

Ernst G. Straus (1922–1983)*

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PROLOGUE

Ernst Gabor Straus was born February 25, 1922, in Munich, Germany. He immigrated to Palestine at the age of 11, and again to the United States in 1941. He received his Ph.D. from Columbia University at age 26; then moved in 1948 to Los Angeles, where he joined the Department of Mathematics at UCLA, and lived with his wife Louise and two sons, Daniel and Paul, until his death on July 12, 1983.

This is the short of it. The long of it is a fascinating story of a young Jewish boy growing up in the shadows of the Nazi rise; a story of readjusting to new life in the upheaval of poverty stricken Palestine; of working for four years with Albert Einstein in Princeton; of an unusually broad and prolific mathematician; a scholar, a thinker, and an avid fighter for human rights.

My main source of information on Ernst is our intensive acquaintance since 1974, our long and frequent conversations on every subject under the sun, and our mathematical collaboration. My other sources are his mathematical papers, his articles and speeches on Einstein, his letters and documents, and last but not least, a large heap of autobiographical notes begun in 1979 and posthumously discovered by his wife Louise.¹

Ernst's autobiography is incomplete in the sense that it covers only his German and Palestinian eras. However, in his writings he would time and again look into later years, sometimes plunging into deep philosophical essays such as "On being a Jew" and "On faith"; thus, indeed, his notes reveal much of his profound convictions and beliefs.

In view of Einstein's Centennial in 1979, and his own failing health due to many years of severe diabetes which resulted in heart, kidney and eye

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problems, Ernst begins his notes as follows: “I have started several times on some autobiographical writing and in my usual dilatory way misplaced it and neglected to continue. Now I feel that the time has come. Partly, it is the shock of the Einstein Centennial, to find myself as one of the few preserved ruins from that glorious past, and partly a general feeling of waning energies. If I don’t do it now I may not live to do it later”.

And he goes on: “Why do it at all? I have two sons who may some day be interested in the world—very different from their own—in which I grew up. I also had the good fortune to meet and collaborate with a number of very interesting people, and no doubt, any part of the record of Einstein’s life is worth preserving...”

Only after paying homage to the Grand Master who had an immeasurable impact on him, does Ernst proceed to describe his ancestors about whom he had a good deal of information. Among them he mentions his grandfather’s grandfather, a famous rabbi who had the reputation of being able to work miracles, called the Baal Shem (Master of the Holy Name) of Michelstadt; he tells about Joseph of Rossheim (Alsace) who was appointed by Emperor Charles V as representative of the Jews of Germany and who corresponded with Martin Luther; and he continues all the way back to Rashi, the famous 11-th century commentator on the Bible. With his typical subtle sense of humor he writes: “Since Rashi claimed that he could trace his ancestry to the 3-rd century Rabbi John the Shoemaker, who in turn claimed that he could trace his ancestry to King David, tracing one’s ancestry to Rashi is considered highly desirable.” It stands to reason that Ernst could not have said much on the mathematical abilities of his rabbinical forefathers. He only mentions that “as I can tell from a computation on the margins of my old Talmud, my Baal Shem ancestor thought that $\sqrt{2} = 7/5$ and $\pi = 3$, quite shocking for a contemporary of Gauss”.

MUNICH

Ernst’s father, Elias Straus, “was full of gentle good humor” and “had an encyclopaedic knowledge of nearly everything”, recalls Ernst. He was a lawyer by profession, but was mainly occupied by Jewish affairs in Munich. As Ernst describes it, “I think it was much closer to his heart than his law practice—namely as a corporate lawyer and service on the board of several companies.”

Ernst’s mother, Rahel Straus, was a woman of “overwhelming energy and strength of personality”, who engaged in “an ever increasing amount of public and private activities.” She studied at Heidelberg where she became

one of the first female medical students at any German university, and practiced medicine until the age of 60. "...it was the greatest of good fortunes to have her as my mother", says Ernst.

