



ELSEVIER

SCIENCE @ DIRECT®

PHYSICS LETTERS B

Physics Letters B 578 (2004) 203–209

[www.elsevier.com/locate/physletb](http://www.elsevier.com/locate/physletb)

# A note on supergravity solutions for partially localized intersecting branes

S. Arapoglu<sup>a</sup>, N.S. Deger<sup>b</sup>, A. Kaya<sup>b</sup>

<sup>a</sup> *Bogazici University, Department of Physics, 34342 Bebek, Istanbul, Turkey*

<sup>b</sup> *Feza Gursey Institute, 81220 Cengelkoy, Istanbul, Turkey*

Received 17 July 2003; accepted 9 August 2003

Editor: H. Georgi

## Abstract

Using the method developed by Cherkis and Hashimoto we construct partially localized  $D3 \perp D5(2)$ ,  $D4 \perp D4(2)$  and  $M5 \perp M5(3)$  supergravity solutions where one of the harmonic functions is given in an integral form. This is a generalization of the already known near-horizon solutions. The method fails for certain intersections such as  $D1 \perp D5(1)$  which is consistent with the previous no-go theorems. We point out some possible ways of bypassing these results.

© 2003 Elsevier B.V. Open access under [CC BY license](http://creativecommons.org/licenses/by/3.0/).

## 1. Introduction

There has been considerable interest in constructing intersecting brane solutions in the past (see [1–3] for review). The problem is completely solvable if one assumes that the solution depends only on overall transverse directions. However, relaxing this condition complicates it considerably. If the metric is chosen to be in some specific form (which is inspired by harmonic function rule [4–6]) then it is easy to see that one of the brane has to be delocalized [7], i.e., its harmonic function is independent of the directions along the other brane's worldvolume. This is not a restriction if the smaller brane is contained in the bigger one; otherwise these type of solutions are said to be *partially localized* (see Fig. 1). Explicit intersections have

been found by further restricting to the near-horizon of the delocalized brane [8–11]. Recently Cherkis and Hashimoto [12] were able to remove this restriction for  $D2 \perp D6(2)$  intersection which allowed them to analyze the system in the near-horizon region of  $D2$  instead of  $D6$  which has some important applications in AdS/CFT duality. This method has been further applied to construct  $D1 \perp NS5(0)$  intersection in [13] and  $D4 \perp D8(4)$  intersection in [14].

The approach of [12], which we adopt in this Letter, is similar to the technique used in [15–17] to prove no-hair theorems for  $p$ -branes. It is a generic feature of intersecting brane configurations that the differential equations involving the metric functions are linear and separable. This lets one to apply Fourier transformation techniques which allows the construction of the harmonic function as an integral expression. This can be evaluated numerically if desired and it is a generalization of the near-horizon solutions given in [10]. (See also [18–20].)

E-mail addresses: arapoglu@boun.edu.tr (S. Arapoglu), deger@gursey.gov.tr (N.S. Deger), kaya@gursey.gov.tr (A. Kaya).

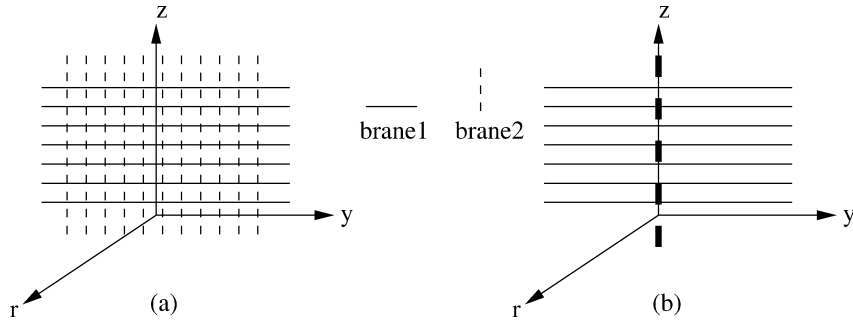


Fig. 1. Delocalized (a) versus partially localized (b) brane intersections. The first brane has the world-volume coordinates  $(x, y)$  and the second one is oriented along  $(x, z)$  directions ( $x$  coordinate is suppressed in the figure). In (a), branes are smeared along  $z$  and  $y$  coordinates, respectively. In (b), only the first brane is smeared and the second brane is located at  $y = 0$ .

