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Distinguishing between $SU(5)$ and flipped $SU(5)$

Ilja Dorsner [a], Pavel Fileviez Pérez [a,b]

[a] *The Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, 34014 Trieste, Italy*
[b] *Pontificia Universidad Católica de Chile, Facultad de Física, Casilla 306, Santiago 22, Chile*

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Abstract

We study in detail the $d = 6$ operators for proton decay in the two possible matter unification scenarios based on $SU(5)$ gauge symmetry. We investigate the way to distinguish between these two scenarios. The dependence of the branching ratios for the two body decays on the fermion mixing is presented in both cases. We point out the possibility to make a clear test of flipped $SU(5)$ through the decay channel $p \to \pi^+ \bar{\nu}$, and the ratio $\tau(p \to K^0 e_\alpha^+)/\tau(p \to \pi^0 e_\alpha^+)$.

1. Introduction

Proton decay [1] is the most dramatic prediction of grand unified theories, where quarks and leptons are at least partially unified. Its signatures have been extensively studied in various theories [2–12] for many years. Recently, in the context of minimal supersymmetric $SU(5)$, the predictions coming from both $d = 5$ and $d = 6$ operators have been studied in order to understand if this model is ruled out [11,12]. Several solutions have been forwarded [13–15] to address this issue. This has renewed the interests of many groups to the important question of the proton stability (for a review see [16]). Similar study, in the context of flipped $SU(5)$, has also been made [17] concluding that the flipped model is out of trouble.

There are several contributions to the decay of the proton. The $d = 4$ and $d = 5$ are the most important in supersymmetric scenarios. In a theory where matter-parity is conserved the $d = 4$ are forbidden, while the $d = 5$ operators can always be suppressed by choosing a particular Higgs sector [18–20]. The less model dependent contributions are the $d = 6$, which we study here in detail.

An extensive study of $d = 6$ operators in the most general way in the context of $SU(5)$ and $SO(10)$ has been preformed in Ref. [21]. There it has been pointed out that it is possible to make a clear test of any grand unified

E-mail address: fileviez@higgs.fis.puc.cl (P. Fileviez Pérez).

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theory with symmetric Yukawa couplings through the decay channels into antineutrinos. However, the particular case of flipped $SU(5)$ has not been studied taking into account the general dependence on fermion mixing (for early analyses see [22–24]). With this work we seek to remedy that. Namely, we investigate all $d = 6$ proton decay operators in two different GUT models based on $SU(5)$. We then confront the signatures of the two unifying schemes pointing out the way to distinguish between them. We also point out the way to test flipped $SU(5)$.

The Letter is organized as follows. In Section 2 we briefly review the key properties of both $SU(5)$ and flipped $SU(5)$ unified theory. Section 3 is devoted to the general discussion of $d = 6$ operators in both scenarios. In Section 4 we specify all the branching ratios for the independent channels for proton decay. That section contains the main results of our work. Finally, we conclude in the last section. Appendix A contains useful decay rate formulas used throughout the manuscript.

2. Matter unification based on $SU(5)$

The smallest special unitary group that contains the Standard Model (SM) gauge group is $SU(5)$. The $SU(5)$ grand unified theory [25,26] is an anomaly free theory, where we have partial matter unification for each family in three representation: **10**, $\bar{\mathbf{5}}$ and **1**. The singlet is identify with the right-handed neutrino. In the SM language we have: $\mathbf{10} = (\mathbf{3}, \mathbf{2}, 1/3) \oplus (\bar{\mathbf{3}}, \mathbf{1}, -4/3) \oplus (\mathbf{1}, \mathbf{1}, 2) = (Q, u^C, e^C)$, $\bar{\mathbf{5}} = (\bar{\mathbf{3}}, \mathbf{1}, 2/3) \oplus (\mathbf{1}, \mathbf{2}, -1) = (d^C, L)$, and $\mathbf{1} = (\mathbf{1}, \mathbf{1}, 0) = \nu^C$, where $Q = (u, d)$ and $L = (\nu, e)$. The off-diagonal part of the gauge fields residing in the **24** of $SU(5)$ is composed of bosons $(X, Y) = (\mathbf{3}, \mathbf{2}, 5/3)$ and their conjugates, which mediate proton decay. X and Y fields have electric charge $4/3$ and $1/3$, respectively.

