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Preface

Symbolic–numerical computations (hereafter abbreviated as *SNC*) make up an important area of rapidly growing popularity. The basic idea is to combine towards a common goal the two rich but distinct bodies of techniques developed in the two fields of symbolic and numerical computing. For a number of highly important computational problems such *Hybrid Methods* lead to more effective algorithms than those based solely on the techniques from one of the fields. Some of the most fundamental problems of symbolic computations are ill conditioned, so that their output is readily corrupted in the realistic case of inprecise input data as well as in floating point numerical computations. Nevertheless, the powerful numerical methods can rapidly produce the desired solution when they are properly employed in combination with symbolic methods. E.g., one can seek help from mathematical programming formulation, preprocess the input data symbolically to improve conditioning, or perform auxiliary numerical computations that reduce the original ill conditioned problem to another problem of a smaller size, so that symbolic methods can handle it efficiently.

An important subject of SNC is the solution of the classical ill conditioned problems of numerical approximation of complex or real roots of a univariate polynomial and a system of multivariate polynomials. These problems are fundamental for symbolic computations, but numerical techniques (combined with some symbolic methods) are the basis for the most successful packages for the univariate polynomial root-finding such as MPSolve and EigenSolve, whereas the need for powerful SNC for multivariate polynomial root-finding is also well understood. Numerical (e.g., Newton-like) methods need good initial approximations to converge to polynomial roots (see [1, pages 651–669] and [2] on some recipes for rescue based on the continuation techniques), whereas symbolic (e.g., Gröbner basis) methods generally suffer from numerical instability.

Two papers in our special issue circumvent these problems. Mourrain and Trebouchet develop a numerically stable construction of normal forms for a zero-dimensional ideal (consisting of a finite number of roots). This construction, proposed by the first author in 1999, has now been advanced, analyzed in depth (including the issues of numerical stability and certification), related to syzigies and commutation, and tested on benchmark polynomial systems.

Zeng exploits the correlation between polynomial root-finding and another highly important problem of SNC, that is the ill conditioned computation of an approximate greatest common divisor (GCD) for a set of univariate or multivariate polynomials. He computes approximate GCDs to eliminate the variables in a multivariate polynomial system. His paper includes detailed analysis of his algorithm and his brief study of numerical stability of the auxiliary matrix computations. His test results demonstrate the power of the resulting root-finder for a polynomial system whose solution set has a positive dimension.

Both root-finding and approximate divisors have deep technical links to computations with structured matrices, which naturally involve SNC as well. Fruitful interaction of computations with structured matrices and polynomials has been consistently advocated by the editors of this issue for decades (see [1–10] and the bibliography therein). Naturally, polynomial root-finding, structured matrices, and related topics are the themes of the present issue. By no means Hybrid methods for SNC lose their power beyond these areas. E.g., besides the latter areas, our special issue treats decomposition of a multivariate polynomial into the sum of squares, sparse multivariate polynomial interpolation, ill conditioned linear systems of equations and determinants, fast Hermite transform, tomographic reconstruction of images, signal templates, and pattern recognition. We summarize the respective papers below. All the important subjects of SNC, however, are by far too numerous to be covered in a single TCS issue.

Bella, Eidelman, Gohberg, and Olshevsky survey their recent results that reveal the correlation between the sequences of polynomials and rank structured (quasiseparable) matrices and employ these correlations to extend the known successful algorithms for Vandermonde matrices to polynomial Vandermonde matrices. Various effective algorithms have been devised in [3–7] based on the correlation between polynomials and structured matrices, but not in the important case of the rank structured matrices, which generalize banded matrices and their inverses, and which are the subject of recently exploded interest. The authors also unify, simplify, and extend the recent eigen-solvers for the rank structured matrices, originally motivated by applications to univariate polynomial root-finding (cf. [9, Chapter 14, pages 219–245]).

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Pan, Grady, Murphy, Qian, Rosholt and Ruslanov combine new and extend old numerical and symbolic techniques of randomization, aggregation, and preconditioning to facilitate some central problems of numerical matrix computations where the input is ill conditioned. Further applications include fundamental algebraic–geometric computations. Analysis and experiments confirm the power of this approach.

Cuyt and Lee present a deterministic numerical algorithm for sparse multivariate polynomial interpolation, which requires no information on the number of terms or the partial degree in each variable.

Li, Nie and Zhi seek a multivariate polynomial GCD by applying the semidefinite relaxation methods to the sparse polynomial optimization problem associated with their task. They adjust and test various known techniques and compare their efficiency.

The sum of squares (SOS) decomposition for nonnegative polynomials has links to Hilbert's 17th problem and applications to optimization, control, dynamical systems, etc. In some applications, e.g., geometric theorem proving, the solution must be exactly verified. Peyrl and Parrilo pass from an approximate numerical solution obtained via semidefinite program to an exact rational output and present an implementation for the computer algebra system Macaulay 2.

Sekigawa treats the problem of finding the nearest polynomial with a zero in a given domain. He proposes a method that reduces the problem to solving algebraic equations.

Sharma analyzes the two most effective continued fraction algorithms for real root isolation and gives the first rigorous estimates for their complexity. Furthermore he modifies one of the algorithms and accelerates it by the factor of n^2 for the nth degree input polynomial.

Emiris and Tsigaridas compute rational isolating points for all real roots of an integer polynomial of degree up to five, show the extension to isolating the real roots of a pair of bivariate polynomials, and effectively implement their algorithms in C++.

Shirayanagi and Sekigawa combine the older numerical stabilization techniques by the first author and Sweedler with Buchberger's algorithm and the method of indeterminate coefficients to yield numerical conversion algorithm for Gröbner basis. The authors' experiments show efficiency of the algorithm in the case of nonrational input coefficients.

Leibon, Rockmore, Park, Taintor and Chirikjian propose fast algorithms for the forward and inverse discrete Hermite, Newton–Cotes, and Euler transforms, study their arithmetic complexity and numerical performance, examine the impact of the choice of the supporting intervals, test the algorithms numerically, and point out some potential applications to Medical Imaging and Cryo-Electron Microscopy.

Normandin, Vajda and Valluri study the matched filtering of a gravitational wave signal by using templates. The authors combine symbolic (analytic) and numeric tools, to design an effective analysis technique well suited for parallel computation. They derive an exact closed-form solution for the inner product between a signal and a template involving the Bessel and Gamma functions and apply numerical techniques for its computation.

Our issue continues the recent SNC publications in [1,2,9,10]. About one half of it is made up of the journal versions of the proceedings papers in [9]. Some papers were submitted too late to pass through the scrutiny of at least two reviewers by the cutoff time for this issue or were not perfectly in the scope of SNC. They have been relegated to the regular issues of the TCS and CAMWA, which maintain interest in publications on SNC, expected to grow with the advance of this important field. We are most grateful to the authors of all submissions, the reviewers, and the TCS management: Prof. Dr. Giorgio Ausiello (Editor-in-Chief), Mick van Gijlswijk and Ella Chen, whose contribution, help, support and cooperation were decisive for the success of the issue.

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