As we will discuss below, this method fails when the overall transverse space,  $(n + 2)$ , is four or higher-dimensional. When  $n > 2$ , for instance, as in the case of  $D0 \perp D4(0)$ , the radial dependence of the metric functions cannot be determined in terms of elementary or known functions. On the other hand, for  $n = 2$  there is a generic spontaneous delocalization when the branes are forced to be placed on top of each other. This is consistent with the previous no-go results for a full localization in such brane systems [15–17].

## 2. Solutions

Let us start with an intersection of two D-branes which has the following metric

$$ds^2 = H_1^{-1/2} H_2^{-1/2} dx^\mu dx^\mu + H_1^{-1/2} H_2^{1/2} d\vec{y} \cdot d\vec{y} + H_1^{1/2} H_2^{-1/2} d\vec{z} \cdot d\vec{z} + H_1^{1/2} H_2^{1/2} d\vec{r} \cdot d\vec{r}, \quad (1)$$

where  $(x^\mu, \vec{y})$  and  $(x^\mu, \vec{z})$  are the world-volume coordinates of the first and the second branes which are characterized by the “harmonic” functions  $H_1(\vec{z}, \vec{r})$  and  $H_2(\vec{y}, \vec{r})$ . Changing the powers of metric functions the same metric can be thought to describe intersection of two M-branes. We follow the usual brane terminology;  $x$  is a common brane coordinate,  $\vec{y}$  and  $\vec{z}$  are relative transverse directions and  $\vec{r}$  coordinates parameterize the overall transverse directions. We assume that the brane functions do not depend on the corresponding brane coordinates. To have a localized

solution one should have

$$\lim_{|\vec{z}| \rightarrow \infty} H_1 \rightarrow 1, \quad (2)$$

$$\lim_{|\vec{y}| \rightarrow \infty} H_2 \rightarrow 1. \quad (3)$$

The harmonic functions satisfy the following differential equations [7]

$$(\partial_{\vec{r}}^2 + H_2 \partial_{\vec{z}}^2) H_1 = q_1 \delta(\vec{r}) \delta(\vec{z}), \quad (4)$$

$$(\partial_{\vec{r}}^2 + H_1 \partial_{\vec{y}}^2) H_2 = q_2 \delta(\vec{r}) \delta(\vec{y}), \quad (5)$$

$$\partial_{\vec{z}} H_1 \partial_{\vec{y}} H_2 = 0, \quad (6)$$

where the branes are assumed to be located at  $\vec{z} = \vec{z} = 0$  and  $\vec{r} = \vec{y} = 0$ , respectively. The last equation indicates that either  $\partial_{\vec{z}} H_1 = 0$  or  $\partial_{\vec{y}} H_2 = 0$ , i.e., one of the branes should be delocalized along the other brane directions. Without loss of generality we take it to be the first brane. Assuming spherical symmetry, (4) gives (up to an irrelevant numerical factor)

$$H_1 = 1 + \frac{q_1}{r^n}, \quad (7)$$

where  $(n + 2)$  is the dimension of the  $\vec{r}$ -space and  $q_1$  is the brane charge. For the special intersection where the second brane is located inside the first one,  $\vec{z}$  coordinates should be ignored. For this case  $H_1$  depends only on  $\vec{r}$  and (6) is satisfied trivially. This corresponds to a full localization. When  $H_1$  is solved as in (7), the solutions of (5) has been studied in certain limits. For instance, near horizon geometries where one can take  $H_1 \sim r^{-n}$  were constructed in [10]. Following [12], to solve (5) exactly, we use a Fourier transformation in the  $\vec{y}$  space to

write

$$\begin{aligned}
 H_2 &= 1 + q_2 \int d^m p e^{i\vec{p}\cdot\vec{y}} H_p(r), \\
 &= 1 + q_2 \int_0^\infty dp \int_0^\pi d\theta (\sin\theta)^{m-2} p^{m-1} \\
 &\quad \times \Omega_{m-2} e^{ipy\cos\theta} H_p(r), \quad (8)
 \end{aligned}$$