The electric charge is a generator of conventional $SU(5)$. However, it is possible to embed the electric charge in such a manner that it is a linear combination of the generators operating in both $SU(5)$ and an extra $U(1)$, and still reproduce the SM charge assignment. This is exactly what is done in a flipped $SU(5)$ [22,27–29]. The matter now unifies in a different manner, which can be obtained from the $SU(5)$ assignment by a flip: $d^C \leftrightarrow u^C$, $e^C \leftrightarrow \nu^C$, $u \leftrightarrow d$ and $\nu \leftrightarrow e$. In the case of flipped $SU(5)$ the gauge bosons responsible for proton decay are: $(X', Y') = (\mathbf{3}, \mathbf{2}, -1/3)$. The electric charge of Y' is $-2/3$, while X' has the same charge as Y. Since the gauge sector and the matter unification differ from $SU(5)$ case, the proton decay predictions are also different [22].

Flipped $SU(5)$ is well motivated from string theory scenarios, since we do not need large representations to achieve the GUT symmetry breaking [29]. Another nice feature of flipped $SU(5)$ is that the dangerous $d = 5$ operators are suppressed due to an extremely economical missing partner mechanism. This allows us to concentrate our attention to the gauge $d = 6$ contributions.

We next analyze the possibility to test two realistic grand unified theories: the $SU(5)$ and flipped $SU(5)$ theory. We make an analysis of the operators in each theory, and study the physical parameters entering in the predictions for proton decay. We do not commit to any particular model for fermion masses, in order to be sure that we can test the grand unification idea.

3. $d = 6$ operators

In the Georgi–Glashow $SU(5)$ matter unification case, the gauge $d = 6$ operators contributing to the decay of the proton are [2–4]:

$$O^{B-L}_{SU(5)} = k_1^2 \epsilon_{ijk} \epsilon_{\alpha\beta} \overline{u^C_{ia}} \gamma^\mu Q_{j\alpha a} \overline{e^C_b} \gamma_\mu Q_{k\beta b}, \tag{1a}$$

$$O^{B-L}_{SU(5)} = k_1^2 \epsilon_{ijk} \epsilon_{\alpha\beta} \overline{u^C_{ia}} \gamma^\mu Q_{j\alpha a} \overline{d^C_{kb}} \gamma_\mu L_{\beta b}. \tag{1b}$$

On the other hand, flipped $SU(5)$ matter unification yields:

$$O^{B-L}_{SU(5)'} = k_2^2 \epsilon_{ijk}\epsilon_{\alpha\beta}\overline{d^C_{ia}}\gamma^\mu Q_{j\beta a}\overline{u^C_{kb}}\gamma_\mu L_{\alpha b}, \tag{2a}$$

$$O^{B-L}_{SU(5)'} = k_2^2 \epsilon_{ijk}\epsilon_{\alpha\beta}\overline{d^C_{ia}}\gamma^\mu Q_{j\beta a}\overline{\nu^C_{b}}\gamma_\mu Q_{k\alpha b}. \tag{2b}$$

In the above expressions $k_1 = g_5 M^{-1}_{(X,Y)}$, and $k_2 = g'_5 M^{-1}_{(X',Y')}$, where $M_{(X,Y)}$ ($M_{(X',Y')}$) $\sim M_{\rm GUT} \approx 10^{16}$ GeV and g_5 (g'_5) are the masses of the superheavy gauge bosons and the couplings at the GUT scale in $SU(5)$ (flipped $SU(5)$) case. i, j and k are the color indices, a and b are the family indices, and $\alpha, \beta = 1, 2$.

In these theories the diagonalization of the Yukawa matrices is given by the following bi-unitary transformations:

$$U_C^{\rm T} Y_U U = Y_U^{\rm diag}, \tag{3}$$

$$D_C^{\rm T} Y_D D = Y_D^{\rm diag}, \tag{4}$$

$$E_C^{\rm T} Y_E E = Y_E^{\rm diag}. \tag{5}$$

Using the operators listed in Eqs. (1), the effective operators for each decay channel in the $SU(5)$ case upon Fierz transformation take the following form in the physical basis [21]:

$$O\big(e^C_\alpha, d_\beta\big)_{SU(5)} = c\big(e^C_\alpha, d_\beta\big)_{SU(5)}\epsilon_{ijk}\overline{u^C_i}\gamma^\mu u_j \overline{e^C_\alpha}\gamma_\mu d_{k\beta}, \tag{6a}$$

