ELSEVIER

Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 14 (2011) 450–459

Procedia Engineering

www.elsevier.com/locate/procedia

The Twelfth East Asia-Pacific Conference on Structural Engineering and Construction

How to Install Sensors for Structural Model Updating?

Lam HF, Yang JH[a] and Hu Q

Department of Building and Construction, City University of Hong Kong

Abstract

Structural model updating can be treated as the process to extract information from measurement for modifying the structural model such that the model calculated responses fit the measured data. The updated model is very important for structural response prediction, structural damage detection and structural control. The location for sensor installation has significant effect on the amount of information that can be extracted from the measured data. This paper presents a methodology for identifying the "optimal" locations to install a given number of sensors on a structure so as to find useful information for structural model updating. The proposed method relies on the information entropy as a measure of the uncertainties associated with the identified model parameters for a given sensor configuration. The larger the value of information entropy, the higher the uncertainty of the identified model parameters will be. As a result, the problem of optimal sensor placement can be transformed to a discrete optimization problem with the information entropy as the objective function and the sensor configuration as the minimization variable. However, the corresponding numerical minimization problem is computational demanding for real structures with many degrees of freedom (DOFs). One of the contributions of this paper is to propose a computational efficient optimization method based on genetic algorithm for solving this minimization problem. A model of typical transmission tower with 40 nodes, 160 elements and 216 DOFs is used as a numerical example to illustrate the proposed methodology. The computational time of the proposed optimization method can be future reduced by making use of parallel computing technologies.

Keywords: Optimal sensor placement, structural health monitoring, information entropy, transmission tower, genetic algorithm.

[a] Corresponding author and presenter: Tel.: +852 97903211; E-mail address: oldsky007@gmail.com.

1877–7058 © 2011 Published by Elsevier Ltd. Open access under CC BY-NC-ND license.
doi:10.1016/j.proeng.2011.07.056

1. Introduction

Structural model updating is important for structural health monitoring, structural control and structural reliability analysis, and many methods have been developed. Due to the problems of modeling error, measurement noise, and "incomplete" measurement, the results of structural model updating are uncertain in nature. Therefore, many methods have been developed following the probabilistic approach. The performance of structural model updating utilizing measured dynamic characteristics depends very much on the quantity and quality of the measured data, which in turn depends on the number of sensors used and their corresponding locations. It is thus critical for researchers or engineers to define the sensor configuration before any field tests.

The problem of optimally placing sensors was perhaps first investigated by Shah and Udwadia (Shah & Udwadia 1978). They considered a linear relationship between small perturbations in a finite dimensional representation of the structural parameters and a finite sample of observations of the system time response, and formulated the optimal sensor locations as an optimization problem to minimize the error in the parameter estimates. Many other researchers have studied the issue of optimally installing a given number of sensors on a target structure for the purpose of model and modal identification. In 2000, Papadimitriou and colleagues (Papadimitriou et al. 2000) introduced the information entropy measure, which is a direct measure of the uncertainties in model parameter estimates that when minimized can be used to determine the optimal sensor locations. Traditional numerical optimization algorithms are not applicable, as this is a discrete optimization problem with only a limited number of sensors. Genetic algorithms have proved to be well suited for the solution of discrete optimization problems, and thus in this paper a computationally efficient optimization method is developed that is based on the genetic algorithm (GA) concept. The main objective of this paper is to address the problem of optimally placing sensors on a typical transmission tower for the purpose of structural model updating.

2. The Bayesian statistical framework

The optimal sensor placement method is developed from the Bayesian statistical framework. Owing to the limited space in this paper, only the main equations are reviewed here. Interesting readers are redirected to reference (Chow et al. 2010) for the detail formulation. Consider a class of structural models **M** with model parameters $\underline{\theta}$, which is used to represent the input-output behavior of a structure. For a specific **M**, let $q(n;\underline{\theta}) \in \mathbf{R}^{N_d}$ be the model output vector at time $t_n = n\Delta t$, where N_d is the total number of DOFs of the system. The system output $\underline{y}(n) \in \mathbf{R}^{N_o}$ at time t_n of the N_o observed DOFs is

