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The strangeness content of the nucleon from effective field theory and phenomenology

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ABSTRACT

We revisit the classical relation between the strangeness content of the nucleon, the pion–nucleon sigma term and the $SU(3)_F$ breaking of the baryon masses in the context of Lorentz covariant chiral perturbation theory with explicit decuplet-baryon resonance fields. We find that a value of the pion–nucleon sigma term of ∼60 MeV is not necessarily at odds with a small strangeness content of the nucleon, in line with the fulfillment of the OZI rule. Moreover, this value is indeed favored by our next-to-leading order calculation. We compare our results with earlier ones and discuss the convergence of the chiral series as well as the uncertainties of chiral approaches to the determination of the sigma terms.

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1. Introduction

We dedicate this study to the interplay between the nucleon sigma terms, $\sigma_{\pi N}$ and σ_s, which are defined as

$$\sigma_{\pi N} = \frac{1}{2M_N} \langle N|\hat{m}(\bar{u}u + \bar{d}d)|N\rangle,$$
$$\sigma_s = \frac{1}{2M_N} \langle N|m_s\bar{s}s|N\rangle. \tag{1}$$

Here, the up, down and strange quarks masses are indicated by m_u, m_d and m_s, respectively, and $\hat{m} = (m_u + m_d)/2$. In the following, we restrict ourselves to the isospin limit, $m_u = m_d = \hat{m}$, with the nucleon states having the Lorentz invariant normalization $\langle N(\mathbf{p}', s')|N(\mathbf{p}, s)\rangle = 2E_N(2\pi)^3\delta(\mathbf{p}' - \mathbf{p})$, where $E_N = \sqrt{M_N^2 + \mathbf{p}^2}$, M_N is the nucleon mass and s and s' are the spin indices.

Both $\sigma_{\pi N}$ and σ_s are interesting observables and their non-vanishing values would clearly indicate that quark masses are not zero and give contribution to the nucleon mass. More precisely, the values of these two sigma terms embody the internal scalar structure of the proton and neutron. If they are small, most of the nucleon mass stems from the confinement of the lightest quarks in typical distances around 1 fm. Another property related to the nucleon scalar structure is the strangeness content of the nucleon, y, which is defined as

$$y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle} = \frac{2\hat{m}}{m_s}\frac{\sigma_s}{\sigma_{\pi N}}. \tag{2}$$

Notice that if the OZI rule (large N_C prediction) were exact then $y = 0$. Besides their role in understanding the mass of the ordinary matter, $\sigma_{\pi N}$ and σ_s are also necessary with respect to theoretical speculations on the origin of dark matter particles based on supersymmetry. An accurate determination of the sigma terms is needed to constrain the parameter space of the underlying supersymmetric models from the experimental bounds in direct searches of weakly interacting dark matter particles [1].

The determination of $\sigma_{\pi N}$ is feasible from πN scattering data due to the low-energy theorem of current algebra [2] that relates the value of the isospin even πN scattering amplitude at the Cheng–Dashen point with the nucleon scalar form factor [3–5]. However, the situation is much more obscure for the strangeness scalar form factor of the nucleon, and then for the phenomenological determination of σ_s as well as of y. Historically [6], the path to escape this end point is based on combining the definitions of Eqs. (1) and (2) as

$$\sigma_{\pi N} = \frac{\sigma_0}{1 - y}, \tag{3}$$

where σ_0 is the nucleon expectation value of the purely octet operator $\bar{u}u + \bar{d}d - 2\bar{s}s$,

$$\sigma_0 = \frac{\hat{m}}{2M_N}\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle. \tag{4}$$

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The point to notice is that the latter operator is the only one in the QCD Lagrangian responsible for the hadronic mass splitting within an $SU(3)$ multiplet. From the experimental values of the lightest baryon octet masses, M_Ξ, M_Σ and M_N, we can then calculate approximately σ_0 by making use of $SU(3)$ flavor symmetry, with the result [6]

$$\sigma_0 = \frac{\hat{m}}{m_s - \hat{m}}(M_\Xi + M_\Sigma - 2M_N) \simeq 27 \text{ MeV}, \qquad (5)$$

where we have used $m_s/\hat{m} = 26(4)$ [7].