where  $q_2$  is the brane charge,  $m$  denotes dimension of the  $\vec{y}$  space, and  $\Omega_{m-2}$  is the volume of the unit  $(m - 2)$ -dimensional sphere with  $\Omega_0 = 1$ . The above formula is valid when  $m > 1$  and for  $m = 1$  the second step is unnecessary. For technical convenience, we first locate the second brane at  $\vec{r} = \vec{r}_0$  and then take  $\vec{r}_0 \rightarrow 0$  limit. Then, from (5) and (8) one finds

$$\begin{aligned}
 &\left[ \frac{d^2}{dr^2} + \frac{n+1}{r} \frac{d}{dr} - p^2 \left( 1 + \frac{q_1}{r^n} \right) \right] H_p(r) \\
 &= q_2 \frac{\delta(r - r_0)}{r^{n+1}}. \quad (9)
 \end{aligned}$$

For each  $m$ , the  $\theta$  integral in (8) can be carried out easily. Therefore, if one can solve (9)  $H_2$  can be determined in an integral form which can be evaluated numerically if wanted. Now let us discuss possible solutions of (9):

$n \geq 3$ : It turns out (9) cannot be solved in terms of elementary functions (at least to our knowledge). Recalling that  $(n + 2)$  is the dimension of the overall transverse space, this corresponds to the intersections like  $D0 \perp D4(0)$  or  $M2 \perp M2(0)$ .

$n = 2$ : The prototype of this case that we will consider is  $D1 \perp D5(1)$  intersection. However, since our arguments are based on the  $r$ -dependence of the harmonic functions (which is fixed by  $n$ ), our conclusions apply intersections like  $D2 \perp D4(1)$  and  $M2 \perp M5(1)$  as well. Even though, there are no-go theorems for the existence of a localized solution [15–17], for completeness we will investigate this case too in order to emphasize the origin of the difficulty. We will also propose some possible ways to resolve this. The solution to (9) which is both regular at  $r = 0$  and  $r = \infty$  can be written as (we demand regularity at

$r = 0$  since we are mainly interested in  $r_0 \rightarrow 0$  limit)

$$H_p(r) = \begin{cases} c_p(r_0)r^{-1}K_\nu(pr), & r > r_0, \\ d_p(r_0)r^{-1}I_\nu(pr), & r < r_0, \end{cases} \quad (10)$$

where  $K_\nu$  and  $I_\nu$  are the modified Bessel function with  $\nu = \sqrt{1 + q_1 p^2}$  and  $[c_p(r_0), d_p(r_0)]$  are constants. The continuity at  $r = r_0$  gives

$$c_p(r_0)K_\nu(pr_0) = d_p(r_0)I_\nu(pr_0). \quad (11)$$

Using this in the condition imposed by the presence of the delta function source at  $r = r_0$  one obtains

$$c_p(r_0)pW\{I_\nu(pr_0), K_\nu(pr_0)\} = q_2r_0^{-2}I_\nu(pr_0), \quad (12)$$

where  $W$  is the Wronskian with respect to the argument which is equal to  $-1/(pr_0)$ . This implies  $c_p(r_0) = -q_2I_\nu(pr_0)/r_0$ . In the  $r_0 \rightarrow 0$  limit  $c_p(r_0) \sim r_0^{(\nu-1)} \rightarrow 0$  which indicates spontaneous delocalization. This is the essence of the trouble in D1/D5 localized solution. Physically, as the separation goes to zero the D1-brane charge spreads over the D5-brane.

