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Reissner–Nordström metric in the Friedman–Robertson–Walker universe

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Abstract

The metric for a Reissner–Nordström black hole in the background of the Friedman–Robertson–Walker universe is obtained. Then we verified it and discussed the influence of the evolution of the universe on the size of the black hole. To study the problem of the orbits of a planet in the expanding universe, we rewrote the metric in the Schwarzschild coordinates system and deduced the equation of motion for a planet.

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1. Introduction

Black holes have been investigated in great depth and detail for more than forty years. However, almost all previous studies have focused on isolated black holes. On the other hand, one cannot rule out the important and more realistic situation in which black holes are actually embedded in the background of universe. Therefore, black holes in non-flat backgrounds form an important topic.

As early as in 1933, McVittie [1] found his celebrated metric for a mass-particle in the expanding universe. This metric gives us an concrete example for a black hole in the non-flat background. It is just the Schwarzschild black hole which is embedded in the Friedman–Robertson–Walker universe although there was no the notion of black hole at that time. In 1993, the multi-black hole solution in the background of de Sitter universe was discovered by Kastor and Traschen [2]. The Kastor–Traschen solution describes the dynamical system of arbitrary number of extreme Reissner–Nordström black holes in the background of de Sitter universe. In 1999, Shiromizu and Gen

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extended it into the spinning version [3]. In 2000, Nayak et al. [4,5] studied the solutions for the Schwarzschild and Kerr black holes in the background of the Einstein universe.

In this Letter, we extend the McVittie's solution into charged black holes. We first deduce the metric for a Reissner–Nordström black hole in the expanding universe; several special cases of our solution are exactly the same as some solutions discovered previously. In the previous work [6] we have applied the asymptotic conditions to derive the Schwarzschild metric in the expanding universe, which is exactly the same as that derived by McVittie by solving the full Einstein equations. That demonstrates the power of this simple and straight-forward approach. In this Letter we follow the same procedure to derive the metric for the Reissner–Nordström black holes in Friedman–Robertson–Walker universe. We then study the influences of the evolution of the universe on the size of the black hole. Finally, in order to study the motion of the planet, we rewrite the metric from the cosmic coordinates system to the Schwarzschild coordinates system.

2. Derivation of the metric

The metric of Reissner–Nordström black hole in the Schwarzschild coordinates system is given by

$$d\tilde{s}^2 = -\left(1 - \frac{2M}{\tilde{r}} + \frac{Q^2}{\tilde{r}^2}\right)d\tilde{t}^2 + \left(1 - \frac{2M}{\tilde{r}} + \frac{Q^2}{\tilde{r}^2}\right)^{-1} d\tilde{r}^2 + \tilde{r}^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right), \tag{1}$$

where M and Q are the mass and charge of the black hole, respectively.

For our purpose we rewrite the metric Eq. (1) in the isotropic spherical coordinates. We assume that $x^0 = v$ and $x^1 = x$. So make variables transformation

$$\tilde{t} = 2v, \qquad \tilde{s} = 2l, \qquad 2\tilde{r} = x\left(1 + \frac{M}{x}\right)^2 - \frac{Q^2}{x^2}, \tag{2}$$

then we can rewrite Eq. (1) as follows

$$dl^2 = -\frac{\left(1 - \frac{M^2}{x^2} + \frac{Q^2}{x^2}\right)^2}{\left[\left(1 + \frac{M}{x}\right)^2 - \frac{Q^2}{x^2}\right]^2} dv^2 + \left[\left(1 + \frac{M}{x}\right)^2 - \frac{Q^2}{x^2}\right]^2 \left(dx^2 + x^2\, d\theta^2 + x^2 \sin^2\theta\, d\phi^2\right). \tag{3}$$

As is known, the metric for the FRW (Friedman–Robertson–Walker) universe is given by

$$dl^2 = -dv^2 + \frac{a^2(v)}{(1 + kx^2/4)^2}\left(dx^2 + x^2\, d\theta^2 + x^2 \sin^2\theta\, d\phi^2\right), \tag{4}$$

where $a(v)$ is the scale factor of the universe and k gives the curvature of space–time as a whole.

Taking account of equations (3) and (4), we set the metric for a Reissner–Nordström black hole embedded in the FRW universe as follows

$$dl^2 = -A^2(v,x)\, dv^2 + B^2(v,x)\left(dx^2 + x^2\, d\theta^2 + x^2 \sin^2\theta\, d\phi^2\right). \tag{5}$$

Then from equation $G_{01} = 0$ one obtains

$$A(v,x) = f(v)\frac{\dot{B}}{2B}, \tag{6}$$

where "·" denotes the derivative with respect to v.

Compare the g_{11} terms in Eqs. (3) and (5), the possible form for the function $B(v,x)$ is

$$B(v,x) = \left[w(v,x) + \frac{q(v)}{x}\right]^2 - \frac{s(v)}{x^2}. \tag{7}$$

We note that the mass and charge of the black hole is concentrated in the singularity. In other words, there is no space distribution for mass and charge. Thus q and s which are related to the mass and charge, respectively, are only the functions of time v.