Ernst's parents had five children, Ernst being by far the youngest. His sisters Isa, Hannah, and Gabriele, and his now deceased brother Peter (this is how they all called him; his real name was Samuel Friedrich) were born in approximately 2-year intervals during 1909–1915. "Then war and revolution intervened in the great world", and Ernst was born in 1922, when his father was 44 and his mother 42, "as an unexpected event marking the return of peace."

Reminiscing about his mother, Ernst is led to the following anecdote: "When I was 5 years old...mother...told me the story of Genesis... 'In the beginning God created the heavens and the earth.' I wanted to know how old God was when he did all this. Mother told me that God was infinitely old since he had lived always and he would live forever. This caused me a sleepless night after which I enunciated my first mathematical theorem: 'If there is an infinite then half of it is also infinite'... I mention all this because...it illustrates clearly the fallacy of the often stated view that conceptual thinking is done in terms of language. Here I was dealing with a concept I had never heard before and the language for thinking about it was absent. Thinking is done in mental images. It is the vexed business of communicating and recording thought which makes language necessary. Quite often the wealth of structure of the language produces expressions that mask the absence of original thought. It is this phenomenon which appears to make language the primary element in thought".

Ernst remembers with pleasure his elementary school in Munich. He skipped the 4-th year of school and entered Gymnasium at the age of 9 after passing a special entrance examination. The Theresiengymnasium he joined was "humanistic", with heavy emphasis on Latin and Greek and little emphasis on science and modern languages. "It taught me for the first time that my thirst for knowledge was not universal and in particular did not include the declension of irregular Latin verbs...", recalls Ernst.

Later, Ernst was transferred to the Wilhelmsgymnasium. There, in 1933, during a course in *Volkskunde* (ethnic studies), his teacher told the class that Hitler was born in Braunau. "I turned to my neighbor...and whispered 'that explains why he likes brown uniforms'", tells Ernst. The smiles following his remark were not overlooked by the Hitlerjugend members in the class, who after school "ceremoniously" slapped his face for "insulting the Führer". His teacher who reprimanded the offenders was threatened, and Ernst had to leave school spending his last three months in Germany with private tutors.

After Hitler took power in 1933, Ernst father's Zionistic outlook made him anticipate the terrible outbreak of Nazi antisemitism and evaluate the situa-

tion clearly. He wanted his family to leave Germany. As it was, he died of cancer shortly after, in June 1933, and the family left for Palestine and settled in Jerusalem.

JERUSALEM

Ernst's first year in Palestine was quite difficult. The primitive conditions, the cultural gap, and his relatively poor Hebrew, became major obstacles in the eyes of young Ernst. There were as yet only a handful of German immigrants in his school, and on the basis of his young age alone he was put back one year. The clash with his teachers and classmates was often bitter.

Things began to improve when the family moved to new quarters in Jerusalem, and Ernst was transferred to a better school. He made friends, and was advanced to the class level which he had lost. "Mainly, the grandeur of Jerusalem, the most exciting city I have seen, took hold of me", writes he.

And he recalls: "Our mathematics teacher, a distinguished scholar in electrical engineering who later became a professor at the Technion in Haifa, had recently come from Germany and his command of Hebrew was shaky. His ability to teach mathematics was also questionable, but it proved a delightful challenge to me. First I had to retranslate his Hebrew into German... Then I had to try to understand the geometric problem. Instead of the standard Euclidean geometry—which he apparently found boring—he would pose weird construction problems, such as to construct a triangle given the length of one side, one altitude and one angle bisector...".