Additionally, with this value for σ_0 and by assuming the OZI rule to hold, so that $y = 0$, one obtains from Eq. (3) the naive estimation $\sigma_{\pi N} \simeq 30$ MeV, that is much smaller than its phenomenological determinations from πN scattering data. For instance, Gasser et al. [5] obtained the canonical result $\sigma_{\pi N} \simeq 45$ MeV [5] in terms of a dispersive analysis of the pre-90s πN elastic scattering data.[1] A partial-wave analysis including the more modern πN database carried out by the George Washington University group [8], resulted in larger values of the pion–nucleon sigma term, $\sigma_{\pi N} = 64(8)$ MeV [9]. Besides that, a study of πN elastic scattering in Lorentz covariant baryon chiral perturbation theory (BχPT) [10] agrees with the dispersive results, which depend on the data set employed [11]. Additionally, it also reveals that modern partial-wave analyses are, in general, more consistent with different scattering phenomenology than the older ones and lead to a relatively large value of the sigma-term, cf. $\sigma_{\pi N} = 59(7)$ MeV [11]. The actual value of $\sigma_{\pi N}$ has important consequences on the strangeness content of the proton since, according to Eq. (3) and the result for σ_0 in Eq. (5), all these values for $\sigma_{\pi N}$ extracted from πN scattering data would imply a very large result for y.

Now, at this point it is important to emphasize that Eq. (5) is an estimate obtained at leading order in an $SU(3)_F$-breaking expansion and the calculation of σ_0 from this equation could be affected by large higher order contributions. The next-to-leading order (NLO) chiral corrections were first calculated by Gasser in Ref. [12]. There he obtained $\sigma_0 = 35(5)$ MeV by employing a chiral model for the meson cloud around the baryon which only considered contributions from the virtual octet baryons. Within the more evolved theoretical framework of BχPT Ref. [13] performed a calculation of the baryon masses and σ_0 in the heavy-baryon (HB) [14] expansion up to next-to-next-to-leading order (NNLO). In this work, the contributions of the decuplet-baryon resonances were not implemented explicitly but through resonance-saturation hypothesis they contributed to several of the many low-energy-constants (LECs) appearing at this order. All in all, they reported the value $\sigma_0 = 36(7)$ MeV, which was almost identical to the NLO result obtained by Gasser 15 years earlier. Later, Ref. [15] also included the decuplet-baryon resonances within HBχPT using a cut-off regularization scheme and still obtained basically the same result for σ_0. One should also notice that $\sigma_{\pi N} = 45$ MeV was taken as input in the analyses of Refs. [13,15], which had a strong influence in the results of Ref. [15].

By employing $\sigma_0 \simeq 35$ MeV from the calculations of Refs. [12,13,15] in Eq. (3) one obtains that $y \simeq 0.2$ and 0.4 for $\sigma_{\pi N} \simeq 45$ MeV and $\simeq 60$ MeV, respectively. In particular, the latter value would imply a strangeness contribution to the mass of the nucleon of ~ 300 MeV. Although not impossible, such a scenario with a strong breaking of the OZI rule is theoretically implausible, moreover after the experimental evidence pointing to a negligible strangeness contribution in other properties of the nucleon such as its electromagnetic structure [16] and spin [17]. Thus, if

one gives credit to these results and translate them into a small value of y, then the present widely accepted value for σ_0 around 35 MeV clearly discredits the relatively large values for $\sigma_{\pi N}$ favored by the most recent analysis of the πN scattering data, cf. $\sigma_{\pi N} = 64(8)$ MeV [9] and $\sigma_{\pi N} = 59(7)$ MeV [11].

It is our aim in this work to emphasize that the situation concerning σ_0 is not settled yet, so that the previous conclusion does not necessarily hold. On one hand, the result of Gasser [12] is based on a model calculation of the meson cloud around the nucleon, whereas Refs. [13,15] might be afflicted by the poor convergence of the chiral series typically shown by HB in the $SU(3)_F$ theory [18,19].