$$\underline{y}(n) = \underline{S}_0 \left[\underline{q}(n;\underline{\theta}) + \underline{e}(n;\underline{\theta}) \right], \tag{1}$$

where $\underline{S}_0 \in \mathbf{R}^{N_o \times N_d}$ is a selection matrix with only one "1" in each row to show the corresponding measured DOF; $\underline{e}(n;\underline{\theta})$ is the prediction error. For the purpose of optimal sensor placement, the N_o observed DOFs are specified using a sensor configuration vector that is expressed as

$$\underline{\delta} = \underline{S}_0 \hat{\underline{u}}, \tag{2}$$

where $\hat{\underline{u}} \in \mathbf{R}^{N_d}$ is a vector with continuous integers from 1 to N_d, $\hat{\underline{u}} = \{1, 2, 3, \ldots, N_d\}$, and $\underline{\delta} \in \mathbf{R}^{N_o}$. It is clear that the sensor configuration vector shows the DOF numbers, at which sensors are installed. The optimal structural model parameters $\underline{\theta}$ are determined by minimizing $J(\underline{\theta})$ with respect to $\underline{\theta}$. $J(\underline{\theta})$ is given by

$$J\left(\underline{\theta}\right)=\frac{1}{NN_o}\sum_{n=1}^{N}\left\|\underline{y}\left(n\right)-\underline{S}_0\underline{q}\left(n;\underline{\theta}\right)\right\|^2 , \tag{3}$$

The posterior PDF of the parameters $\underline{\theta}$ can be obtained using the asymptotic approximation

$$p\left(\underline{\theta}\middle|\underline{\delta},D,\mathbf{M}\right)=c\,J\left(\underline{\theta}\right)^{-\frac{NN_o-1}{2}}\pi\left(\underline{\theta}\right), \tag{4}$$

Assuming a non-informative prior distribution $\pi(\underline{\theta})$ and a large number of observed data points N, which is usually the case in dynamic tests, the posterior PDF peaks markedly at $\hat{\underline{\theta}}$, which is the optimal parameter $\underline{\theta}$. In the globally identifiable case, the posterior PDF can be approximated by a weighted sum of multivariate Gaussian distributions with a mean $\hat{\underline{\theta}}$ and a covariance matrix $\mathrm{A}^{-1}\left(\hat{\underline{\theta}}\right)$, where $\mathrm{A}\left(\hat{\underline{\theta}}\right)$ matrix is the Hessian matrix of the function $g(\underline{\theta})=\left[(NN_o-1)\ln J(\underline{\theta})\right]/2$ evaluated at $\hat{\underline{\theta}}$.

3. Information entropy

Assuming that the most probable value of $\underline{\theta}$ is $\hat{\underline{\theta}}$, which can be identified by maximizing the posterior PDF in Equation (4). The information entropy H provides a unique measure of the uncertainties associated with the estimates of $\underline{\theta}$, depends on $\hat{\underline{\theta}}$, D and M, and more important is the sensor configuration $\underline{\delta}$. It can be expressed as the mathematical expectation of the function $-\ln p\left(\underline{\theta}\middle|\hat{\underline{\theta}},\underline{\delta},\mathbf{M}\right)$ with respect to $\underline{\theta}$. Based on the asymptotic approximation, the information entropy can be simplified as:

$$H\left(\hat{\underline{\theta}},\underline{\delta},\mathbf{M}\right)=\frac{1}{2}N_{\underline{\theta}}\left[\ln(2\pi)+1+\ln\hat{\sigma}^2\right]-\frac{1}{2}\ln\det Q(\underline{\delta},\hat{\underline{\theta}}) \tag{5}$$

Owing to the space limitation in this conference paper, detail expression of the Q matrix will not be given here. Interesting readers are redirected to reference (Jaynes 1978) for the detail formulation. Now, the optimal sensor placement problem for a given model class can be converted into a minimization problem with the information entropy given in Equation (5) as the objective function and the sensor configuration vector $\underline{\delta}$ as the minimization variables. However, direct minimization of the information entropy may causes numerical problem as its order of magnitude can be very large. To solve this problem, only the relative values of H are considered in this study. Consider a reference sensor configuration $\underline{\delta}_0$ with an information entropy of H_0. From Equation (5), the change in information entropy for the sensor configurations $\underline{\delta}$ and $\underline{\delta}_0$ can be expressed as

$$H-H_0=\frac{1}{2}\ln\frac{\det Q(\underline{\delta}_0,\hat{\underline{\theta}})}{\det Q(\underline{\delta},\hat{\underline{\theta}})}. \tag{6}$$