In 1935 Ernst was transferred again to a new secondary school, Beth Hakerem, where he became the star pupil. His description of his new school is full of praise: "... we were the beneficiaries of the flight of intellectuals from Hitler's Germany which had brought scholars to Jerusalem who were happy to be able to use part of their knowledge as high school teachers... The school was ideal in almost every way... My mathematics teacher was Divshah Amira who had started her thesis research in Goettingen under Hilbert working on the foundations of geometry—though she finished her thesis in Geneva. She taught geometry from notes she had prepared from Hilbert's *Grundlagen der Geometrie*, a curious emphasis on axiomatic formalism which I would not now recommend. Yet in her hands it worked... Many of the traditional high school teachers were critical of Amira's approach, and when Otto Toeplitz came from Germany they appointed him... to investigate... Amira's method. Since Toeplitz spoke no Hebrew she had to pick a pair of German speaking pupils—me and my best friend. We spent a very pleasant two hours with Toeplitz talking about all kinds of mathematics

and he wrote an enthusiastic report... I cannot overstress the debt I owe to Mrs. Amira...”.

Ernst never graduated from high school. Instead, he skipped the last year and after passing special entrance examinations he was admitted to the Mathematics Department of the Hebrew University. At that time Ernst's views on the Jewish-Arab conflict and on Zionism were already quite determined. These views had probably began to form at the age of 12–13 under the “powerful impact” of a small volume of articles by Gandhi from his journal *Young India*. As he put it, “I was fascinated by the thought that there was a rational and moral way to wage a conflict, that it was possible to win your enemy rather than defeat him...”. In 1940, while a student at the Hebrew University, he became involved in the political activities of the League for Jewish-Arab Reconciliation and Collaboration and similar organizations. He developed strong feelings against nationalism of any kind: “I have hardly made it a secret that my attitude towards all nationalism including the Jewish one is not positive”.

In describing his political activities during those days, he writes: “I was in my second year at the Hebrew University and World War II had started in Europe. My interests and energies were centered on the hope for a binational Palestine of peace and friendship between Arabs and Jews... My mother saw the danger to my safety which this cause involved, and also that my education was being neglected and likely to become more precarious as war came closer. She therefore started to urge me to go to America, where my sister Hannah and her husband were already living in New York, and my brother was studying for his Ph.D. ...”.

Ernst's pacifistic and anti-nationalistic convictions led him in later years to raise his voice time after time against America's aggression in Vietnam, the Soviet invasion of Afghanistan, the Israeli interference in Lebanon, and many other issues.

NEW YORK AND PRINCETON

On February 1941, after three semesters at the Hebrew University, Ernst left Palestine on his way to New York. In the fall of that year he became a graduate student at Columbia University, where he received his Master's in June of 1942, and continued towards a doctorate. Within a period of two years he completed all his course requirements—including an unforgettable seminar with Jacques Hadamard who had arrived as a refugee from Paris—when in 1944 Bruns, an old German mathematician, proposed Ernst's

name to Albert Einstein as a replacement for his talented assistant Valentine Bergmann.

Ernst met Einstein in New York, they talked for a while on various physical and mathematical questions, and Ernst found himself hired. When Einstein reached for the phone, recalls Ernst, "the enormity of the thing struck me, and I said, 'I must tell you, I know absolutely no relativity theory'... 'That doesn't matter. I know relativity theory...'". shrugged Einstein.

That fall, Ernst married Louise Miller, a Columbia graduate student in mathematics, and moved to the Institute for Advanced Study in Princeton. He spent the next four years as Einstein's assistant, working with him daily on several aspects of Einstein's unified field theory. These, no doubt, were Ernst's most glorious years. There, Einstein's universal genius had an everlasting imprint on him, shaping his rare scientific and humanistic standards.

During their continuous and intimate collaboration, Einstein and Straus wrote three joint papers on the influence of the expansion of space on the gravitation fields surrounding the individual stars [1, 2], and on a generalization of the relativistic theory of gravitation [3]. At the same time Ernst remained nominally a student of the Mathematics Department at Columbia, and his dissertation, "Some Results in Einstein's Unified Field Theory" [4], earned him his Ph.D. in June of 1948.