Inserting Eq. (7) into Eq. (6), we obtain

$$A(v,x) = \frac{\frac{\dot{w}f}{w} + (w\dot{q} + \dot{w}q)\frac{f}{w^2x} + \frac{q\dot{q}f}{w^2x^2} - \frac{\dot{s}f}{2w^2x^2}}{\left(1 + \frac{q}{wx}\right)^2 - \frac{s}{w^2x^2}}. \tag{8}$$

In the case of $v = \text{const}$ and the asymptotically flat conditions, $A(v = \text{const}, x)$ should be reduced to the $\sqrt{-g_{00}}$ term in Eq. (3). Thus comparing Eq. (8) with the $\sqrt{-g_{00}}$ term in Eq. (3), we infer the following identities should always hold

$$\begin{aligned} &\frac{\dot{w}f}{w} = 1, \\ &(w\dot{q} + \dot{w}q)\frac{f}{w^2x} = 0, \\ &\frac{q\dot{q}f}{w^2x^2} = -\left(\frac{q}{wx}\right)^2, \\ &-\frac{\dot{s}f}{2w^2x^2} = \frac{s}{w^2x^2}, \end{aligned} \tag{9}$$

namely

$$\dot{w}f = w, \qquad \dot{q}f = -q, \qquad \dot{s}f = -2s. \tag{10}$$

From Eq. (10) we obtain

$$w = \frac{b(v)}{\sqrt{1 + kx^2/4}}, \qquad f = \frac{b}{\dot{b}}, \qquad q = \frac{M}{b}, \qquad s = \frac{Q^2}{b^2}, \tag{11}$$

where M and Q are two integration constant which are related to the mass and charge of the black hole; $b(v)$ is an arbitrary function which is related to the scale factor of the universe; the form of ω is obtained by inspecting the McVittie's solution,

$$dl^2 = -\frac{\left(\frac{\sqrt{a}}{\sqrt{1+kx^2/4}} - \frac{M/\sqrt{a}}{x}\right)^2}{\left(\frac{\sqrt{a}}{\sqrt{1+kx^2/4}} + \frac{M/\sqrt{a}}{x}\right)^2} dv^2 + \left(\frac{\sqrt{a}}{\sqrt{1+kx^2/4}} + \frac{M/\sqrt{a}}{x}\right)^4 \left(dx^2 + x^2\,d\theta^2 + x^2\sin^2\theta\,d\phi^2\right), \tag{12}$$

where $a = a(v)$.

Substituting Eqs. (7), (8), (11) in Eq. (5), we obtain our final metric for the Reissner–Nordström black hole in the background of FRW universe

$$\begin{aligned} dl^2 = {} & -\frac{\left[1 - \frac{M^2}{a^2x^2}(1 + kx^2/4)\right]^2 + \frac{Q^2}{a^2x^2}(1 + kx^2/4)}{\left[\left(1 + \frac{M}{ax}\sqrt{1 + kx^2/4}\right)^2 - \frac{Q^2}{a^2x^2}(1 + kx^2/4)\right]^2} dv^2 \\ & + \frac{a^2}{(1 + kx^2/4)^2}\left[\left(1 + \frac{M}{ax}\sqrt{1 + kx^2/4}\right)^2 - \frac{Q^2}{a^2x^2}\left(1 + kx^2/4\right)\right]^2 \left(dx^2 + x^2\,d\theta^2 + x^2\sin^2\theta\,d\phi^2\right), \end{aligned} \tag{13}$$

where we have made a variable replacement $b(v)^2 \to a(v)$. Eq. (13) is derived from Eq. (7) and when $a(v) = \text{const}$, $k = 0$ Eq. (13) restores Eq. (3). So Eq. (3) satisfies Eq. (7).

For the Reissner–Nordström–de Sitter metric, we have $a = e^{Ht}$, $k = 0$, so Eq. (13) becomes

$$dl^2 = -\frac{\left[1 - \frac{M^2}{a^2x^2} + \frac{Q^2}{a^2x^2}\right]^2}{\left[\left(1 + \frac{M}{ax}\right)^2 - \frac{Q^2}{a^2x^2}\right]^2} dv^2 + a^2\left[\left(1 + \frac{M}{ax}\right)^2 - \frac{Q^2}{a^2x^2}\right]^2 \left(dx^2 + x^2\,d\theta^2 + x^2 \sin^2\theta\, d\phi^2\right). \tag{14}$$