Let s^2 be the geometrical mean of the eigenvalues of the covariance matrix $\hat{\sigma}^2(\hat{\underline{\theta}})Q(\underline{\delta},\hat{\underline{\theta}})^{-1}$ of the distribution $p\left(\underline{\theta}\middle|\hat{\underline{\theta}},\underline{\delta},\mathbf{M}\right)$, which represents the overall spread of the distribution about the mean value of the structural model parameters. For the two distributions corresponding to the sensor configurations $\underline{\delta}$ and $\underline{\delta}_0$, the parameter-uncertainty ratio can be shown as

$$\frac{s}{s_0}=\exp\left(\frac{H-H_0}{N_{\underline{\theta}}}\right), \tag{7}$$

In this study, this ratio is employed to measure the change in uncertainty between two sensor configurations. By selecting $\underline{\delta}_0$ to be the full sensor configuration, the minimum value of the ratio in Equation (7) is unity. In the numerical case study, s_0 corresponds to the case in which all horizontal DOFs of the model are instrumented.

4. Optimization method for optimal sensor placement

A computationally efficient numerical optimization method is developed based on a genetic algorithm (GA) to significantly reduce the computational time required in searching for the "optimal" sensor configuration. Consider the general case of optimally placing N_s sensors on a structural system of N_d DOFs. In total, $N_t = C_{N_s}^{N_d}$ different sensor configurations are required for an exhaustive search, where C is the binomial combinatorial coefficient. For structures with a large number of DOFs, an exhaustive search would be extremely time consuming or even impossible. The proposed optimization method offers an alternative means of obtaining the optimal or near optimal solutions by exploring just a small portion of the total parameter space. The proposed optimization method repeatedly modifies a population of individual solutions. At each step, the method selects individuals from the current population according to their objective function values to be parents, and uses them to produce children for the next generation. The population then converges toward an optimal solution over successive generations.

In the original genetic algorithm, the minimization variables must be encoded into bit strings (i.e., chromosomes with "0"s and "1"s). In the problem of sensor placement, the minimization variables are the sensor locations and the size of the parameter space varies for different structures and different number of available sensors. The mapping between the physical minimization variables and the chromosomes is one big difficulty in the application of genetic algorithm in addressing the optimal sensor placement problem. In order to overcome this difficulty, an individual that represents a particular sensor configuration consists of N_s elements, each of which is used to represent the instrumented (or measured) DOF, is adopted in the proposed optimization method. This is the same as the sensor configuration vector given in Equation (2). For instance, the individual [3 15 21] represents a case with three sensors that are installed at DOFs 3, 15, and 21. Furthermore, the encoding procedure is not employed in the proposed method, and therefore, the genetic algorithm operators (e.g., crossover and mutation operators) are applied directly on the individuals.