At this point, after working for years with the Grand Master and mingling with numerous celebrated physicists like Robert Oppenheimer and Wolfgang Pauli, one could not avoid asking why Ernst went back to mathematics. The answer is, of course, that Ernst never actually left mathematics. First, his work with Einstein was highly mathematical; and then, during his years at the Institute he met and associated with many of the greatest mathematicians of the time (not to mention Einstein's closest friend, Kurt Goedel). In fact, the conflict of choosing between mathematics and physics was quite legitimate even in the eyes of Einstein who always told Ernst that as a young man he hesitated between becoming a mathematician or a physicist. In contrast to Einstein who perceived himself as somebody who had a "nose" for physics, Ernst decided in 1947 that he had a "nose" for number theory.

Ernst made his decision to leave Princeton early in 1948, and in his remaining months at the Institute he became very active mathematically. A breathtaking example of the mathematical involvement that Ernst experienced at Princeton during his last months there, is given in his unpublished 1983 account, "The Elementary Proof of the Prime Number Theorem", describing the evolution of this famous proof by Erdős and Selberg: "Winter and spring of 1948 were a very exciting time in number theory at Princeton. Siegel was back from Germany, such relatively recent arrivals as A. Selberg, S. Chowla and Paul Turán were at the Institute. Most stimulating for me was

the presence of my—by then already long-term friend—Paul Erdős... This led to a number of interesting and stimulating discussions including the now celebrated and still unsettled Erdős-Straus conjecture:

$$\frac{4}{n} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

has a solution in positive integers x, y, z , for all integers $n > 1$.

Turán who was eager to catch up with the mathematical developments that had happened during the war, talked with Selberg about his sieve method and his now famous inequality. He tried to talk Selberg into giving a seminar, ... but Selberg ... declined. He suggested that Turán give the seminar ... This Turán did for a small group of us, including Chowla, Erdős and myself and, I think one or two others ... After the lecture ... Erdős said ‘I think that you can also derive $p_{n+1}/p_n \rightarrow 1$ from this inequality.’ Some skepticism was expressed ... In any case within an hour or two Erdős had discovered an ingenious derivation from Selberg’s inequality. After presenting an outline of his proof to the Turán seminar group, Erdős met Selberg in the hall and told him ... He [Selberg] said: ‘You must have made a mistake, because with this result I can get an elementary proof of the Prime Number Theorem and I have convinced myself that my inequality is not powerful enough for that.’

With that, excitement immediately reached a fever pitch. Erdős and Selberg checked and rechecked every step of their respective proofs and by about 10 PM they had convinced one another. An impromptu lecture was arranged in Fuld Hall. Since my wife was coming from New York ... the whole group (of nearly 50 people) was kind enough to wait until midnight. Then Selberg and Erdős in succession produced their results. There appeared to me to be an atmosphere of great joy, even elation in the room. Many of us, including myself, had the feeling of attending an important historical event ...”.

Despite his growing mathematical interest, Ernst continued to be fully involved in his work with Einstein until he left Princeton. Before his departure from the Institute Einstein wrote about him: “I like to work with him so much that I anticipate with greatest regret the day when he will leave ...”.

LOS ANGELES

In the fall of 1948 Ernst and his wife Louise moved to Los Angeles, where Ernst joined the Mathematics Department at UCLA as Instructor. He became Assistant Professor in 1950, Associate Professor in 1955, and Professor in 1960. His older son Daniel Albert (named after Einstein) was born in 1954,

and the youngest, Paul Elias (named after Ernst's father), in 1957.

During his long career, Ernst had at least 151 publications (some might have been lost), on a surprising variety of subjects including number theory, analytic functions, combinatorics, geometry, and algebra. His current publication list consists of 139 research papers, his 1949 English translation of *Partial Differential Equations in Physics* by A. Sommerfeld [6], two undergraduate texts on analytic geometry with Paul J. Kelly [67, 68], the coediting with William J. LeVeque of a Proceedings Volume in Number Theory [72], one book review [131], 6 articles on Einstein [22, 23, 117–119, 126], and a lovely note in honor of Paul Erdős [143].

