increases slowly with increase of angle of incidence till $\theta_0 = 25^\circ$, then its value decreases till $\theta_0 = 45^\circ$. After $\theta_0 = 45^\circ$, the reflection coefficient $Z_3$ again increases rapidly with angle of incidence to attain its maximum value and thereafter it decreases and vanishes at $\theta_0 = 90^\circ$. The value of reflection coefficient $Z_4$ increases very slowly with increase in the angle of incidence till $\theta_0 = 34^\circ$ and then it decreases gradually to the value zero at grazing incidence. We found that the reflection coefficients $Z_2$ and $Z_3$ are very small as compared to $Z_1$ in the entire range of angle of incidence. Therefore, we have plotted each of them by magnifying $10^5$ times their original values. At the grazing incidence, all the reflected waves are found to disappear except the reflected coupled longitudinal wave propagating with speed $V_1$.

In Fig. 3, we have shown the variation of modulus values of transmission coefficients with the angle of incidence of coupled longitudinal wave propagating with velocity $V_1$. We observed that the amplitude ratios $Z_5$ and $Z_6$ of transmitted coupled longitudinal waves attain their maximum values at normal incidence and then their values decrease gradually to zero at the grazing incidence. Both the amplitude ratios $Z_7$ and $Z_8$ increase with increase in angle of incidence and attain their maximum values near $\theta_0 = 70^\circ$. Thereafter, they decrease and vanish at grazing incidence. However, the maximum value of $Z_8$ is greater than the maximum value of $Z_7$. Also, it is noted that the modulus of transmission coefficient $Z_8$ corresponding to the transmitted coupled longitudinal wave propagating with velocity $V'_1$ is contributing significantly as compared to all other transmission coefficients which are very small. The variation of $Z_6$ and $Z_7$ with angle of incidence are plotted after magnifying their original values by the multiples of $10^3$ and $10^5$ respectively.

Figs. 4 and 5 depict the variation of energy ratios of reflected and transmitted waves respectively, with the angle of incidence of coupled longitudinal wave propagating with velocity $V_1$. It can be seen from Fig. 4 that the energy carried by reflected coupled longitudinal wave propagating with velocity $V_1$ is maximum in comparison to energy carried along with other reflected waves. In Fig. 5, we note that the transmitted coupled longitudinal wave and transmitted coupled transverse wave propagating with velocities $V'_2$ and $V'_3$ respectively, have minimum energy in comparison to other transmitted waves. It has been verified that $\sum_{i=1}^{8} E_i = 1$ at each angle of incidence, showing that there is no dissipation of energy at the interface. In Fig. 5, the dotted line

![Fig. 2. Variation of modulus of reflection coefficients with angle of incidence of coupled longitudinal wave propagating with velocity $V_1$. (Curve I: $Z_1$, Curve II: $Z_2 \times 10^5$, Curve III: $Z_3 \times 10^5$, Curve IV: $Z_4$).](image-url)
represents the energy ratio carried by the transmitted longitudinal displacement wave at the plane interface between two micropolar elastic solid half-spaces.

Fig. 6 depicts the variation of reflection coefficients with non-dimensional frequency \( \frac{\omega}{\omega_0} \) when coupled longitudinal wave traveling with speed \( V_1 \) strikes the interface at 45° angle of incidence. We observed from this figure that the reflection coefficients \( Z_2 \) and \( Z_3 \) are influenced with frequency in the range \( 1.9 \leq \frac{\omega}{\omega_0} \leq 4.5 \), while for higher values of \( \frac{\omega}{\omega_0} \), these coefficients are almost independent of \( \frac{\omega}{\omega_0} \) and bear different constant
values. The reflection coefficients $Z_1$ and $Z_4$ are least affected by $\omega/\omega_0$ and remain almost constant in the entire range. The pattern of reflection coefficient $Z_2$ is similar to that of $Z_3$. Both these coefficients decrease in the influential range and approach to zero as $\omega/\omega_0$ takes larger and larger values. We have plotted the reflection coefficients $Z_1$, $Z_2$, $Z_3$ and $Z_4$ by magnifying their original values by the factor of 10, $10^3$, $10^3$ and 10 respectively.