Now we would like to point out two possible ways of resolving this difficulty although we could not establish a clear cut result. Firstly, there may be a subtlety in taking  $r_0 \rightarrow 0$  limit. Namely, a localized intersection when branes are coincident may not be continuously reached from a separated brane configuration. If so, then one should solve (5) directly without assuming any separation between the branes. In this case, one finds that  $H_p(\vec{r})$  in (8) obeys

$$\left[ \partial_{\vec{r}}^2 - p^2 \left( 1 + \frac{q_1}{|\vec{r}|^2} \right) \right] H_p(\vec{r}) = q_2\delta(\vec{r}). \quad (13)$$

Fourier expanding  $H_p(\vec{r})$  as

$$H_p(\vec{r}) = \int d^4v e^{i\vec{r}\cdot\vec{v}} h_p(\vec{v}), \quad (14)$$

(13) gives

$$\begin{aligned}
 &(2\pi)^4(|\vec{v}|^2 + |\vec{p}|^2)h_p(\vec{v}) \\
 &+ 4\pi^2 p^2 q_1 \int d^4v' \frac{h_p(\vec{v}')}{|\vec{v} - \vec{v}'|^2} = -q_2. \quad (15)
 \end{aligned}$$

Unfortunately, we could not solve this integral equation. However, in principle, there may exist well-behaved solutions which might have important implications for the moduli space of the D1/D5 system.

One possible way is to find a series solution by iteration which would be identical to an expansion in powers of  $q_1$ . Secondly, there may be a smooth solution away from the delta function source. For this purpose, we set the right-hand side of Eq. (9) to zero. Then using the solution for  $H_p(r)$  which decays as  $r \rightarrow \infty$ , (8) becomes

$$H_2 = 1 + q_2 \int_0^\infty dp c_p (yr)^{-1} J_1(py) K_\nu(pr). \quad (16)$$

At this point, the constant  $c_p$  is completely arbitrary (which may also depend on  $q_1$ ). However, it should satisfy the following two conditions for a localized solution. Obviously, (16) should yield a finite D1-brane charge which can be calculated from

$$\int_\Sigma *(dt \wedge dx \wedge dH_2^{-1}), \quad (17)$$

where  $*$  is the Hodge dual and the integral is taken over a 7-dimensional closed surface  $\Sigma$  surrounding the D1-brane which can be taken as  $(\lim_{y \rightarrow \infty} y^3 \Omega_3 d^4 r + \lim_{r \rightarrow \infty} r^3 \hat{\Omega}_3 d^4 y)$ , where  $\Omega_3$  and  $\hat{\Omega}_3$  are the unit spheres in  $\vec{y}$  and  $\vec{r}$  spaces, respectively. The other condition on  $c_p$  is that for  $q_1 = 0$ , i.e.,  $\nu = 1$ , (16) should give a single D1-brane solution. However, it turns out to be quite difficult to satisfy both conditions. For example, it is easy to see that choosing  $c_p = p^3$ , (16) gives  $H_2 \sim 1 + 1/(y^2 + r^2)^3$  when  $q_1 = 0$  which is precisely the harmonic function for a single D1-brane. Moreover, D1-brane is localized inside the D5-brane, i.e.,  $H_2 \rightarrow 1$  as  $y \rightarrow \infty$ . Nevertheless, the metric has a pathologic divergence as one approaches the D5-brane horizon at  $r = 0$ . To see this let us consider the integral (16) for large  $p$ . In this case,  $\nu \sim p\sqrt{q_1}$ . For fixed  $r$ , the modified Bessel function has the following limiting behavior

$$\lim_{\rightarrow \infty} K_\nu(pr) = \sqrt{\frac{\pi}{2p}} (1+r^2)^{-1/4} e^{-p\eta(r)}, \quad (18)$$

where  $\eta(r) = \sqrt{1+r^2} + \ln r - \ln(1 + \sqrt{1+r^2})$ . One can see that there is a positive constant  $b$  (which depends on the D5-brane charge  $q_1$ ) such that  $\eta > 0$  when  $r > b$ ,  $\eta < 0$  when  $r < b$  and  $\eta = 0$  when  $r = b$ . Therefore, the integral (16) converges for  $r > b$  but diverges when  $r \leq b$ . Note that this is similar to a

delta function type singularity. Due to this pathologic behavior, the total D1-charge diverges.