A suitable approach that also includes explicitly the contributions from the decuplet-baryon resonances is a Lorentz covariant formulation of BχPT with a consistent power-counting via the extended-on-mass-shell renormalization (EOMS) scheme [20]. The relativistic corrections that results in this approach, in a way preserving the exact analytical properties of the Green functions, have been shown to tame the poorly convergent series of the HB expansion in baryonic observables as important as the magnetic moments [18,21] or masses [19,22]. Moreover, once a prescription is taken to treat the problem of the interacting Rarita–Schwinger fields [23], this scheme is straightforwardly applicable to include the contributions of the decuplet-baryon resonances [21]. In this work we calculate σ_0 up to NLO using Lorentz covariant BχPT renormalized in the EOMS prescription and including explicitly the effects of the decuplet. We compare the results with those obtained in the HB expansion and estimate systematic higher-order effects through a partial calculation of NNLO pieces. All together, we find the remarkable result that the value of σ_0 becomes larger so that the modern experimental determinations of $\sigma_{\pi N} \sim 60$ MeV are then consistent with a small strangeness content of the nucleon, or with a small OZI rule violation. A first indication that the decuplet contributions could help to solve the strangeness puzzle concerning a relatively large $\sigma_{\pi N}$ was given by the HB calculation in Ref. [24]. Indirectly, this was also the case in Ref. [25] where very large and negative values of σ_s were obtained when demanding $\sigma_{\pi N} = 45$ MeV, indicating a larger σ_0.

2. Calculation

The expressions for the sigma terms can be obtained either from the explicit calculation of the scalar form factor of the nucleon at $q^2 = 0$ or applying the Hellmann–Feynman theorem to the chiral expansion of its mass,

$$\sigma_{\pi N} = \hat{m}\frac{\partial M_N}{\partial \hat{m}} = \frac{m_\pi^2}{2}\left(\frac{1}{m_\pi}\frac{\partial}{\partial m_\pi} + \frac{1}{2m_K}\frac{\partial}{\partial m_K} + \frac{1}{3m_\eta}\frac{\partial}{\partial m_\eta}\right)M_N$$
$$+ \mathcal{O}(p^4),$$
$$\sigma_s = m_s\frac{\partial M_N}{\partial m_s} = \left(m_K^2 - \frac{m_\pi^2}{2}\right)\left(\frac{1}{2m_K}\frac{\partial}{\partial m_K} + \frac{2}{3m_\eta}\frac{\partial}{\partial m_\eta}\right)M_N$$
$$+ \mathcal{O}(p^4). \qquad (6)$$

We follow the latter strategy since the explicit expressions for the baryon masses in the different schemes treated in this Letter can be directly obtained using the Appendix of Ref. [19]. Thus, the chiral expansion of the sigma terms up to NLO from Eq. (6) is written as,

$$\sigma_{\pi N} = -4(2b_0 + b_D + b_F)\frac{m_\pi^2}{2}$$
$$+ \frac{1}{(4\pi F_\phi)^2}\sum_{\phi=\pi,K,\eta}\left(\xi_{N,\phi}^{(B)}\Sigma_\pi^{(B)}(m_\phi) + \xi_{N,\phi}^{(T)}\Sigma_\pi^{(T)}(m_\phi)\right)$$
$$+ \mathcal{O}(p^4),$$

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[1] For a detailed exposition of the dispersive methods for obtaining $\sigma_{\pi N}$ from the analytic continuation of the πN scattering amplitude to the Cheng–Dashen point see Refs. [3–5].
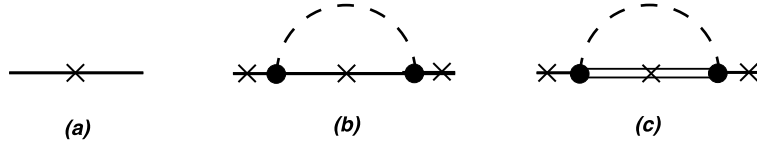
Fig. 1. Feynman diagrams contributing to the nucleon mass up to $\mathcal{O}(p^3)$ in BχPT. The internal solid lines correspond, in general, to any octet baryon, double lines to decuplet-baryon resonances and dashed lines to mesons. The black dots indicate 1st-order couplings while crosses are insertions of $\mathcal{O}(p^2)$ operators given by the LECs b_0, b_D and b_F responsible for the leading $SU(3)_F$ breaking of the baryon-octet masses.