His remarkable broad knowledge, his personal charm and his insatiable desire to discuss almost any kind of mathematics with almost anybody at almost any time, led him to collaborate with an amazing number of 70 coauthors—from students to distinguished mathematicians—the record being 19 joint papers with Erdős. His scientific qualities and strong love for education endowed him with a long list of 23 students who received their Ph.D. under his instruction between 1958 and 1978.

Ernst's unshakable moral convictions, his undeterred commitment to human rights, his uncompromising devotion to principles, and his rhetorical powers, directed him towards serving on important University of California committees such as Academic Freedom Committee (statewide, 1961–64; UCLA Chairman, 1963–64), University Welfare Committee (1964–66), UCLA Legislative Assembly (1967–68, 1971–73), and Privilege and Tenure Committee (1972–75, Chairman 1972–74). He was also the 1965–66 president of the UCLA Chapter of the American Association of University Professors, and an Appointed Member of the Ethics Committee of the American Mathematical Society. In addition, he was on the editorial board of the *Pacific Journal of Mathematics* (1951–64), serving as Managing Editor in 1954–59.

For all his mathematical and professional contributions, Ernst was promoted in 1977 to an Above-Scale Status which according to the policy of the University of California is granted to “scholars and teachers of the highest distinction, whose work has been internationally recognized and acclaimed. Such recognition is comparable...to appointment at other institutions to named chairs.”

Ernst's scientific work was also acknowledged by the National Science Foundation which supported him from 1960 until his death. When his NSF grant was last renewed in 1982 he renounced his summer salary, claiming that in view of the growing difficulties in obtaining such support the funds should be spent amongst younger, more needy mathematicians.

Many agree that Ernst's most striking work was on the arithmetic and algebraic properties of analytic functions. David Cantor—a UCLA professor and a former Ph.D. student and collaborator of Ernst [69, 132, 141]—had the

following account on this aspect of Ernst's work at a UCLA Memorial Service held in October 1983: "...It was this work which first established him as an independent, world-class number theorist... In his first paper on the subject ([7]), published in 1950, Ernst introduced the use of Riemann surfaces into this subject, and among many other things, gave a new proof of the transcendence of numbers of the form e^a , where a is nonzero and algebraic. It introduced the idea, of which he made much fruitful use in later years, of considering the Taylor expansion of an analytic function around several points simultaneously... This first paper showed that this field, which he liked to call 'the study of arithmetic of analytic functions' was worthy of study on its own account. It has had a substantial influence on numerous works of other mathematicians, sometimes without their recognizing so... It recently had a major resurgence, especially in the French School of Diophantine Approximation...

The same sorts of ideas occur in many other papers of Ernst, such as on PV and Salem numbers. He was quite interested in extending his results to p -adic analytic functions and his intuition on these matters was quite remarkable..."

Ernst's study of the arithmetic of analytic functions continued to occupy him for many years (e.g. [8, 52, 57, 76]) and reached its peak in his celebrated papers "Differential Rings of Meromorphic Functions" [80], and "Differential Rings of Analytic Functions of a Non Archimedean Variable" [82].

Another area in which Ernst was a world known expert is combinatorics. As Erdős conceives it, Ernst's most original and profound contribution in this area is their joint work with Ron Graham, Peter Montgomery, Bruce Rothschild and Joel Spencer [85, 97, 98] on Euclidean Ramsey Theorems. Ernst had a crucial role in developing this new field, and Erdős in his reminiscences (to appear in the Proceedings of the 1984 Edmonton Number Theory Conference, *Rocky Mountain Journal*, June 1985) writes: "...I hope that these results, and even more the problems, will outlive the authors... by centuries..."