![Graph of energy ratios](image1)

**Fig. 5.** Variation of energy ratios of transmitted waves with angle of incidence of coupled longitudinal wave propagating with velocity $V_1$. (Curve I: $E_5$, Curve II: $E_6 \times 10^3$, Curve III: $E_7 \times 10^3$, Curve IV: $E_8$).

![Graph of reflection coefficients](image2)

**Fig. 6.** Variation of modulus of reflection coefficients with frequency ratio ($\omega/\omega_0$) when coupled longitudinal wave propagating with velocity $V_1$ is incident at an angle $\theta_0 = 45^\circ$. (Curve I: $Z_1 \times 10$, Curve II: $Z_2 \times 10^3$, Curve III: $Z_3 \times 10^3$, Curve IV: $Z_4 \times 10$).
In Fig. 7, we have plotted the transmission coefficients against frequency ratio \((\omega/\omega_0)\) when coupled longitudinal wave traveling with velocity \(V_1\) strikes the interface at 45° angle of incidence. The transmission coefficients \(Z_5\) and \(Z_6\) exhibit reverse behaviour as \(\omega/\omega_0\) takes values greater than 1.8 (i.e. \(\omega/\omega_0 > 1.8\)). The transmission coefficient \(Z_7\) decreases in the range \(1.9 \leq \omega/\omega_0 \leq 4.0\) and then becomes constant for higher values of \(\omega/\omega_0\). The transmission coefficient \(Z_8\) remains almost constant in the entire range. The transmission coefficient \(Z_7\) is very small in comparison to other transmission coefficients. We have plotted the transmission coefficient \(Z_7\) by magnifying its original value by the factor \(10^3\).

In Fig. 8, we have shown the variation of modulus values of reflection coefficients versus angle of incidence of coupled transverse wave propagating with velocity \(V_3\). We observed from this figure that the behaviour of reflection coefficients \(Z_{10}^1\) and \(Z_{10}^2\) are found to be almost similar. Both the reflection coefficients begin with certain finite values, then their values increase with increase in angle of incidence till \(\theta_0 = 26°\) and \(\theta_0 = 31°\) respectively, thereafter they decrease till \(\theta_0 = 47°\) and \(\theta_0 = 58°\), respectively, after that both the reflection coefficients increase sharply. The reflection coefficient \(Z_{10}^3\) begins with its minimum value at 1° angle of incidence then it increases with increase in the angle of incidence and achieves its maximum value 0.1183 at 66° angle of incidence. The behaviour of reflection coefficient \(Z_{10}^4\) is found to be monotonically decreasing in the range \(1° \leq \theta_0 \leq 35°\), approaches to zero at \(\theta_0 = 35°\) and then it increases afterwards.

Fig. 9 shows the variation of modulus of amplitude ratios corresponding to transmitted waves with angle of incidence of coupled transverse wave propagating with speed \(V_3\). It can be seen that the transmission coefficients \(Z_{10}^5\) and \(Z_{10}^6\) have similar pattern in the entire range. Both the transmission coefficients increase with increase in the angle of incidence. The value of reflection coefficient \(Z_{10}^7\) corresponding to the transmitted coupled transverse wave propagating with velocity \(V_3\) is found to be dominant among all other transmitted waves. The transmission coefficient \(Z_{10}^6\) is very small and so, we have plotted it by magnifying its original value with the factor of \(10^3\). The transmission coefficient \(Z_{10}^8\) begins with its maximum value at 1° angle of incidence and then decreases till \(\theta_0 = 64°\) and afterwards it increases slightly.