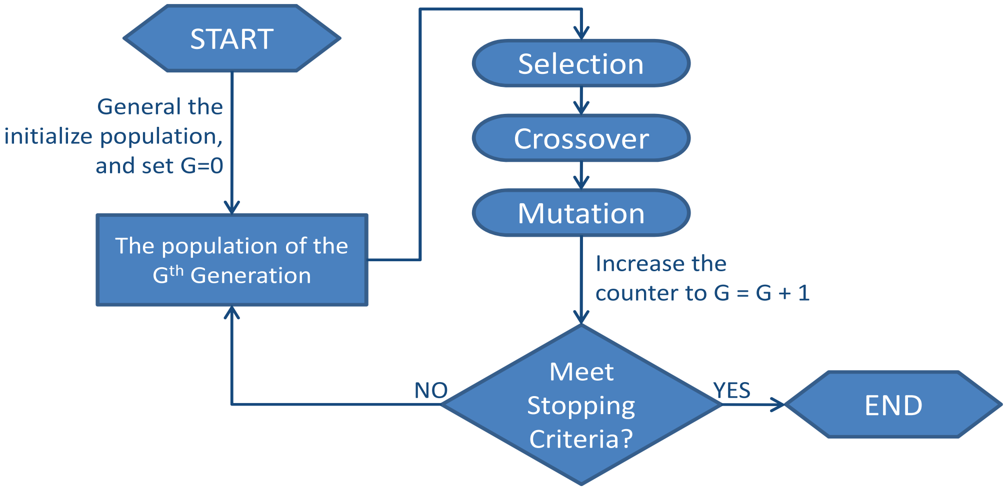


Figure 1. Flow chart of the proposed numerical optimization method

Figure 1 briefly illustrates the procedure of the proposed optimization method. To start with, an initial population of genetic strings of a fixed size is randomly generated to represent various possible sensor configurations. The parameter-uncertainty ratio in Equation (7) is the fitness function to be minimized in the optimization process. The raw fitness scores returned by the fitness function are converted to selection indexes within a given range before entering the selection process. This conversion is based on the rank of each individual, which is determined by the position of an individual in the sorted raw fitness scores (the rank of the fittest individual is 1, the next fittest is 2, and so on). The selection index of an individual with rank n is directly proportional to $1/\sqrt{n}$, and the sum of the selection indexes of the entire population equals the number of parents needed to create the next generation. The selection process then uses the selection indexes to select the parents of the next generation from the current generation. The use of selection indexes removes the effect of the spread of the raw fitness scores. A higher probability of selection is assigned to individuals with a higher selection index value, or a lower value of the ratio s/s_0 . An individual can be selected more than once as a parent, in which case it contributes its genes to more than one child. The selection process lays out a line on which each individual corresponds to a segment of the line with a length proportional to its selection index value. The method then selects parents by moving along the line in steps of equal size, allocating parents from the sections on which it lands. Consequently, an individual with a larger selection index will have a greater chance of being selected.

In each generation, the two parents with the best two fitness values automatically survive to the next generation. The rest of the parents are then bred using the crossover and mutation operators with pre-defined probabilities. Crossover operators specify how two parents are combined to form a child for the next generation. Consider the two parents P_1 and P_2 as an example.

$$P_1=\left[1\ 2\ 3\ 4\ 5\ 6\right],\ P_2=\left[\mathrm{a\ b\ c\ d\ e\ f}\right]. \tag{8}$$

A random binary vector is generated for selecting genes from the two parents according to the rule that the genes are from the first (second) parent if the corresponding elements are equal to 1 (0). Thus, if the binary vector is [1 1 0 0 1 0], then the child will be

$$\text{Child}=\left[1\ 2\ \mathrm{c\ d}\ 5\ \mathrm{f}\right]. \tag{9}$$

Crossover enables the method to extract the best genes from different individuals and recombine them into potentially superior children. Mutation children are created by introducing random changes into a single parent within a feasible range. The mutation process adds to the diversity of a population, and thus reduces the chance that the optimization process will become trapped in local optimal regions. With each successive generation, the population is selectively updated and moves closer to the optimal solution. The process continues from one generation to the next until any one of the stopping conditions of the method (the maximum number of generations specified or the specified time interval in which there is no improvement in the objective function) is fulfilled.

One of the advantages of the proposed optimization method is that all function evaluations in a given generation can be carried out independently (i.e., the calculations of the selection indexes for individuals in a given population). As a result, parallel computing technique can be applied to the proposed method to further reduce the required computational time. Note that multi-core processors are commonly used nowadays in desktop and even laptop computers (i.e., Dual-core and Quad-core processors). This advantage is particular useful and make the entropy-based optimal sensor placement approach practical in real situations. Although it is unlikely, it is possible to have an individual with two or more sensors installed in the same DOF (e.g., [3 15 15 21] has two sensors install at DOF 15). Since the information can be obtained by this individual (with only three measured DOFs) is much less than others (with four measured DOFs), it will not be able to survive in the next generation. Therefore, the proposed algorithm does not have any mechanism to prevent this kind of individual from generating.