Two papers by Erdős and Straus—one of them in collaboration with John Selfridge—were supposed to be in Ernst's possession shortly before his death, but were apparently lost. In fact, apart from these lost papers, at the time of his death Ernst was working on 7 different articles [145–151], with 9 collaborators. Other papers may have been lost as well, since good order was not one of Ernst's most prominent virtues.

Ernst's extraordinarily broad mathematical scope makes it quite difficult to give a complete account of his work. But let me briefly mention his articles with his friend, colleague, and former teacher at the Hebrew University Theodore Motzkin on combinatorics and a variety of analytic problems (e.g. [24, 47, 58]); his papers in geometry with Paul Kelly (e.g. [25, 40, 66]); his

papers on convexity with Fred Valentine [13, 14]; his papers with Sato on entire functions and interpolation (e.g. [52, 57, 61]); his highly technical articles in analytic number theory with Gregori Kolesnik [112, 123, 139]; and our own joint work on numerical ranges, numerical radii and submultiplicativity of operator norms, which has appeared mainly in *Linear Algebra and Its Applications* (e.g. [110, 115, 129]), and which apparently played some role in the nomination of Ernst to Associate Editor of the Report Section of this journal in 1983.

EPILOGUE

At Ernst's UCLA Memorial Service last fall, Paul Kelly of Santa Barbara—who first met Ernst when they were both visiting the Institute for Advanced Study in 1955—gave the following touching and modest description of his acquaintance and collaboration with Ernst: "...I would think of a problem and I would ask Ernst about it. He would usually suggest some attack on it. I would use his idea and get some results, occasionally actually solving it. Then I would turn it to Ernst... He would then generalize it, and from that point on I never understood it...". And he goes on: "When I had an opportunity to write a book on a new geometry... Ernst was extremely supportive and acted as a backup for me... so I felt like I had God in my corner... To me he was just absolutely a marvelous person. He was the way I think human beings were meant to be, bright, kind, humane, humorous, knowledgeable, gentle. I just don't know how you could design anybody any better than that. Maybe a little weak on sports... I think I felt about Ernst in the way Ernst felt about Einstein...".

Responses to Ernst's unexpected decease of a massive heart attack at the age of 61 came in multitudes from former students, collaborators and friends alike. His 1966 Ph.D. student John Ewell writes: "Mere words fail to express the sense of grief which I feel over Professor Straus' death...". Krishnaswami Alladi, who was the last student to receive his Ph.D. from Ernst in 1978, cabled: "On behalf of Mathematical Sciences let me pay tribute to Professor Straus, my great teacher, rare combination of mathematical eminence with largeness of heart...".

Steve Diliberto of Berkeley, an old collaborator of Ernst [9, 10], put it this way: "...How lucky I felt to have known him. Such a warm, sensitive—and very clever fellow... Because of all the mathematicians there is none for whom I can say so quickly and surely 'there is a truly good person. I am better for knowing him.'".

Finally, Ray Redheffer of UCLA—another old friend and collaborator [51, 144]—chose to express his reflections at Ernst’s Memorial by simply quoting the following moving lines from a sonnet by Edna St. Vincent Millay:

There are a hundred places where I fear
 To go,—so with his memory they brim.
 And entering with relief some quiet place
 Where never fell his foot or shone his face
 I say, “There is no memory of him here!”
 And so stand stricken, so remembering him.

Let Ernst’s memory be blessed.

LIST OF PUBLICATIONS

1. (With Albert Einstein) The influence of the expansion of space on the gravitation fields surrounding the individual stars, *Rev. Modern Phys.* 17:120–124 (1945).
2. (With Albert Einstein) Corrections and additional remarks to our paper: The influence of the expansion of space on the gravitation fields surrounding the individual stars, *Rev. Modern Phys.* 18:148–149 (1946).
3. (With Albert Einstein) A generalization of the relativistic theory of gravitation, II, *Ann. of Math.* 47:731–741 (1946).
4. Some results in Einstein’s unified field theory, *Rev. Modern Phys.* 21:414–420 (1949).
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