Figs. 10 and 11 depict the variation of modulus of energy ratios of reflected and transmitted waves respectively, against the angle of incidence of coupled transverse wave propagating with velocity \(V_3\). It can be seen from these figures that maximum amount of energy of incident coupled transverse wave is being carried by transmitted coupled transverse wave traveling with velocity \(V'_3\), and only a small amount of energy is being
carried by all other reflected and transmitted waves in this case. Also, it has been verified that 
\[ P_i = \frac{1}{2} \quad E_i = \frac{1}{2} a t \]
each angle of incidence in the range \( 0 < \theta_0 \leq 66^\circ \).

In Fig. 12, we have shown the variation of modulus of reflection coefficients with frequency ratio \( \frac{\omega}{\omega_0} \) when coupled transverse wave propagating with velocity \( V_3 \) is incident at \( \theta_0 = 45^\circ \). It can be seen that the reflection coefficient \( Z'_1 \) starts with some non-zero value at \( \frac{\omega}{\omega_0} = 1.5 \) and then increases to attain its maximum value at \( \frac{\omega}{\omega_0} = 2.2 \), beyond which it decreases smoothly. Here, the reflection coefficient \( Z'_2 \) is monotonically decreasing in nature, while the reflection coefficient \( Z'_3 \) possesses reverse behaviour to it, in the entire range. The reflection coefficient \( Z'_4 \) decreases in the range \( 1.5 \leq \frac{\omega}{\omega_0} \leq 2 \), and then it increases very frequently.
Fig. 10. Variation of energy ratios of reflected waves with angle of incidence of coupled transverse wave propagating with velocity $V_3$. (Curve I: $E_1 \times 10^5$, Curve II: $E_2 \times 10^{11}$, Curve III: $E_3 \times 10^5$, Curve IV: $E_4 \times 10^9$).

Fig. 11. Variation of energy ratios of transmitted waves with angle of incidence of coupled transverse wave propagating with velocity $V_3$. (Curve I: $E_5 \times 10^5$, Curve II: $E_6 \times 10^8$, Curve III: $E_7$, Curve IV: $E_8 \times 10^5$).

Fig. 13 shows the variation of modulus of transmission coefficients with frequency ratio when coupled transverse wave traveling with velocity $V_3$ is incident at an angle $\theta_0 = 45^\circ$. We observed from this figure that the value of transmission coefficient $Z'_6$ increases with increase in the frequency ratio ($\omega/\omega_0$). The transmission coefficient $Z'_6$ begins with some non-zero value and it increases sharply in the range $1.5 \leq \omega/\omega_0 \leq 2.3$, then it decreases gradually with $\omega/\omega_0$. The transmission coefficients $Z'_7$ and $Z'_8$ exhibit reverse behaviour to each other.
as \( \omega_0/\omega_0 \) takes value greater than 1.5. The values of transmission coefficients \( Z_6' \) and \( Z_8' \) are small in comparison to the value of transmission coefficient \( Z_7' \). We have plotted \( Z_6' \) and \( Z_8' \) after magnifying its original value 10 times.