$n = 1$ : Eq. (9) can be solved in terms of confluent hypergeometric functions  $U(a, b, r)$  and  $M(a, b, r)$ . The solution which decays at large  $r$  and regular at  $r = 0$  can be written as

$$H_p(r) = \begin{cases} c_p(r_0) e^{-pr} U(1 + q_1 p/2, 2, 2pr), & r > r_0, \\ d_p(r_0) e^{-pr} M(1 + q_1 p/2, 2, 2pr), & r < r_0. \end{cases} \quad (19)$$

The continuity and discontinuity conditions at  $r = r_0$  give

$$c_p(r_0)U = d_p(r_0)M, \quad (20)$$

$$c_p(r_0)(2p)W\{M, U\} = q_2 r_0^{-2} e^{pr_0} M, \quad (21)$$

where  $U$  and  $M$  have the same arguments given in (19) and  $W$  is the Wronskian. From the last relation  $c_p(r_0)$  can be fixed as

$$c_p(r_0) = -q_2(2p)\Gamma(1 + q_1 p/2)M e^{-pr_0}. \quad (22)$$

Unlike D1/D5 case, the constant  $c_p$  has a smooth  $r_0 \rightarrow 0$  limit in which it becomes (up to an irrelevant numerical factor)

$$c_p = q_2 q_1 p^2 \Gamma(q_1 p/2). \quad (23)$$

Now we focus on specific examples. For  $D3 \perp D5(2)$  it is possible to delocalize D3 or D5 branes. When D5-brane is delocalized inside D3-brane, D5-brane has the world-volume coordinates  $(\vec{x}, \vec{y})$  and D3-brane has  $(\vec{x}, \vec{z})$ .  $H_1$  is the harmonic function of the D5-brane. In this case,  $m = 3$  and the  $\theta$  integral in (8) can be calculated easily, which results

$$H_2 = 1 + q_2 \int_0^\infty dp p^3 q_1 \Gamma(q_1 p/2) y^{-1} \sin(py) e^{-pr} \times U(1 + q_1 p/2, 2, 2pr). \quad (24)$$

Note that, as  $y \rightarrow \infty$ ,  $H_2 \rightarrow 1$ , which means that D3-branes are localized inside D5-branes. On the other hand, as  $q_1 \rightarrow \infty$  we have

$$H_2 = 1 + q_2 \int_0^\infty dp p^2 y^{-1} \sin(py) e^{-pr} U(1, 2, 2pr) = 1 + \frac{2q_2}{(r^2 + y^2)^2}, \quad (25)$$

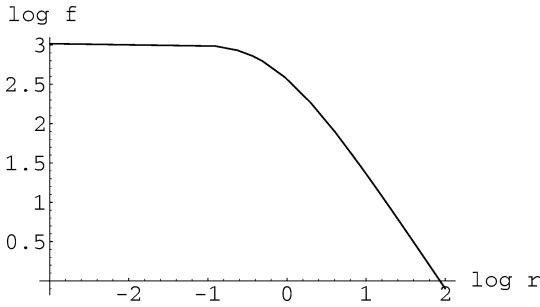


Fig. 2. Log–Log plot of the function  $f(r)$  (28) for  $D3 \perp D5(2)$  intersection when D5-brane is delocalized along D3.

which is precisely the single D3-brane solution. To obtain the near horizon geometry, we use the fact

$$\begin{aligned} \lim_{a \rightarrow \infty} \Gamma(1 + a - b)U(a, b, z/a) \\ = 2z^{\frac{1}{2} - \frac{1}{2}b} K_{b-1}(2\sqrt{z}), \end{aligned} \quad (26)$$

and the three-dimensional Fourier transform of  $K_1$ . Defining a new radial coordinate  $\rho^2 = q_1 r$  and sending  $q_1 \rightarrow \infty$  while keeping  $\rho$  fixed (which is the near horizon limit) we obtain

$$H_2 = q_1 \frac{6\pi q_2}{(y^2 + 4\rho^2)^{5/2}}. \quad (27)$$

The overall  $q_1$  factor can be scaled away in the metric (this is standard in taking near horizon limits) and this is exactly the near horizon solution constructed in [10,20]. Therefore, (24) gives a background smoothly interpolating between the asymptotically flat and near horizon regions. To see this more explicitly, one can numerically integrate (24). Let us define

$$f(r) = kr^{5/2} [H_2(y = 0, r) - 1], \quad (28)$$

where  $k$  is a normalization constant. From Fig. 2, it is possible to see the behavior of the function  $H_2(y = 0, r)$  both in the near horizon and asymptotic infinity which is clearly consistent with (27) and (25).