$$O\big(e_\alpha, d^C_\beta\big)_{SU(5)} = c\big(e_\alpha, d^C_\beta\big)_{SU(5)}\epsilon_{ijk}\overline{u^C_i}\gamma^\mu u_j \overline{d^C_{k\beta}}\gamma_\mu e_\alpha, \tag{6b}$$

$$O\big(\nu_l, d_\alpha, d^C_\beta\big)_{SU(5)} = c\big(\nu_l, d_\alpha, d^C_\beta\big)_{SU(5)}\epsilon_{ijk}\overline{u^C_i}\gamma^\mu d_{j\alpha} \overline{d^C_{k\beta}}\gamma_\mu \nu_l, \tag{6c}$$

$$O\big(\nu^C_l, d_\alpha, d^C_\beta\big)_{SU(5)} = c\big(\nu^C_l, d_\alpha, d^C_\beta\big)_{SU(5)}\epsilon_{ijk}\overline{d^C_{i\beta}}\gamma^\mu u_j \overline{\nu^C_l}\gamma_\mu d_{k\alpha}, \tag{6d}$$

where

$$c\big(e^C_\alpha, d_\beta\big)_{SU(5)} = k_1^2\Big[V_1^{11}V_2^{\alpha\beta} + (V_1 V_{UD})^{1\beta}\big(V_2 V_{UD}^\dagger\big)^{\alpha 1}\Big], \tag{7a}$$

$$c\big(e_\alpha, d^C_\beta\big)_{SU(5)} = k_1^2 V_1^{11} V_3^{\beta\alpha}, \tag{7b}$$

$$c\big(\nu_l, d_\alpha, d^C_\beta\big)_{SU(5)} = k_1^2 (V_1 V_{UD})^{1\alpha}(V_3 V_{EN})^{\beta l}, \quad \alpha = 1 \text{ or } \beta = 1, \tag{7c}$$

$$c\big(\nu^C_l, d_\alpha, d^C_\beta\big)_{SU(5)} = 0. \tag{7d}$$

In the case of flipped $SU(5)$ (see Eqs. (2)) the effective operators are

$$O\big(e^C_\alpha, d_\beta\big)_{SU(5)'} = c\big(e^C_\alpha, d_\beta\big)_{SU(5)'}\epsilon_{ijk}\overline{u^C_i}\gamma^\mu u_j \overline{e^C_\alpha}\gamma_\mu d_{k\beta}, \tag{8a}$$

$$O\big(e_\alpha, d^C_\beta\big)_{SU(5)'} = c\big(e_\alpha, d^C_\beta\big)_{SU(5)'}\epsilon_{ijk}\overline{u^C_i}\gamma^\mu u_j \overline{d^C_{k\beta}}\gamma_\mu e_\alpha, \tag{8b}$$

$$O\big(\nu_l, d_\alpha, d^C_\beta\big)_{SU(5)'} = c\big(\nu_l, d_\alpha, d^C_\beta\big)_{SU(5)'}\epsilon_{ijk}\overline{u^C_i}\gamma^\mu d_{j\alpha} \overline{d^C_{k\beta}}\gamma_\mu \nu_l, \tag{8c}$$

$$O\big(\nu^C_l, d_\alpha, d^C_\beta\big)_{SU(5)'} = c\big(\nu^C_l, d_\alpha, d^C_\beta\big)_{SU(5)'}\epsilon_{ijk}\overline{d^C_{i\beta}}\gamma^\mu u_j \overline{\nu^C_l}\gamma_\mu d_{k\alpha}, \tag{8d}$$

where

$$c\big(e^C_\alpha, d_\beta\big)_{SU(5)'} = 0, \tag{9a}$$

$$c\big(e_\alpha, d^C_\beta\big)_{SU(5)'} = k_2^2\big(V_4 V_{UD}^\dagger\big)^{\beta 1}\big(V_1 V_{UD} V_4^\dagger V_3\big)^{1\alpha}, \tag{9b}$$

$$c\big(\nu_l, d_\alpha, d^C_\beta\big)_{SU(5)'} = k_2^2 V_4^{\beta\alpha}\big(V_1 V_{UD} V_4^\dagger V_3 V_{EN}\big)^{1l}, \quad \alpha = 1 \text{ or } \beta = 1, \tag{9c}$$

$$c\big(\nu^C_l, d_\alpha, d^C_\beta\big)_{SU(5)'} = k_2^2\Big[\big(V_4 V_{UD}^\dagger\big)^{\beta 1}\big(U_{EN}^\dagger V_2\big)^{l\alpha} + V_4^{\beta\alpha}\big(U_{EN}^\dagger V_2 V_{UD}^\dagger\big)^{l1}\Big], \quad \alpha = 1 \text{ or } \beta = 1. \tag{9d}$$