Parfitt and Eringen (1969) have shown that in an infinite micropolar elastic medium there exist four basic waves: an independent longitudinal displacement wave, an independent longitudinal micro-rotational wave, and two sets of coupled transverse waves. In this paper, we have discussed the propagation of plane waves in
an infinite electro-microelastic solid medium. Electro-microelastic medium is a microstretch material under electric interactions. We found that in an infinite electro-microelastic medium, there exist two sets of coupled longitudinal wave, an independent longitudinal micro-rotational wave, and two sets of coupled transverse waves. Each set of coupled longitudinal wave consists of a longitudinal displacement wave and a longitudinal microstretch wave, influenced by the electric effect. The remaining waves in the electro-microelastic solid are the same waves as found in micropolar elasticity. In fact, in the absence of microstretch and electric effects, the two sets of coupled longitudinal waves reduce to a longitudinal displacement wave of micropolar elasticity. Reflection and refraction phenomena of elastic waves at a plane interface between two different electro-microelastic solid half-spaces have been investigated. By neglecting the electric and microstretch effects from two different electro-microelastic solid half-spaces in perfect contact, the problem of reflection and refraction of elastic waves at a plane interface between two different micropolar elastic solid half-spaces is recovered. The two sets of reflected and refracted coupled transverse waves are not influenced by microstretch and electric properties. These waves are common at electro-microelastic/electro-microelastic interface (say $I_1$) and micropolar/micropolar interface (say $I_2$). There appear two new waves (a reflected coupled longitudinal wave propagating at speed $V'_2$ and a refracted coupled longitudinal wave propagating at speed $V'_1$) during reflection and refraction of elastic waves at interface $I_1$, not observed in the case of interface $I_2$. However, the amplitude ratios of these new waves are very small in comparison to other reflected and refracted waves. The variation of amplitude ratios corresponding to common reflected and refracted waves with angle of incidence at a fixed frequency are found to be similar, when looked in case of incidence of a coupled longitudinal wave, and in case of incidence of a coupled transverse wave separately. Interestingly, the variation of amplitude ratios with angle of incidence corresponding to the reflected and refracted coupled longitudinal waves propagating at speeds $V_1$ and $V'_1$ respectively, are found to be the same at the interface $I_1$ as that of reflected and refracted longitudinal displacement waves at the interface $I_2$. Now the question arises: When there is no significant difference in the amplitude ratios of common reflected and refracted waves, and also two more amplitude ratios corresponding to the new reflected and refracted waves are appearing at the interface $I_1$ that are not appearing at the interface $I_2$, then how does the energy relation balances at the interface $I_1$ and $I_2$? The answer to this question is: We have then computed the energies of various reflected and refracted waves at the interface $I_1$ as well as at the interface $I_2$. We found that (i) there is a significant effect of electric and microstretch properties on the energy ratio of refracted longitudinal displacement wave and (ii) the energies carried by the new reflected and refracted waves are very small (of order of $10^{-6}$ and $10^{-3}$ respectively), when coupled longitudinal wave is made incident at the interface $I_1$. It is seen that the microstretch and electric effects lower down the energy of refracted longitudinal displacement wave. This difference is being covered by the energies of new waves. This is how the balance of energy relation could be satisfied. Further, it is also to be noted that when coupled transverse wave is made incident at the interfaces $I_1$ and $I_2$ there is no significant effect of electric and microstretch properties is noticed neither on the amplitude ratios nor on the energy ratios of various reflected and refracted waves. In this case, the energy carried by the new reflected and refracted waves are very very small (of order of $10^{-11}$ and $10^{-8}$ respectively) and consequently, the new waves have not created any problem in satisfying the energy balance relation.

7. Conclusions

In this paper, we have presented the wave propagation in an infinite electro-microelastic solid medium. It has been found that there exist five plane waves propagating with five distinct phase velocities namely, two coupled longitudinal waves, which are influenced by the electric effect, two coupled transverse waves, and an independent longitudinal micro-rotational wave. Reflection and transmission phenomena of plane waves at a plane interface between two different electro-microelastic solid half-spaces in perfect contact are also investigated. We conclude that

(i) When coupled longitudinal (transverse) wave is incident normally, the reflection and transmission of only coupled longitudinal (transverse) waves take place.

(ii) At grazing incidence of coupled longitudinal wave, no reflection and transmission of waves take place and the same wave propagates along the interface.

(iii) The angle $\theta_0 = 67^\circ$ is found to be the critical angle when coupled transverse wave is incident.
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Appendix A