In the  $D3 \perp D5(2)$  intersection when D3-brane is delocalized instead of D5-brane,  $H_1$  in (7) becomes the harmonic function of the D3-brane which has the world-volume coordinates  $(\vec{x}, \vec{y})$ . It is easy to see that the space transverse to D5-brane located inside the D3-brane is one-dimensional thus we have  $m = 1$ .

From the first line of (8) one obtains

$$\begin{aligned} H_2 = 1 + q_2 \int_0^\infty dp p^2 q_1 \Gamma(q_1 p/2) \cos(py) \\ \times e^{-pr} U(1 + q_1 p/2, 2, 2pr). \end{aligned} \quad (29)$$

In this solution, delocalization of D5-branes inside D3-branes, i.e., the fact that  $y \rightarrow \infty$ ,  $H_2 \rightarrow 1$ , is guaranteed by the Riemann–Lebesgue theorem. On the other hand, it is easy to see that as  $q_1 \rightarrow 0$  one obtains  $H_2 = 1 + q_2/(y^2 + r^2)$  which gives the solution for a single D5-brane. To obtain the near horizon limit, we define  $\rho^2 = q_1 r$ , let  $q_1 \rightarrow 0$  while keeping  $\rho$  fixed and use (26) to get

$$H_2 = \frac{2\pi q_2}{(y^2 + 4\rho^2)^{3/2}}. \quad (30)$$

In this expression an overall factor of  $q_1$  is ignored. Thus (29) gives a solution which interpolates between the asymptotically flat and near horizon regions.

Finally, we consider  $M5 \perp M5(3)$  intersection in  $D = 11$ . (The same results also apply to  $D4 \perp D4(2)$  intersection of type IIA theory). Let us remind that one of the harmonic functions is given by (7) with  $n = 1$  corresponding to a smeared M5-brane. The relative transverse space of the other M5-brane located inside the smeared one is two-dimensional. Thus  $m = 2$  and  $H_2$  can be calculated from (8) to give

$$\begin{aligned} H_2 = 1 + q_2 \int_0^\infty dp p^3 q_1 \Gamma(q_1 p/2) J_0(py) \\ \times e^{-pr} U(1 + q_1 p/2, 2, 2pr). \end{aligned} \quad (31)$$

As  $y \rightarrow \infty$ ,  $H_2 \rightarrow 1$  hence one of the M5-branes is localized inside the other one. On the other hand, it is easy to see that as  $q_1 \rightarrow 0$  we have  $H_2 = 1 + q_2/2(r^2 + y^2)^3$  which is the solution for a single M5-brane. Taking the near horizon limit by keeping  $\rho^2 = q_1 r$  fixed as  $q_1 \rightarrow \infty$  we obtain (ignoring an overall  $q_1$  factor)

$$H_2 = \frac{8q_2}{(y^2 + 4\rho^2)^2}. \quad (32)$$

This shows that the solution given by the integral (31) smoothly interpolates between the asymptotically flat and near horizon regions.

From these examples we see that when the overall transverse space is three-dimensional (which corresponds to  $n = 1$ ) it is possible to obtain smooth solutions in an integral form for partially localized brane intersections. Therefore, for higher dimensions with  $n > 1$ , it is possible to smear some directions in the overall transverse space and reduce the problem to the  $n = 1$  case. For instance, in D1/D5 system smearing one direction we get

$$H_2 = 1 + q_2 \int_0^\infty dp p^4 q_1 \Gamma(q_1 p/2) y^{-1} J_1(py) \times e^{-pr} U(1 + q_1 p/2, 2, 2pr). \quad (33)$$

In the near horizon limit defined by  $q_1 \rightarrow \infty$  with fixed  $\rho^2 = q_1 r$ , we get

$$H_2 = \frac{32q_2}{(4\rho^2 + y^2)^3} \quad (34)$$

which is in agreement with the previously constructed solution given in [10].