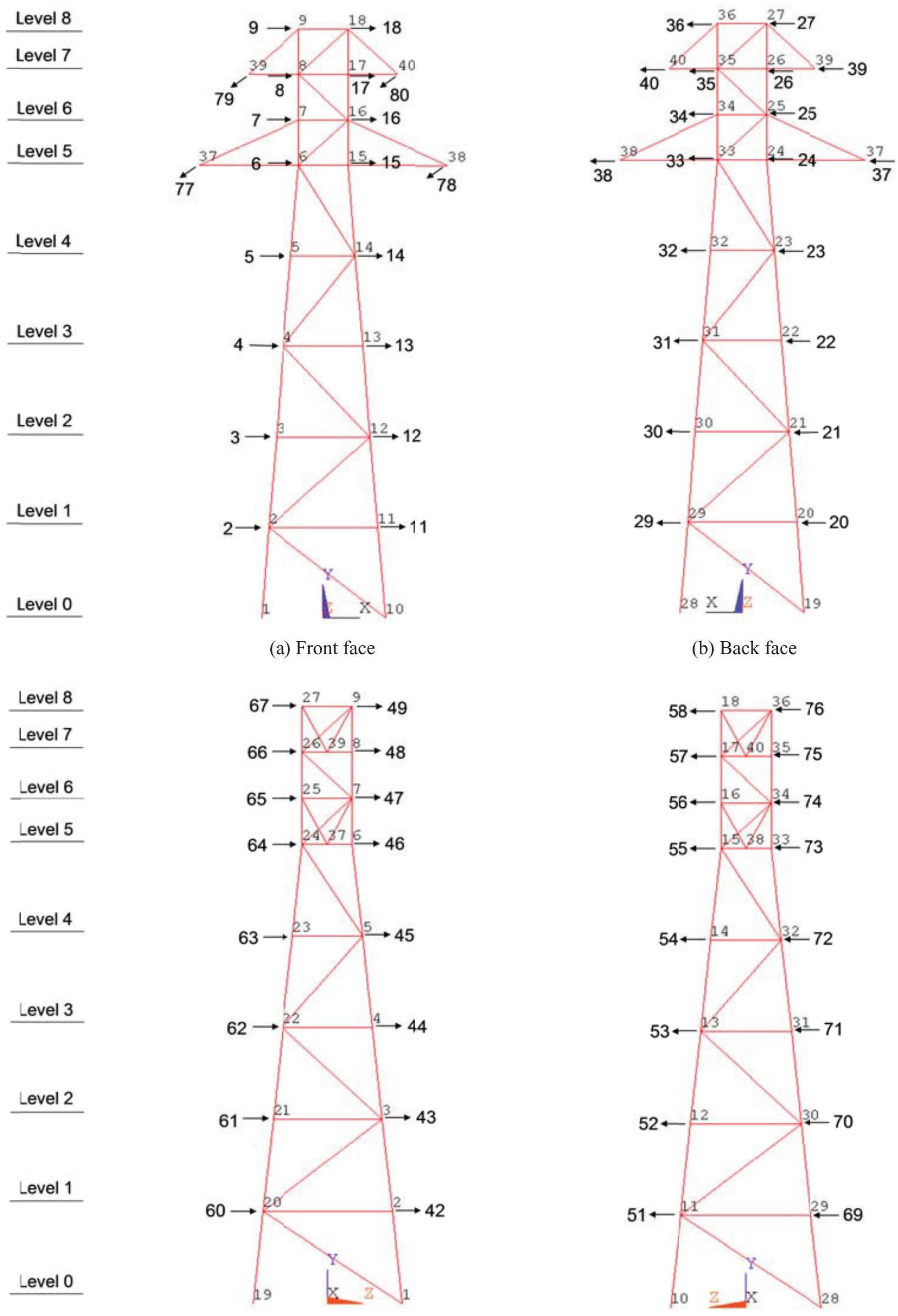
5. Numerical case study

To numerically test the method, a three-dimensional finite element (FE) model of a transmission tower is developed in ANSYS® (see Figure 2). In the case study, only the stiffness of the braces is considered to be uncertain and is thus included in the identification process. For each level, the stiffness of the braces on the front and back faces is assumed to be the same, as is the stiffness of the braces on the left and right faces. Following this arrangement, only two model parameters are required to scale the stiffness of the braces at each level. For the purpose of optimal sensor configuration, all of the model parameters are assumed to be unity, and the system is assumed to have ambient excitation at all translational DOFs. Although the structure has a large number of DOFs, only translational DOFs are considered for possible sensor installation in this case study.