The non-zero entries of the matrix \([a_{ij}]\) of Eq. (42) are given as,

\[
\begin{align*}
  a_{11} &= 1, a_{12} = \left( \lambda + (2\mu + K)(1 - v_{21}^2 \sin^2 \theta_0) - \frac{\lambda_0 v_{21} \varepsilon}{k^4_2} \right) / (D_1 v_{21}^4), \\
  a_{1p} &= D_2 \sin \theta_0 \sqrt{1 - v_{p1}^2 \sin^2 \theta_0} / (D_1 v_{p1}), \\
  a_{15} &= - \left( \lambda' + (2\mu' + K') (1 - v_{11}^2 \sin^2 \theta_0) - \frac{\lambda_0 v_{11} \varepsilon'}{k^4_1} \right) / (D_1 v_{11}^4), \\
  a_{16} &= - \left( \lambda' + (2\mu' + K') (1 - v_{21}^2 \sin^2 \theta_0) - \frac{\lambda_0 v_{21} \varepsilon'}{k^4_1} \right) / (D_1 v_{21}^4), \\
  a_{1i} &= D_2 \sin \theta_0 \sqrt{1 - v_{r1}^2 \sin^2 \theta_0} / (D_1 v_{r1}), \\
  a_{21} &= \sin \theta_0 \cos \theta_0, \\
  a_{22} &= \sin \theta_0 \sqrt{1 - v_{21}^2 \sin^2 \theta_0} / v_{21}, \\
  a_{23} &= - \left( \mu(1 - 2v_{31}^2 \sin^2 \theta_0) + K(1 - v_{31}^2 \sin^2 \theta_0) - \frac{K \eta_3}{k^4_3} \right) / (D_2 v_{31}^4), \\
  a_{24} &= - \left( \mu(1 - 2v_{41}^2 \sin^2 \theta_0) + K(1 - v_{41}^2 \sin^2 \theta_0) - \frac{K \eta_4}{k^4_4} \right) / (D_2 v_{41}^4), \\
  a_{27} &= \left[ \mu'(1 - 2v_{31}^2 \sin^2 \theta_0) + K'(1 - v_{31}^2 \sin^2 \theta_0) - \frac{K' \eta_3}{k^4_3} \right] / (D_2 v_{31}^4), \\
  a_{28} &= \left[ \mu'(1 - 2v_{41}^2 \sin^2 \theta_0) + K'(1 - v_{41}^2 \sin^2 \theta_0) - \frac{K' \eta_4}{k^4_4} \right] / (D_2 v_{41}^4), \\
  a_{33} &= \sqrt{1 - v_{31}^2 \sin^2 \theta_0} / v_{31}, \\
  a_{34} &= \eta_4 \sqrt{1 - v_{41}^2 \sin^2 \theta_0} / (\eta_4 v_{41}), \\
  a_{37} &= \gamma \eta_3 \sqrt{1 - v_{31}^2 \sin^2 \theta_0} / (\gamma \eta_3 v_{31}), \\
  a_{38} &= \gamma \eta_4 \sqrt{1 - v_{41}^2 \sin^2 \theta_0} / (\gamma \eta_4 v_{41}), \\
  a_{41} &= \cos \theta_0, \\
  a_{42} &= \zeta_2 \sqrt{1 - v_{21}^2 \sin^2 \theta_0} / (\zeta_1 v_{21}), \\
  a_{45} &= \left( \xi_0 - \frac{\lambda_0^2}{1 + \xi^2} \right) \xi_1 \sqrt{1 - v_{11}^2 \sin^2 \theta_0} / \left( \left( \xi_0 - \frac{\lambda_0^2}{1 + \xi^2} \right) v_{11} \xi_1 \right).
\end{align*}
\]
\[ a_{46} = \left( x_0^2 - \frac{x_0^2}{1 + \chi^2} \right) V_{i2}^2 \left( 1 - v_{21}^2 \sin^2 \theta_0 \right) / \left( \left( x_0^2 - \frac{x_0^2}{1 + \chi^2} \right) v_{21}^2 v_1' \right), \]
\[ a_{51} = a_{52} = \sin \theta_0, \quad a_{5p} = -\sqrt{1 - v_{21}^2 \sin^2 \theta_0} / v_{p1}, \quad a_{5j} = -\sin \theta_0, \]
\[ a_{5l} = -\sqrt{1 - v_{21}^2 \sin^2 \theta_0} / V_{i1}', \quad a_{61} = \cos \theta_0, \quad a_{62} = \sqrt{1 - v_{21}^2 \sin^2 \theta_0} / v_{21}, \]
\[ a_{6p} = \sin \theta_0, \quad a_{6j} = \sqrt{1 - v_{j1}^2 \sin^2 \theta_0} / V_{j1}', \quad a_{6i} = -\sin \theta_0, \quad a_{73} = 1, \]
\[ a_{74} = \eta_4 / \eta_3, \quad a_{77} = -\eta_3 / \eta_3, \quad a_{78} = -\eta_4 / \eta_3, \quad a_{81} = 1, \quad a_{82} = \zeta_2 / \zeta_1, \]
\[ a_{85} = -\zeta_1 / \zeta_1, \quad a_{86} = -\zeta_2 / \zeta_1, \quad p = 3, 4; \quad j = 5, 6; \quad i = 7, 8; \]
\[ D_1 = \left[ \lambda + (2 \mu + K) \cos^2 \theta_0 - \frac{\lambda_0 \eta_3}{k_1^2} \right], \quad D_2 = 2 \mu + K, \quad v_{m1} = \frac{V_m}{V_1} (m = 2, 3, 4), \]
\[ v_{n1}' = \frac{V_n'}{V_1} (n = 1, 2, 3, 4), \quad V_{51}' = v_{i1}', \quad V_{61}' = v_{21}', \quad V_{71}' = v_{31}', \quad V_{81}' = v_{41}', \]
and the column matrix \([M] = [-1, \sin \theta_0 \cos \theta_0, 0, \cos \theta_0, -\sin \theta_0, \cos \theta_0, 0, -1]^T\).