We use the subscripts $SU(5)$ and $SU(5)'$ to distinguish the two scenarios. The mixing matrices $V_1 = U_C^\dagger U$, $V_2 = E_C^\dagger D$, $V_3 = D_C^\dagger E$, $V_4 = D_C^\dagger D$, $V_{UD} = U^\dagger D$, $V_{EN} = E^\dagger N$ and $U_{EN} = E_C^\dagger N_C$. The quark mixing is given by $V_{UD} = U^\dagger D = K_1 V_{\rm CKM} K_2$, where K_1 and K_2 are diagonal matrices containing three and two phases, respectively. The leptonic mixing $V_{EN} = K_3 V_l^D K_4$ in case of Dirac neutrino, or $V_{EN} = K_3 V_l^M$ in the Majorana case. V_l^D and V_l^M are the leptonic mixing matrices at low energy in the Dirac and Majorana case, respectively.

Notice that in general to predict the lifetime of the proton in $SU(5)$, due to the presence of $d = 6$ operators, we have to know k_1, V_1^{1b}, V_2, V_3, while in flipped $SU(5)$ we have to know k_2, V_1^{1b}, V_3, V_4 and U_{EN}. In addition, we have to know three diagonal matrices containing CP violating phases, K_1, K_2 and K_3, in the case that the neutrino is Majorana. In the Dirac case there is an extra matrix with two more phases.

From the above equations, we see that there are no decays into ν^C in $SU(5)$, and in flipped $SU(5)$ into e^C, since these are singlets in the corresponding scenarios.

4. Flipped $SU(5)$ versus $SU(5)$

There are only seven independent relations for all coefficients of the gauge $d = 6$ operators contributing to nucleon decay [21]. Therefore, if we want to test a grand unified theory, the number of physical quantities entering in the proton decay amplitude must be less than that. This is important to know in order to see if it is possible to test a GUT scenario.

Since we cannot distinguish between the neutrino flavors in the proton decay experiments, in order to compute the branching ratios into antineutrinos we have to sum over all of them. Using the expressions in Appendix A, and the following relations,

$$\sum_{l=1}^{3} c\big(\nu_l, d_\alpha, d_\beta^C\big)^*_{SU(5)} c\big(\nu_l, d_\gamma, d_\delta^C\big)_{SU(5)} = k_1^4 \big(V_1^* V_{UD}^*\big)^{1\alpha} (V_1 V_{UD})^{1\gamma} \delta^{\beta\delta}, \tag{10a}$$

$$\sum_{l=1}^{3} c\big(\nu_l, d_\alpha, d_\beta^C\big)^*_{SU(5)'} c\big(\nu_l, d_\gamma, d_\delta^C\big)_{SU(5)'} = k_2^4 \big(V_4^*\big)^{\beta\alpha} V_4^{\delta\gamma}, \tag{10b}$$

we can write down the ratios between the lifetimes in both theories for the decays into antineutrinos. They are:

$$\frac{\tau(p \to K^+ \bar\nu)^{SU(5)'}}{\tau(p \to K^+ \bar\nu)^{SU(5)}} = \frac{k_1^4}{k_2^4} \frac{A_1^2 |(V_1 K_1 V_{\rm CKM})^{11}|^2 + A_2^2 |(V_1 K_1 V_{\rm CKM})^{12}|^2}{A_1^2 |V_4^{21}|^2 + A_2^2 |V_4^{12}|^2 + A_1 A_2 ((V_4^*)^{21} V_4^{12} + (V_4^*)^{12} V_4^{21})}, \tag{11a}$$

$$\frac{\tau(p \to \pi^+ \bar\nu)^{SU(5)'}}{\tau(p \to \pi^+ \bar\nu)^{SU(5)}} = \frac{k_1^4}{k_2^4} \frac{|(V_1 K_1 V_{\rm CKM})^{11}|^2}{|V_4^{11}|^2}, \tag{11b}$$