For the single-sensor case, an exhaustive search requires only 72 function evaluations and it is not necessary to apply the proposed optimization method. The information entropy measures for the various sensor locations for this case as calculated by Equation (7) and summarized in Table 1. It can be observed from the table that the information entropy measure varies greatly, with the smallest value being about 8.10 (at DOFs 67 and 27, see Figure 2) and the largest value about 38.30 (at DOFs 40 and 80). The large difference in the information entropy measure highlights the importance of selecting a suitable location for sensor installation. Note again that the smaller the information entropy measure s/s_0, the lower the uncertainty in the identification results and the better the location for installing a sensor.

Table 1: Information entropy measure for a sensor at different DOFs

measured DOF	67	27	5	45	32	72	58	18
entropy measure	8.1031	8.1031	8.1665	8.1665	8.2239	8.2239	8.245	8.245
measured DOF	14	54	23	63	76	36	49	9
entropy measure	8.3598	8.3598	8.4237	8.4237	8.4388	8.4388	8.5652	8.5652
measured DOF	30	70	3	43	22	62	13	53
entropy measure	9.2339	9.2339	9.2607	9.2607	9.443	9.443	9.5022	9.5022
measured DOF	21	61	12	52	31	71	4	44
entropy measure	9.5466	9.5466	9.5904	9.5904	9.7338	9.7338	9.8281	9.8281
measured DOF	66	26	57	17	64	24	73	33
entropy measure	11.4401	11.4401	11.5788	11.5788	11.6253	11.6253	12.1192	12.1192
measured DOF	75	35	55	15	11	51	20	60
entropy measure	12.1201	12.1201	12.1224	12.1224	12.1702	12.1702	12.2888	12.2888
measured DOF	48	8	2	42	46	6	29	69
entropy measure	12.295	12.295	12.6988	12.6988	12.7158	12.7158	12.8097	12.8097
measured DOF	74	34	47	7	65	25	56	16
entropy measure	14.8729	14.8729	15.1388	15.1388	15.8692	15.8692	16.0981	16.0981
measured DOF	37	77	38	78	39	79	40	80
entropy measure	24.5756	24.5756	24.685	24.685	37.8736	37.8736	38.2967	38.2967



(a) Front face (b) Back face

(c) Left face (d) Right face

Figure 2: Nodes and DOFs of the transmission tower model

According to the results in Table 1, the good sensor locations follow a pattern, and the transmission tower can be divided into several parts according to the value of the information entropy measure. The information entropy measure of the DOFs at Levels 4 and 8 (see Figure 2) of the tower ranges from 8.10 to 8.56, from which it can be concluded that the best sensor locations are at the top and middle parts of the structure if only one sensor is available. Levels 2 and 3 can be regarded as "second-best" on the structure for sensor placement. Levels 1, 5, 6, and 7 of the tower form a possible "third-best" location for sensor placement. The remaining part runs from DOF 37 to DOF 80, which are the DOFs at the cross arms of the transmission tower, thus implying that the cross arms are the worst locations for sensor placement when only one sensor is available.

To better understand the sensor configuration with respect to the geometry of the structure, the possible DOFs are divided into nine groups based on their locations. Groups 1 to 8 refer to the DOFs on Levels 1 to 8, respectively, excluding the DOFs at the cross-arms. The DOFs on the cross-arms are included in Group 0 (i.e., DOFs 37, 38, 39, 40, 77, 78, 79 and 80).

It must be remembered that the results for single sensor placement may not apply to cases with double or multiple sensors. The proposed optimization method is thus required when more than one sensor are available for the measurement. We summarized the entropy measures for the best fifty sensor configurations for the two-sensor case in Figure 3. Each sensor configuration corresponds to two points in Figure 3. This figure provides a graphical way for researchers to clearly identify the groups (or levels) that are favor for sensor installation by simply looking at the left most points on the figure. It is clear that Groups (or Levels) 4 and 8 are the most suitable locations for installing two sensors. Levels 2 and 3 are also good locations if two sensors are available.