**Appendix B**

The non-zero entries of matrix \([b_{ij}]\) in Eq. (60) are given by
\[ b_{11} = \left[ \lambda + (2 \mu + K) (1 - v_{11}^2 \sin^2 \theta_0) - \frac{\lambda_0 \eta_1}{k_1^2} \right] / (D_2 v_{11}^2), \]
\[ b_{12} = \left[ \lambda + (2 \mu + K) (1 - v_{22}^2 \sin^2 \theta_0) - \frac{\lambda_0 \eta_2}{k_2^2} \right] / (D_2 v_{22}^2), \]
\[ b_{13} = \sin \theta_0 \cos \theta_0, \quad b_{14} = \sin \theta_0 \sqrt{1 - v_{44}^2 \sin^2 \theta_0} / v_{44}, \]
\[ b_{15} = - \left[ \lambda + (2 \mu' + K') (1 - v_{11}^2 \sin^2 \theta_0) - \frac{\lambda_0 \eta_1}{k_1^2} \right] / (D_2 v_{11}^2), \]
\[ b_{16} = - \left[ \lambda + (2 \mu' + K') (1 - v_{22}^2 \sin^2 \theta_0) - \frac{\lambda_0 \eta_2}{k_2^2} \right] / (D_2 v_{22}^2), \]
\[ b_{17} = D_2' \sin \theta_0 \sqrt{1 - v_{11}^2 \sin^2 \theta_0} / (V_{i1}' D_2), \]
\[ b_{21} = D_2 \sin \theta_0 \sqrt{1 - v_{11}^2 \sin^2 \theta_0} / (D_3 v_{11}), \]
\[ b_{22} = D_2 \sin \theta_0 \sqrt{1 - v_{22}^2 \sin^2 \theta_0} / (D_3 v_{22}), \quad b_{23} = -1, \]
\[ b_{24} = - \left[ \mu (1 - 2 v_{44}^2 \sin^2 \theta_0) + K (1 - v_{44}^2 \sin^2 \theta_0) - \frac{K \eta_4}{k_4^2} \right] / (D_3 v_{44}^2), \]
\[ b_{2j} = D_2' \sin \theta_0 \sqrt{1 - v_{j1}^2 \sin^2 \theta_0} / (V_{j1}' D_3). \]
\[ b_{27} = \frac{\mu' (1 - 2v_{33}^2 \sin^2 \theta_0) + K' (1 - v_{33}^2 \sin^2 \theta_0) - \frac{K' \eta_3}{k_3^2}}{(D_3 v_{33}^2)}, \]
\[ b_{28} = \frac{\mu' (1 - 2v_{43}^2 \sin^2 \theta_0) + K' (1 - v_{43}^2 \sin^2 \theta_0) - \frac{K' \eta_3}{k_4^2}}{(D_3 v_{43}^2)}, \]
\[ b_{33} = \cos \theta_0, \quad b_{34} = \eta_4 \sqrt{1 - v_{43}^2 \sin^2 \theta_0} / (\eta_3 v_{43}), \]
\[ b_{37} = \gamma' \eta_3 \sqrt{1 - v_{33}^2 \sin^2 \theta_0} / (\gamma \eta_3 v_{33}), \quad b_{38} = \gamma' \eta_4 \sqrt{1 - v_{43}^2 \sin^2 \theta_0} / (\gamma \eta_3 v_{43}), \]