$$\frac{\tau(n \to K^0 \bar\nu)^{SU(5)'}}{\tau(n \to K^0 \bar\nu)^{SU(5)}} = \frac{k_1^4}{k_2^4} \frac{A_3^2 |(V_1 K_1 V_{\rm CKM})^{11}|^2 + A_2^2 |(V_1 K_1 V_{\rm CKM})^{12}|^2}{A_3^2 |V_4^{21}|^2 + A_2^2 |V_4^{12}|^2 + A_3 A_2 ((V_4^*)^{21} V_4^{12} + (V_4^*)^{12} V_4^{21})}, \tag{11c}$$

where

$$A_1 = \frac{2 m_p}{3 m_B} D, \tag{12a}$$

$$A_2 = 1 + \frac{m_p}{3 m_B} (D + 3F), \tag{12b}$$

$$A_3 = 1 + \frac{m_n}{3 m_B} (D - 3F). \tag{12c}$$

The same procedure can be done for the decays into charged leptons:

$$\frac{\tau(p\to\pi^0 e_\beta^+)^{SU(5)'}}{\tau(p\to\pi^0 e_\beta^+)^{SU(5)}}=\frac{k_1^4}{k_2^4}\frac{|V_1^{11}V_3^{1\beta}|^2+|V_1^{11}V_2^{\beta 1}+(V_1K_1V_{\rm CKM}K_2)^{11}(V_2K_2^*V_{\rm CKM}^\dagger K_1^*)^{\beta 1}|^2}{|(V_4K_2^*V_{\rm CKM}^\dagger K_1^*)^{11}(V_1K_1V_{\rm CKM}K_2V_4^\dagger V_3)^{1\beta}|^2}, \tag{13}$$

$$\frac{\tau(p\to K^0 e_\beta^+)^{SU(5)'}}{\tau(p\to K^0 e_\beta^+)^{SU(5)}}=\frac{k_1^4}{k_2^4}\frac{|V_1^{11}V_3^{2\beta}|^2+|V_1^{11}V_2^{\beta 2}+(V_1K_1V_{\rm CKM}K_2)^{12}(V_2K_2^*V_{\rm CKM}^\dagger K_1^*)^{\beta 1}|^2}{|(V_4K_2^*V_{\rm CKM}^\dagger K_1^*)^{21}(V_1K_1V_{\rm CKM}K_2V_4^\dagger V_3)^{1\beta}|^2}. \tag{14}$$

Eqs. (11), (13), and (14) are the most general equations that we could write in the two scenarios and will help in future to distinguish between them if proton decay is found. In other words, for a given model of fermion masses, using the above equations we could see the difference in the predictions for proton decay. Unfortunately, as one can appreciate, the branching ratios depend on too many unknown factors, including the new CP violating phases. (These, in principle, could be defined in a particular model for CP violation.) Therefore, it is impossible to test those scenarios in general through the decay of the proton unless we known the flavor structure of the SM fermions.

Since we cannot make clear predictions in the most general case, let us consider special cases in these two matter unification scenarios based on $SU(5)$ and compare them.

4.1. $SU(5)$ with $Y_U = Y_U^{\rm T}$

In $SU(5)$, if $Y_U = Y_U^{\rm T}$, we have $U_C = U K_u$, where K_u is a diagonal matrix containing three CP violating phases. Therefore, we get:

$$\sum_{l=1}^{3} c\big(\nu_l, d_\alpha, d_\beta^C\big)^*_{SU(5)} c\big(\nu_l, d_\gamma, d_\delta^C\big)_{SU(5)} = k_1^4\big(V_{\rm CKM}^*\big)^{1\alpha}\big(K_2^*\big)^{\alpha\alpha}(V_{\rm CKM})^{1\gamma}K_2^{\gamma\gamma}\delta^{\beta\delta}. \tag{15}$$

In this case, as has been shown [21], the clean channels, i.e., the channels that we have to look to test this scenario, are:

$$\Gamma\big(p\to K^+\bar\nu\big) = k_1^4\big[A_1^2\big|V_{\rm CKM}^{11}\big|^2 + A_2^2\big|V_{\rm CKM}^{12}\big|^2\big]C_1, \tag{16a}$$

$$\Gamma\big(p\to \pi^+\bar\nu\big) = k_1^4\big|V_{\rm CKM}^{11}\big|^2 C_2, \tag{16b}$$