Table 1
Values of the $\mathcal{O}(p^2)$ LECs b_D and b_F determined from the baryon octet mass splittings in the different BχPT approaches considered in this Letter.

	Tree level $\mathcal{O}(p^2)$	Octet $\mathcal{O}(p^3)$		Octet + Decuplet $\mathcal{O}(p^3)$	
		HB	Covariant	HB–SSE	Covariant
b_D [GeV^{-1}]	0.060(4)	0.061(4)	0.061(4)	0.315(4)	0.161(4)
b_F [GeV−1]	−0.213(2)	−0.502(2)	−0.420(2)	−0.704(2)	−0.502(2)

$$\sigma_s = -4(b_0 + b_D - b_F)\left(m_K^2 - \frac{m_\pi^2}{2}\right)$$
$$+ \frac{1}{(4\pi F_\phi)^2} \sum_{\phi=\pi,K,\eta} \left(\xi_{N,\phi}^{(B)} \Sigma_s^{(B)}(m_\phi) + \xi_{N,\phi}^{(T)} \Sigma_s^{(T)}(m_\phi)\right)$$
$$+ \mathcal{O}(p^4). \tag{7}$$

The first line in these formulas corresponds to the LO contribution given at tree-level by the same $\mathcal{O}(p^2)$ LECs that appear in the chiral expansion of the baryon masses. While b_0 provides an $SU(3)_F$-singlet contribution that cannot be disentangled from the bulk mass of the octet baryons, the LECs b_D and b_F induce a splitting of octet-baryon masses (tree-level in diagram *(a)* in Fig. 1) which gives rise to the GMO relation [19]. The second lines enclose the NLO or $\mathcal{O}(p^3)$ corrections that stem from the loop topologies shown in Fig. 1*(b)* and *(c)*. Thus, the effect of virtual octet (B) and decuplet (T) baryons is explicitly accounted for. Their contributions are weighted by the coefficients $\xi_{N,\phi}^{(X)}$, which are combinations of $SU(3)$ Clebsch–Gordan coefficients and the meson–baryon couplings D, F (octet contributions) and $\mathcal{C}$ (decuplet contributions). The loop functions $\Sigma_a^{(X)}$ depend, exclusively, on the mass of the virtual pseudoscalar meson and on the ones of the octet and decuplet baryons in the chiral limit, M_B and M_T respectively. Strictly speaking, the $SU(3)_F$ breaking of the baryon masses in these loops, which are represented by the crosses in Fig. 1*(b)* and *(c)*, are contributions that start at NNLO or $\mathcal{O}(p^4)$.

For the baryon masses we use the results obtained in Ref. [19] in Lorentz covariant BχPT up to $\mathcal{O}(p^3)$ in EOMS. The chiral loops contain divergences and analytic pieces breaking the power-counting formula [10] that are removed in dimensional regularization by the proper redefinition of the bare LECs (EOMS scheme [20]). The contributions of the decuplet baryons are included taking the octet and decuplet masses in the chiral limit of approximately the same order. Namely, the octet and decuplet contributions are considered on the same footing for power-counting purposes and no specific expansion in $\delta = (M_T - M_B)$ is performed. The HB formulas [24] can be always recovered from the renormalized covariant results by taking the non-relativistic expansion $M_B \sim M_T \sim \Lambda_{\chi SB}$. In particular, the HB results within the small-scale-expansion (SSE) [26], that it is used to include explicitly the decuplet resonances,[2] are retrieved once the HB expansion is performed in our results [25].

For the numerical values of the couplings, we use $D = 0.80$ and $F = 0.46$ [27]. The decuplet coupling $\mathcal{C}$ can be fixed from the $\Delta(1232) \to \pi N$ decay rate, giving $\mathcal{C} = 1.0$ [21]. However,