Using the proposed optimization method, the optimal sensor locations for cases with 3 and 4 sensors are identified and summarized in Figures 4 and 5, respectively. Similar to Figure 3, each sensor configuration has a value of information entropy measure, and corresponds to points along the same vertical line in the figures. The figures clearly show that Groups (or Levels) 2, 3, 4, and 8 are always the best sensor locations. Note that these figures do not contain all of the possible sensor configurations due to the huge number of possible combinations. There are $C_3^{72} = 59,640$ possible sensor configurations for the three-sensor case, for example. The proposed numerical optimization method provides a convenient and feasible way to estimate sensor configurations that are close to optimal.

6. Conclusions

With the help of the proposed genetic algorithm based optimization method, the entropy-based optimal sensor placement approach is successfully applied to solve the sensor placement problem of a typical transmission tower. When only one sensor is available, the optimal location for this sensor is at the top (Level 8) of the tower. When two sensors are available, it is best to place one at the top (Level 8) and the other in the middle (Levels 2, 3 or 4) of the tower. Such conclusions cannot be extended to cases with more than two sensors due to the complexity of the sensor configurations. However, it can be concluded that the top (Level 8) and the middle (Levels 2, 3, and 4) of the tower are generally good locations for sensor placement. The cross arms of the tower should be avoided for sensor installation if the purpose of model updating is to identify the stiffness of the braces, because they always give large information entropy values, which result in greater uncertainty in the model identification results. The proposed optimization method significantly reduces the required computational power in identifying the optimal or close-to-optimal sensor configurations. As the function evaluations in a given generation is independent to each other, parallel computing techniques can be easily adopted to further increase the efficiency of the proposed method. The numerical case study in a commonly used desktop computer shows that the computational time can be reduced by about 40% with one additional processor.

Acknowledgements

The work described in this paper was fully supported by a grant from the City University of Hong Kong (7002589). This generous support is gratefully acknowledged.