where

$$C_1 = \frac{(m_p^2-m_K^2)^2}{8\pi m_p^3 f_\pi^2}A_L^2|\alpha|^2, \tag{17a}$$

$$C_2 = \frac{m_p}{8\pi f_\pi^2}A_L^2|\alpha|^2(1+D+F)^2. \tag{17b}$$

Notice that we have two expressions for k_1, which are independent of the unknown mixing matrices and the CP violating phases. Therefore, it is possible to test $SU(5)$ grand unified theory with symmetric up Yukawa matrices through these two channels [21]. Notice that these results are valid for any unified model based on $SU(5)$ with $Y_U = Y_U^{\rm T}$. For example, this includes the case of minimal SUSY $SU(5)$ with two extra Higgses in the fundamental and antifundamental representations. The case of modified missing doublet SUSY $SU(5)$ model [30,31] is also included in our analysis.

4.2. Renormalizable flipped $SU(5)$

In renormalizable flipped $SU(5)$ we have $Y_D = Y_D^{\mathrm{T}}$, so $D_C = DK_d$, where K_d is a diagonal matrix containing three CP violating phases. In this case the coefficients entering in the proton decay predictions are:

$$\sum_{l=1}^{3} c\left(\nu_l, d_\alpha, d_\beta^C\right)^*_{SU(5)'} c\left(\nu_l, d_\gamma, d_\delta^C\right)_{SU(5)'} = k_2^4 K_d^{\beta\beta}\delta^{\beta\alpha}\left(K_d^*\right)^{\delta\delta}\delta^{\delta\gamma}, \tag{18a}$$

$$\left|c\left(e_\alpha, d_\beta^C\right)\right|^2 = k_2^4\left|V_{\mathrm{CKM}}^{1\beta}\right|^2\left|\left(V_1 V_{UD} V_4^\dagger V_3\right)^{1\alpha}\right|^2 = k_2^4\left|V_{\mathrm{CKM}}^{1\beta}\right|^2\left|\left(U_C^\dagger E\right)^{1\alpha}\right|^2. \tag{18b}$$

Using these equations we get the following relations:

$$\Gamma\left(p \to \pi^+\bar{\nu}\right) = k_2^4 C_2, \tag{19a}$$

$$\Gamma\left(p \to \pi^0 e_\alpha^+\right) = \frac{1}{2}\Gamma(p \to \pi^+\bar{\nu})\left|V_{\mathrm{CKM}}^{11}\right|^2\left|\left(U_C^\dagger E\right)^{1\alpha}\right|^2, \tag{19b}$$

$$\frac{\Gamma(p \to K^0 e_\alpha^+)}{\Gamma(p \to \pi^0 e_\alpha^+)} = 2\frac{C_3}{C_2}\frac{|V_{\mathrm{CKM}}^{12}|^2}{|V_{\mathrm{CKM}}^{11}|^2}, \tag{19c}$$

where:

$$C_3 = \frac{(m_p^2 - m_K^2)^2}{8\pi f_\pi^2 m_p^3} A_L^2 |\alpha|^2 \left[1 + \frac{m_p}{m_B}(D - F)\right]^2. \tag{20}$$

Notice that in this case, $\Gamma(p \to K^+\bar{\nu}) = 0$, and $\Gamma(n \to K^0\bar{\nu}) = 0$. In Eq. (19c) we assume $(U_C^\dagger E)^{1\alpha} \neq 0$.

We can say that the renormalizable flipped $SU(5)$ can be verified by looking at the channel $p \to \pi^+\bar{\nu}$, and using the correlation stemming from Eq. (19c). This is a nontrivial result and can help us to test this scenario, if proton decay is found in the next generation of experiments. It is one of the main results of this work. If this channel is measured, we can know the predictions for decays into charged leptons using Eq. (19b) for a given model for fermion masses. Therefore, it is possible to differentiate between different fermion mass models.

Note the difference between Eqs. (16b) and (19a); there appears a suppression factor for the channel $p \to \pi^+\bar{\nu}$ in the case of $SU(5)$.

Since the nucleon decays into K mesons are absent in the case of flipped $SU(5)$, that is an independent way to distinguish this model from $SU(5)$, where these channels are always present.