there is some evidence from LQCD that this coupling is somewhat smaller [22]. Indeed, an $SU(3)_F$-average among the different decuplet-to-octet pionic decay channels gives $\mathcal{C} = 0.85 \pm 0.15$, that is the value we use.[3] As mentioned above, the $\mathcal{O}(p^2)$ LECs b_D and b_F are determined using the experimental baryon-octet mass splittings. Their values for the different BχPT schemes analyzed in this Letter can be found in Table 1 [19]. For the meson decay constant we also take the $SU(3)_F$-average $F_\phi \equiv 1.17 f_\pi$ with $f_\pi = 92.4$ MeV. Variations in these values of D, F, C and F_ϕ were discussed in Ref. [19] and do not influence the final results once their correlations are taken into account. For the masses of the pseudoscalar mesons we use $m_\pi \equiv m_{\pi^\pm} = 139$ MeV, $m_K \equiv m_{K^\pm} = 494$ MeV, while for the baryon masses in the loops we use the chiral-limit baryon masses obtained at LO, $M_B^{(1)} = 1.151$ GeV and $M_T^{(1)} = 1.382$ GeV. The mass of the η meson is fixed with the Gell-Mann–Okubo mass relation, $3m_\eta^2 = 4m_K^2 - m_\pi^2$ which is accurate enough up to the order we work.

Finally, we restrain our analysis to $\mathcal{O}(p^3)$ despite of the fact that the extension of formulas to $\mathcal{O}(p^4)$ accuracy is straightforward, albeit affected by a dramatic loss of predictability, and have been reported in the literature [13,28–31]. At the latter order, 15 new LECs contribute to the baryon masses and sigma-terms. Eight of them correspond to $\mathcal{O}(p^2)$ operators which appear through diagrams with the topology of a tadpole (see Ref. [28] for details). These also contribute to the chiral expansion of the meson–baryon scattering amplitudes, although their LECs have not been determined yet from the associated experimental data or LQCD results. The other loop diagrams appearing at this order are the ones at $\mathcal{O}(p^3)$ but with the $SU(3)_F$ breaking of the baryon masses in the loop taken into account by insertions of the $\mathcal{O}(p^2)$ LECs b_0, b_D and b_F (crosses in the diagrams *(b)* and *(c)* of Fig. 1). The remaining 7 LECs correspond to $\mathcal{O}(p^4)$ operators and they renormalize the loop divergences appearing at this order. Therefore, a quantitative analysis of the sigma terms at NNLO without any further assumption on the values of the LECs (such as Large N_c constraints [29] or resonance saturation hypothesis estimates [13]) is affected, at present, by a large uncertainty. On the other hand, a promising source of theoretical information on the values of the LECs is becoming available through LQCD calculations. An application in this direction within EOMS BχPT at $\mathcal{O}(p^3)$ and $\mathcal{O}(p^4)$ can be found in [19] and [30,31], respectively.

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[2] In the SSE one furthermore considers $\delta \sim p$.

[3] Note that the value for $\mathcal{C}$ of the present work is different from the one often used in HB calculations [14]. In these papers, a convention for the "vielbein" that is related to ours by a factor of 2 is employed. Moreover, the value we use in this Letter is different to the one used in [19], explaining the slightly different decuplet results obtained here and there.

Table 2
Value of the LEC b_0 and of the observables related to the strangeness of the nucleon, y and σ_s, obtained in Lorentz covariant BχPT including decuplet contributions and using the phenomenological determinations of $\sigma_{\pi N}$ as input.

	$b_0^{\text{Expt.}}$ [GeV^{-1}]	y	σ_s [MeV]
$\sigma_{\pi N} = 45(7)$ MeV	$-0.79(9)$	$-0.28(13)(10)$	$-150(80)(60)$
$\sigma_{\pi N} = 59(7)$ MeV	$-0.97(9)$	$0.02(13)(10)$	$16(80)(60)$

Nevertheless, the analysis of part of the $\mathcal{O}(p^4)$ corrections can be useful to asses the convergence of the chiral series and to give a credible estimate on the systematic error to the $\mathcal{O}(p^3)$ results on the sigma terms due to the truncation of their chiral expansions. Indeed, we have calculated explicitly the respective corrections arising from the $SU(3)_F$ breaking of the baryon masses in the loops **(b)** and **(c)** of Fig. 1. The divergences have been renormalized in the EOMS scheme and the uncertainty on the unknown values of the $\mathcal{O}(p^4)$ LECs has been explored by varying the renormalization scale in the interval $0.7 \text{ GeV} \leqslant \mu \leqslant 1.3 \text{ GeV}$. The maximal contribution obtained for these corrections in both, the octet and decuplet diagrams, is quoted as our theoretical uncertainty. That is,

$$\Delta\sigma_{\pi N}^{\text{HB}} \simeq 20 \text{ MeV}, \qquad \Delta\sigma_s^{\text{HB}} \simeq 140 \text{ MeV},$$
$$\Delta\sigma_{\pi N}^{\text{EOMS}} \simeq 6 \text{ MeV}, \qquad \Delta\sigma_s^{\text{EOMS}} \simeq 60 \text{ MeV}. \tag{8}$$