Another way of reducing the power of  $r$  in  $H_1$  is to consider other Ricci flat spaces in the transverse part, however this may not be sufficient alone. For example, for D1/D5, one can replace four-dimensional flat  $\vec{r}$  coordinates in (1) with a Taub-NUT space. Note that no-go theorem does not apply with this modification. In this case, the field equations (4)–(6) become

$$\nabla_{\text{TN}}^2 H_1 = q_1 \delta_{\text{TN}}, \quad (35)$$

$$(\nabla_{\text{TN}}^2 + H_1 \partial_{\vec{y}}^2) H_2 = q_2 \delta(\vec{y}) \delta_{\text{TN}}, \quad (36)$$

where  $\nabla_{\text{TN}}^2$  is the Laplacian and  $\delta_{\text{TN}}$  is the covariant delta function of the Taub-NUT space which has the metric

$$ds^2 = \left[1 + \frac{2m}{r}\right] (dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)) + \left[1 + \frac{2m}{r}\right]^{-1} (4m)^2 \left(d\psi + \frac{1}{2} \cos\theta d\phi\right)^2. \quad (37)$$

For  $H_1 = H_1(r)$ , away from the source (35) becomes

$$\left[1 + \frac{2m}{r}\right]^{-1} \left(\frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r}\right) H_1 = 0. \quad (38)$$

This has the solution

$$H_1 = 1 + \frac{q_1}{r}, \quad (39)$$

which precisely obeys (35) with the source term. Now, recall that  $r$  dependence of  $H_1$  was  $1/r^2$  when the transverse space was flat. So we achieved our goal and reduced the its power by one. To find the harmonic function  $H_2$ , we first put D1-brane at  $r = r_0$  in Taub-NUT space. Writing  $H_2$  as in (8), (36) becomes

$$\left[\frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} - p^2 \left(1 + \frac{q_1}{r}\right) \left(1 + \frac{2m}{r}\right)\right] H_p(r) = q_2 \frac{\delta(r - r_0)}{r^2}. \quad (40)$$

This can be solved in terms of confluent hypergeometric functions, and the solution which decays at large  $r$  and regular at  $r = 0$  can be found as

$$H_p(r) = \begin{cases} c_p(r_0) e^{-pr} r^{-1 + \frac{\mu}{2}} U(mp + \frac{q_1 p + \mu}{2}, \mu, 2pr), & r > r_0, \\ d_p(r_0) e^{-pr} r^{-1 + \frac{\mu}{2}} M(mp + \frac{q_1 p + \mu}{2}, \mu, 2pr), & r < r_0, \end{cases} \quad (41)$$

where  $\mu = 1 + \sqrt{1 + 8mp^2 q_1}$ . Using the conditions imposed by the delta function source, it is easy to obtain

$$c_p(r_0) = q_2 \frac{\Gamma[mp + \frac{q_1 p + \mu}{2}]}{\Gamma[\mu]} r_0^{-1 + \mu/2} p^{\mu-1} M e^{-pr_0}. \quad (42)$$

In the  $r_0 \rightarrow 0$  limit, we have  $c_p \rightarrow 0$  implying spontaneous delocalization. So, even though the  $r$  dependence of  $H_1$  in (39) is lowered by using Taub-NUT space, still it is not possible to construct a localized D1  $\perp$  D5(1) intersection.

### 3. Conclusions

In this Letter we obtained partially localized supergravity solutions for D3  $\perp$  D5(2), D4  $\perp$  D4(2) and M5  $\perp$  M5(3) intersections where the overall transverse space is three-dimensional. It is clear that, as in the case of D2/D6 intersection studied in [12], our solutions exhibit richer behavior in the decoupling limit compared to the completely delocalized or partially localized but near-horizon solutions [10].

When  $n > 2$ , we could not succeed in solving the radial differential equation. Yet the delocalization

phenomenon is expected to occur [16,17]. For these cases smearing the overall transverse dimensions until  $n = 1$  is an option. In principle, intersections with  $n \leq 0$  can also be analyzed as above. However, since the asymptotic geometry is not flat they are not considered in this paper.