5. Conclusions

We have investigated in model independent way the predictions coming from the gauge $d = 6$ operators in the two possible matter unification scenarios based on $SU(5)$ gauge symmetry. We write down the most general ratios between the lifetimes in $SU(5)$ and flipped $SU(5)$ theory for each channel, providing the way to distinguish between them. We find that in general it is very difficult to test flipped $SU(5)$. However, in the case of renormalizable flipped $SU(5)$ model, the decay channel $p \to \pi^+\bar{\nu}$, which is a clean channel, and the ratio $\tau(p \to K^0 e_\alpha^+)/\tau(p \to \pi^0 e_\alpha^+)$ could be used to test this theory. If the decay of the proton is found in future, our results will be useful to analyze the predictions in these theories.

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Appendix A

Using the chiral Lagrangian techniques (see Ref. [32]), the decay rate of the different channels due to the presence of the gauge $d = 6$ operators are given by:

$$\Gamma\big(p \to K^+ \bar{\nu}\big) = \frac{(m_p^2 - m_K^2)^2}{8\pi m_p^3 f_\pi^2} A_L^2 |\alpha|^2 \sum_{i=1}^{3} \left| \frac{2m_p}{3m_B} Dc\big(\nu_i, d, s^C\big) + \left[1 + \frac{m_p}{3m_B}(D+3F)\right] c\big(\nu_i, s, d^C\big) \right|^2, \quad \text{(A.1)}$$

$$\Gamma\big(p \to \pi^+ \bar{\nu}\big) = \frac{m_p}{8\pi f_\pi^2} A_L^2 |\alpha|^2 (1+D+F)^2 \sum_{i=1}^{3} \big|c\big(\nu_i, d, d^C\big)\big|^2, \quad \text{(A.2)}$$

$$\Gamma\big(p \to \eta e_\beta^+\big) = \frac{(m_p^2 - m_\eta^2)^2}{48\pi f_\pi^2 m_p^3} A_L^2 |\alpha|^2 (1+D-3F)^2 \Big\{ \big|c\big(e_\beta, d^C\big)\big|^2 + \big|c\big(e_\beta^C, d\big)\big|^2 \Big\}, \quad \text{(A.3)}$$

$$\Gamma\big(p \to K^0 e_\beta^+\big) = \frac{(m_p^2 - m_K^2)^2}{8\pi f_\pi^2 m_p^3} A_L^2 |\alpha|^2 \left[1 + \frac{m_p}{m_B}(D-F)\right]^2 \Big\{ \big|c\big(e_\beta, s^C\big)\big|^2 + \big|c\big(e_\beta^C, s\big)\big|^2 \Big\}, \quad \text{(A.4)}$$

$$\Gamma\big(p \to \pi^0 e_\beta^+\big) = \frac{m_p}{16\pi f_\pi^2} A_L^2 |\alpha|^2 (1+D+F)^2 \Big\{ \big|c\big(e_\beta, d^C\big)\big|^2 + \big|c\big(e_\beta^C, d\big)\big|^2 \Big\}, \quad \text{(A.5)}$$

$$\Gamma\big(n \to K^0 \bar{\nu}\big) = \frac{(m_n^2 - m_K^2)^2}{8\pi m_n^3 f_\pi^2} A_L^2 |\alpha|^2 \sum_{i=1}^{3} \left| c\big(\nu_i, d, s^C\big) \left[1 + \frac{m_n}{3m_B}(D-3F)\right] - c\big(\nu_i, s, d^C\big) \left[1 + \frac{m_n}{3m_B}(D+3F)\right] \right|^2, \quad \text{(A.6)}$$

$$\Gamma\big(n \to \pi^0 \bar{\nu}\big) = \frac{m_n}{16\pi f_\pi^2} A_L^2 |\alpha|^2 (1+D+F)^2 \sum_{i=1}^{3} \big|c\big(\nu_i, d, d^C\big)\big|^2, \quad \text{(A.7)}$$

$$\Gamma(n \to \eta \bar{\nu}) = \frac{(m_n^2 - m_\eta^2)^2}{48\pi m_n^3 f_\pi^2} A_L^2 |\alpha|^2 (1+D-3F)^2 \sum_{i=1}^{3} \big|c\big(\nu_i, d, d^C\big)\big|^2, \quad \text{(A.8)}$$

$$\Gamma\big(n \to \pi^- e_\beta^+\big) = \frac{m_n}{8\pi f_\pi^2} A_L^2 |\alpha|^2 (1+D+F)^2 \Big\{ \big|c\big(e_\beta, d^C\big)\big|^2 + \big|c\big(e_\beta^C, d\big)\big|^2 \Big\}. \quad \text{(A.9)}$$

In the above equations m_B is an average baryon mass satisfying $m_B \approx m_\Sigma \approx m_\Lambda$, D, F and α are the parameters of the chiral Lagrangian, and all other notation follows [32]. Here all coefficients of four-fermion operators are evaluated at M_Z scale. A_L takes into account renormalization from M_Z to 1 GeV. $\nu_i = \nu_e, \nu_\mu, \nu_\tau$ and $e_\beta = e, \mu$.