This explicit calculation of higher-order pieces already confirms the expectation that the convergence in the covariant approach is substantially better than the one obtained in the HB case [18,19].

3. Results

From the discussion above and Eq. (7), it is clear that the only unknown parameter in the chiral expansion of the sigma terms up to NLO is the LEC b_0. In the analysis that follows we calculate it by taking two phenomenological determinations of $\sigma_{\pi N}$, namely, $\sigma_{\pi N} \simeq 45(7)$ MeV [5] and $\sigma_{\pi N} \simeq 59(7)$ MeV [11], and using Eq. (7). As a result we obtain the values for y and σ_s shown in the third and fourth columns of Table 2, respectively. Notice that we do not make use of LQCD results to fix our free parameters and deliberately we only use experimental information.

The new contributions given by the decuplet baryons are not negligible producing a ~ 10 MeV rise on σ_0, compared to the situation when only the lightest octet of baryons is included in the intermediate states. This result is very important for sigma-term physics and it has a strong impact on statements about the strangeness content of the nucleon based on the value of $\sigma_{\pi N}$, cf. Eq. (3). As we can see in Table 2, the determination of σ_s depends strongly on the value of the pion–nucleon sigma term due to a relative large factor $\sim m_s/(2\hat{m}) \simeq 13$ between the two observables. Hence, a variation of 10 MeV in $\sigma_{\pi N}$ causes changes in σ_s spanning more than 100 MeV. This factor also amplifies the uncertainty on the latter observable that propagates from the relatively small error of the former and it makes very difficult to give predictions of σ_s with the ballpark accuracy of few tens of MeV. Two errors are quoted for y and σ_s in the last two columns of Table 2. The first stems from the propagation of the uncertainty in $\sigma_{\pi N}$ and the last from the estimated $\mathcal{O}(p^4)$ uncertainty in our calculation, Eq. (8).

In order to appreciate the improvement in the chiral expansion that results by employing Lorentz covariant BχPT in the EOMS we compare our results for $y = 0$, quite close to the last line value in Table 2, with the HBχPT calculations with/without the decuplet-baryon resonances in Table 3. As we can see, the corrections to the LO result on σ_0 studied are large. This occurs despite that the discrepancy of the Gell-Mann–Okubo equation is correctly predicted in any of these schemes and, in fact, the description of the experimental octet mass splittings improves at $\mathcal{O}(p^3)$ [19]. As already anticipated by the calculation of the $\mathcal{O}(p^4)$ pieces in Eqs. (8), the $SU(3)_F$ HB expansion has severe problems of convergence in the description of the sigma terms at $\mathcal{O}(p^3)$. The huge central value and errors of σ_0 for the HB–SSE expansion has to be regarded as a clear manifestation of these problems. Another one is the large variation in the value of σ_0 between HB and HB–SSE. On the contrary, for the covariant calculation the difference between the calculations excluding/including the explicit decuplet of baryon resonances is only of around 10 MeV, much smaller than the difference between the LO and NLO results in the purely octet formulation of the theory. This indicates a stabilization of the final outcome at the $\mathcal{O}(p^3)$ value for the covariant case. These conclusions are consistent with those derived from the analyses of other observables [18,19,21] that also indicated similar problems of convergence for the HB studies in the $SU(3)_F$ sector. Similar comments can be done concerning the value of b_0 and its variations when comparing with the different levels of sophistication in the calculation. E.g. one observes a change in b_0 between HB and HB–SSE in Table 3 that is a factor 3 times larger than for the covariant calculations.