\[ b_{41} = \sqrt{1 - v_{13}^2 \sin^2 \theta_0} / v_{13}, \quad b_{42} = \zeta_2 \sqrt{1 - v_{23}^2 \sin^2 \theta_0} / (\zeta_1 v_{23}), \]
\[ b_{45} = \left( \frac{\zeta_0 - \frac{\gamma' \eta_2}{1 + \chi^2}}{1 + \chi^2} \right) \sqrt{1 - v_{13}^2 \sin^2 \theta_0} / \left( \left( \frac{\zeta_0 - \frac{\gamma' \eta_2}{1 + \chi^2}}{1 + \chi^2} \right) v_{13} \right), \]
\[ b_{46} = \left( \frac{\zeta_0 - \frac{\gamma' \eta_2}{1 + \chi^2}}{1 + \chi^2} \right) \sqrt{1 - v_{23}^2 \sin^2 \theta_0} / \left( \left( \frac{\zeta_0 - \frac{\gamma' \eta_2}{1 + \chi^2}}{1 + \chi^2} \right) v_{23} \right), \]
\[ b_{51} = b_{52} = \sin \theta_0, \quad b_{53} = -\cos \theta_0, \quad b_{54} = -\sqrt{1 - v_{43}^2 \sin^2 \theta_0} / v_{43}, \quad b_{55} = -\sin \theta_0, \]
\[ b_{56} = -\sqrt{1 - v_{13}^2 \sin^2 \theta_0} / v_{13}, \quad b_{57} = -\sqrt{1 - v_{23}^2 \sin^2 \theta_0} / v_{23}, \]
\[ b_{58} = -\sin \theta_0, \quad b_{60} = \sin \theta_0, \quad b_{73} = 1, \quad b_{74} = \eta_4 / \eta_3, \]
\[ b_{77} = -\eta_3 / \eta_3, \quad b_{78} = -\eta_4 / \eta_3, \quad b_{81} = 1, \quad b_{82} = \zeta_2 / \zeta_1, \quad b_{85} = -\zeta_1 / \zeta_1, \quad b_{86} = -\zeta_2 / \zeta_1, \]
\[ p = 3, 4; \quad j = 5, 6; \quad i = 7, 8; \quad \text{where} \quad V'_{53} = v_{13}^i, \quad V'_{65} = v_{13}^i, \quad V'_{73} = v_{33}^i, \quad V'_{83} = v_{33}^i, \quad v_{m3} = \frac{V_m}{V_3} (m = 1, 2, 4), \quad v_{n3}^i = \frac{V_n^i}{V_3} (n = 1, 2, 3, 4), \quad D_3 = \left[ \mu \cos 2\theta_0 + K \cos^2 \theta_0 - \frac{K \eta_3}{k_3^2} \right], \]
and the column matrix \[ [N] = [\sin \theta_0 \cos \theta_0, 1, \cos \theta_0, 0, -\cos \theta_0, \sin \theta_0, -1, 0]^t. \]

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