References

[1] J.C. Pati, A. Salam, Phys. Rev. D 8 (1973) 1240;
J.C. Pati, A. Salam, Phys. Rev. Lett. 31 (1973) 661.
[2] S. Weinberg, Phys. Rev. Lett. 43 (1979) 1566;
S. Weinberg, Phys. Rev. D 22 (1980) 1694;
S. Weinberg, Phys. Rev. D 26 (1982) 287.
[3] F. Wilczek, A. Zee, Phys. Rev. Lett. 43 (1979) 1571.
[4] N. Sakai, T. Yanagida, Nucl. Phys. B 197 (1982) 533.
[5] S. Dimopoulos, S. Raby, F. Wilczek, Phys. Lett. B 112 (1982) 133.
[6] A.J. Buras, J.R. Ellis, M.K. Gaillard, D.V. Nanopoulos, Nucl. Phys. B 135 (1978) 66.
[7] P. Nath, A.H. Chamseddine, R. Arnowitt, Phys. Rev. D 32 (1985) 2348.
[8] R. Arnowitt, A.H. Chamseddine, P. Nath, Phys. Lett. B 156 (1985) 215.

[9] J. Hisano, H. Murayama, T. Yanagida, Nucl. Phys. B 402 (1993) 46.
[10] V. Lucas, S. Raby, Phys. Rev. D 55 (1997) 6986.
[11] H. Murayama, A. Pierce, Phys. Rev. D 65 (2002) 055009.
[12] T. Goto, T. Nihei, Phys. Rev. D 59 (1999) 115009.
[13] B. Bajc, P. Fileviez Pérez, G. Senjanović, Phys. Rev. D 66 (2002) 075005.
[14] B. Bajc, P. Fileviez Pérez, G. Senjanović, hep-ph/0210374.
[15] D. Emmanuel-Costa, S. Wiesenfeldt, Nucl. Phys. B 661 (2003) 62.
[16] J.C. Pati, hep-ph/0407220.
[17] J.R. Ellis, D.V. Nanopoulos, J. Walker, Phys. Lett. B 550 (2002) 99.
[18] R. Arnowitt, P. Nath, Phys. Rev. D 49 (1994) 1479.
[19] M.E. Gomez, J. Rizos, K. Tamvakis, Phys. Rev. D 59 (1999) 015015.
[20] K.S. Babu, S.M. Barr, Phys. Rev. D 48 (1993) 5354.
[21] P. Fileviez Pérez, Phys. Lett. B 595 (2004) 476.
[22] S.M. Barr, Phys. Lett. B 12 (1982) 219.
[23] J.R. Ellis, J.S. Hagelin, S. Kelley, D.V. Nanopoulos, Nucl. Phys. B 311 (1988) 1.
[24] J.R. Ellis, J.L. Lopez, D.V. Nanopoulos, K.A. Olive, Phys. Lett. B 308 (1993) 70.
[25] H. Georgi, S.L. Glashow, Phys. Rev. Lett. 32 (1974) 438.
[26] S. Dimopoulos, H. Georgi, Nucl. Phys. B 193 (1981) 150.
[27] A. De Rujula, H. Georgi, S.L. Glashow, Phys. Rev. Lett. 45 (1980) 413.
[28] J.P. Derendinger, J.E. Kim, D.V. Nanopoulos, Phys. Lett. B 139 (1984) 170.
[29] I. Antoniadis, J.R. Ellis, J.S. Hagelin, D.V. Nanopoulos, Phys. Lett. B 194 (1987) 231.
[30] A. Masiero, D.V. Nanopoulos, K. Tamvakis, T. Yanagida, Phys. Lett. B 115 (1982) 380.
[31] B. Grinstein, Nucl. Phys. B 206 (1982) 387.
[32] M. Claudson, M.B. Wise, L.J. Hall, Nucl. Phys. B 195 (1982) 297;
S. Chadha, M. Daniel, Nucl. Phys. B 229 (1983) 105.