The main result of our study is having shown that $\sigma_{\pi N} \sim 60$ MeV is perfectly compatible with a rather accurate fulfillment of the OZI rule and, hence, with a small strangeness content of the nucleon. In other words, there is no argument against relatively large values of $\sigma_{\pi N}$ based on the OZI rule, as σ_0 can be afflicted by important systematics as those driven by properly accounting of the relativistic corrections and the explicit inclusion of the decuplet resonances. Other outcome of our work is that the value of σ_0 obtained using the experimental baryon-octet mass splittings, Lorentz covariant BχPT in EOMS up to $\mathcal{O}(p^3)$ with explicit decuplet-baryon resonances as degrees of freedom, favors $\sigma_{\pi N} \sim 60$ MeV. However, for this result to hold higher order corrections should be under control. Indeed, this seems to be the case as indicated by our calculation of a sub-set of known $O(p^4)$ diagrams. Unfortunately, first complete studies of LQCD results at $\mathcal{O}(p^4)$ in $SU(3)$ BχPT [29–32] are not conclusive on this respect yet, as large variations in the sigma terms are found between different strategies. Ref. [29] concludes $\sigma_{\pi N} = 32 \pm 2$ MeV, $\sigma_s = 22 \pm 20$ MeV and $y \simeq 0.05 \pm 0.04$, while Ref. [31] results with $\sigma_{\pi N} = 46(2)(12)$ and $\sigma_s = 157(25)(68)$. Further work in this direction, ideally including more observables, experimental data and LQCD results to tackle the large number of unknown LECs appearing at $\mathcal{O}(p^4)$, will be necessary to settle this question.

We also show two results from LQCD without employing BχPT, Ref. [33] obtained $\sigma_{\pi N} = 39(4)\binom{+18}{-7}$ MeV, $\sigma_s = 33(14)\binom{+23}{-24}$ MeV and $y = 0.20(7)\binom{+13}{-17}$, while Ref. [34] determines y with the value $y = 0.135(46)$. These direct LQCD calculations clearly suggest a

Table 3
Values of σ_0 and the $\mathcal{O}(p^2)$ LEC b_0 given by the exact fulfillment of the OZI rule for the different BχPT approaches considered in this Letter.

	Tree level $\mathcal{O}(p^2)$	Octet $\mathcal{O}(p^3)$		Octet + Decuplet $\mathcal{O}(p^3)$	
		HB	Covariant	HB–SSE	Covariant
σ_0 [MeV]	27	58(23)	46(8)	89(23)	58(8)
b_0^{OZI} [GeV^{-1}]	-0.274	$-0.90(15)$	$-0.70(5)$	$-1.52(15)$	$-0.95(5)$

small value for y and a subsequent contribution to the nucleon mass due to strangeness of around the same size as from the lightest quark masses. Other calculations supplying LQCD results with different formulations of BχPT are [32,35,36]. In these studies a small value for y results, compatible with our own determination in the last line of Table 2. There is also a tendency in the LQCD results favoring the phenomenological determination $\sigma_{\pi N} \simeq$ 45(7) MeV [5], although within present uncertainties the result of Ref. [33] is compatible at the level of one sigma with the larger value $\sigma_{\pi N} = 59(7)$ MeV [11].

4. Conclusions

In summary, we have revisited an old empirical relation between the strangeness content of the nucleon and the pion–nucleon sigma term in the context of covariant BχPT and employing only phenomenological information. Earlier estimates of σ_0 made at different levels of accuracy in χPT agreed on that a small violation of the OZI rule in the nucleon requires a value of $\sigma_{\pi N}$ close to $\sim$35 MeV. A long-standing puzzle [37] has arisen from sustained experimental evidence pointing to a value of this quantity close to 60 MeV, reinforced by the values obtained from modern πN databases. We have shown that the previous calculations of σ_0 are afflicted by important systematic effects, in particular those given by relativistic corrections and by the omission of the decuplet resonances. Once these are incorporated, we obtain a larger σ_0 so that a relatively large value of $\sigma_{\pi N}$ is not necessarily inconsistent with a negligible strangeness content of the nucleon as currently indicated by experiment and LQCD. In fact, our calculation at NLO in Lorentz covariant BχPT in the EOMS with explicit decuplet-baryon resonances favors this scenario.

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