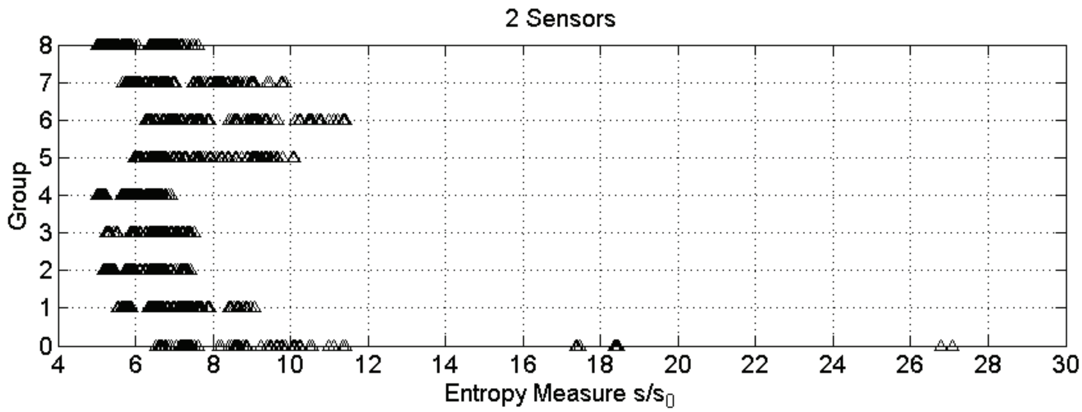


Figure 3: Sensor locations for the two-sensor case

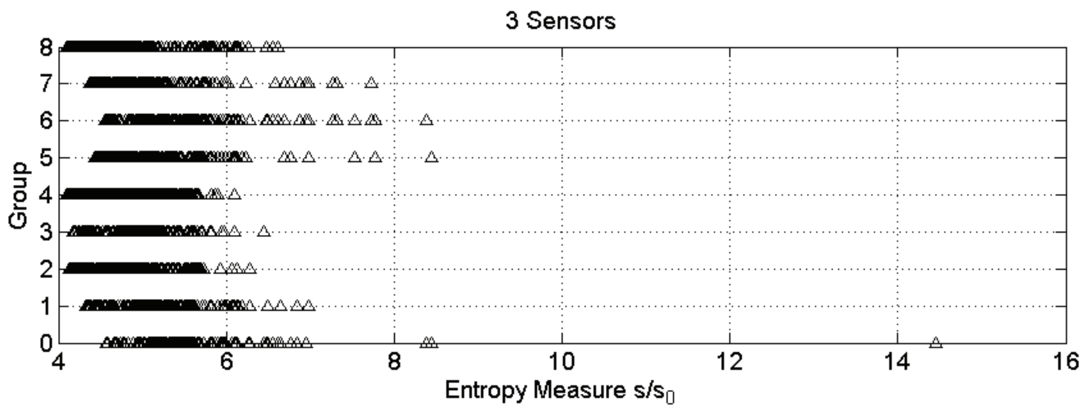


Figure 4: Sensor locations for the three-sensor case

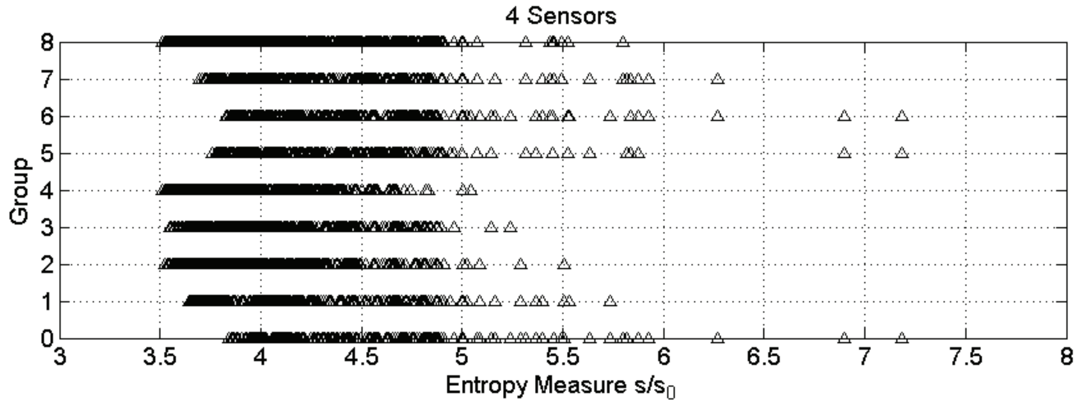


Figure 5: Sensor locations for the four-sensor case

References

[1] Beck JL and Katafygiotis LS. *Updating models and their uncertainties – Bayesian statistical framework*. Journal of Engineering Mechanics. 124(4), 1998, pp. 455-461.

[2] Chow HM, Lam HF, Yin T and Au SK, *Optimal sensor configuration of a typical transmission tower for the purpose of structural model updating, Structural Control and Health Monitoring*, In press; 2010

[3] Heredia-Zavoni E and Esteva L. *Optimal instrumentation of uncertain structural systems subject to earthquake ground motions*. Earthquake Engineering and Structural Dynamics. 27, 1998, pp. 343-362.

[4] Jaynes ET. *Where do we stand on maximum entropy?* The Maximum Entropy Formalism. Levine RD and Tribus M eds. Cambridge: MIT Press; 1978

[5] Katafygiotis LS, Papadimitriou C and Lam HF. *A probabilistic approach to structural model updating*. International Journal of Soil Dynamics and Earthquake Engineering. 17, 1998 pp. 795-507.

[6] Lam HF, Ko JM and Wong CW. *Localization of damaged structural connections based on experimental modal and sensitivity analysis*. Journal of Sound and Vibration. 210(1), 1998, pp. 91-115.

[7] Papadimitriou C, Beck JL and Au SK. *Entropy-based optimal sensor location for structural model updating*. Journal of Vibration and Control. 6(5), 2000, pp. 781–800.

[8] Shah P and Udwadia FE. *A methodology for optimal sensor locations for identification of dynamic systems*. Journal of Applied Mechanics. 45, 1978, pp. 188–196.