For intersections with four-dimensional transverse space, the primary example being  $D1 \perp D5(1)$ , we observed that the method fails, implying a delocalization which is consistent with the no-go theorems [15–17]. To overcome this problem we highlighted two possible ways. Namely, one can solve the integral equation (15) or find a suitable  $c_p$  in (16). However, these seem to be quite difficult to come up with. On the other hand, smearing one transverse dimension we obtained a valid supergravity solution (33). The field theoretic meaning of neither this nor the near horizon version given in [10] is not clear to us. This needs further investigation. We also tried to construct a localized solution by replacing the flat transverse space with Taub-NUT which unfortunately did not improve the situation. It would be interesting to consider other Ricci flat manifolds.

Recently  $D3 \perp D5(2)$  intersection has received a lot of interest after [21]. In the approach that we employed we were forced to delocalize one of the branes. Although this may still be useful for the purposes of [21], a fully localized solution would probably be more appropriate.

Finally, in [12],  $D2/D6$  intersection was obtained by starting from an M2-brane which contained Taub-NUT space in the transverse part. Similarly,  $D4/D6$  system can be studied by considering an M5-brane whose two of the world-volume coordinates embedded holomorphically into a Taub-NUT space [9,22]. It would be interesting to construct this solution which might give some clue for a more general intersection ansatz.

## Acknowledgements

We would like to thank A. Hashimoto for an e-mail correspondence on D1/D5 intersection. We also would like to thank T. Rador for discussions.

## References

- [1] J.P. Gauntlett, hep-th/9705011.
- [2] H. Lu, C.N. Pope, T.R. Tran, K.W. Xu, Nucl. Phys. B 511 (1998) 98, hep-th/9708055.
- [3] D.J. Smith, Class. Quantum Grav. 20 (2003) R233, hep-th/0210157.
- [4] A.A. Tseytlin, Nucl. Phys. B 475 (1996) 149, hep-th/9604035.
- [5] K. Behrndt, E. Bergshoeff, B. Janssen, Phys. Rev. D 55 (1997) 3785, hep-th/9604168.
- [6] J.P. Gauntlett, D.A. Kastor, J. Traschen, Nucl. Phys. B 478 (1996) 544, hep-th/9604179.
- [7] H. Lu, C.N. Pope, Int. J. Mod. Phys. A 13 (1998) 4425, hep-th/9710155.
- [8] N. Itzhaki, A.A. Tseytlin, S. Yankielowicz, Phys. Lett. B 432 (1998) 298, hep-th/9803103.
- [9] A. Hashimoto, JHEP 9901 (1999) 018, hep-th/9812159.
- [10] D. Youm, Nucl. Phys. B 556 (1999) 222, hep-th/9902208.
- [11] A. Loewy, Phys. Lett. B 463 (1999) 41, hep-th/9903038.
- [12] S.A. Cherkis, A. Hashimoto, JHEP 0211 (2002) 036, hep-th/0210105.
- [13] D.A. Lowe, H. Nastase, S. Ramgoolam, hep-th/0303173.
- [14] H. Nastase, hep-th/0305069.
- [15] S. Surya, D. Marolf, Phys. Rev. D 58 (1998) 124013, hep-th/9805121.
- [16] D. Marolf, A.W. Peet, Phys. Rev. D 60 (1999) 105007, hep-th/9903213.
- [17] A. Gomberoff, D. Kastor, D. Marolf, J. Traschen, Phys. Rev. D 61 (2000) 024012, hep-th/9905094.
- [18] A. Fayyazuddin, D.J. Smith, JHEP 9904 (1999) 030, hep-th/9902210.
- [19] A. Fayyazuddin, D.J. Smith, JHEP 0010 (2000) 023, hep-th/0006060.
- [20] A. Fayyazuddin, JHEP 0303 (2003) 033, hep-th/0207129.
- [21] A. Karch, L. Randall, JHEP 0106 (2001) 063, hep-th/0105132.
- [22] S.A. Cherkis, hep-th/9906203.