We will show in the next section that Eq. (14) can be reduced to the familiar form in Schwarzschild coordinates. When $H = 0$, Eq. (14) restores the Reissner–Nordström metric as given by Eq. (3). When $Q = 0$, Eq. (13) becomes

$$dl^2 = -\frac{\left(1 - \frac{M}{ax}\sqrt{1 + kx^2/4}\right)^2}{\left(1 + \frac{M}{ax}\sqrt{1 + kx^2/4}\right)^2} dv^2 + \frac{a^2}{(1 + kx^2/4)^2}\left(1 + \frac{M}{ax}\sqrt{1 + kx^2/4}\right)^4 \left(dx^2 + x^2\,d\theta^2 + x^2 \sin^2\theta\, d\phi^2\right). \tag{15}$$

It is just the McVittie solution. Another special case of our solution is for the extreme Reissner–Nordström black hole, $M = Q$, in the de Sitter universe. In this case, Eq. (14) is reduced to a special case of the Kastor and Traschen solution [2] for a single black hole,

$$dl^2 = -\frac{1}{\left(1 + \frac{2M}{ax}\right)^2} dv^2 + a^2\left(1 + \frac{2M}{ax}\right)^2 \left(dx^2 + x^2\,d\theta^2 + x^2 \sin^2\theta\, d\phi^2\right). \tag{16}$$

In Eq. (16), $a = e^{Ht}$. It describes one extreme Reissner–Nordström black hole in the de Sitter universe.

3. Further discussion on the metric

In this section we will verify that Eq. (13) satisfies Einstein–Maxwell equations. The Einstein–Maxwell equations may be written as

$$G_{\mu\nu} = 8\pi(T_{\mu\nu} + E_{\mu\nu}),$$
$$F_{\mu\nu} = A_{\mu;\nu} - A_{\nu;\mu},$$
$$F^{\mu\nu}_{;\nu} = 0, \tag{17}$$

where $T_{\mu\nu}$ and $E_{\mu\nu}$ are the energy momentum for the perfect fluid and electromagnetic fields, respectively, which are defined by

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu + p g_{\mu\nu},$$
$$E_{\mu\nu} = \frac{1}{4\pi}\left(F_{\mu\alpha}F^\alpha_\nu - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right), \tag{18}$$

where ρ and p are the energy density and pressure. U_μ is the 4-velocity of the particles. $F_{\mu\nu}$ and A_μ are the tensor and the potential for electromagnetic fields.

Input the components of the metric, Eq. (13), to the Maple software package, we obtain the Einstein tensor $G_{\mu\nu}$ and the energy momentum tensor $T_{\mu\nu}$ and $E_{\mu\nu}$ for the perfect fluid and the electromagnetic fields

$$T^0_0 = \rho, \qquad T^1_1 = T^2_2 = T^3_3 = p,$$
$$2\pi E^0_0 = 2\pi E^1_1 = \frac{Q^2(1 + kx^2/4)}{x^4a^4\left[\left(1 + \frac{M}{ax}\sqrt{1 + kx^2/4}\right)^2 - \frac{Q^2}{a^2x^2}(1 + kx^2/4)\right]^4},$$
$$2\pi E^2_2 = 2\pi E^3_3 = -\frac{Q^2(1 + kx^2/4)}{x^4a^4\left[\left(1 + \frac{M}{ax}\sqrt{1 + kx^2/4}\right)^2 - \frac{Q^2}{a^2x^2}(1 + kx^2/4)\right]^4}. \tag{19}$$

Substituting the above components of electromagnetic tensor in the second equation of Eq. (18), we obtain the non-vanishing components of electromagnetic tensor $F_{\mu\nu}$

$$F^{01} = \frac{(1+kx^2/4)^{3/2}Q}{x^2a^3\left[1-\frac{M^2}{a^2x^2}(1+kx^2/4)+\frac{Q^2}{a^2x^2}(1+kx^2/4)\right]} \times \frac{1}{\left[\left(1+\frac{M}{ax}\sqrt{1+kx^2/4}\right)^2-\frac{Q^2}{a^2x^2}(1+kx^2/4)\right]^2}. \tag{20}$$

Then substituting Eq. (20) in the second equation of Eq. (17), we obtain the non-vanishing components of the potential A_μ

$$A_0 = \int F^{01} g_{00} g_{11}\, dx. \tag{21}$$

It is a straightforward work to verify that Eq. (20) also satisfies the last equation in Eq. (17). Since $G^{\mu\nu}_{;\nu} = 0$ always holds, thus from Einstein equations we have $0 = T^{\mu\nu}_{;\nu} + E^{\mu\nu}_{;\nu}$. On the other hand, we have the relation

$$\begin{aligned} 4\pi E^{\mu\nu}_{;\nu} &= F^{\mu\alpha}F^{\nu}_{\alpha;\nu} + F^{\mu\alpha}_{;\nu}F^{\nu}_{\alpha} - \frac{1}{2}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta} = F^{\mu\alpha}F^{\nu}_{\alpha;\nu} + \frac{1}{2}g^{\mu\rho}F^{\nu\sigma}(F_{\rho\sigma;\nu} - F_{\rho\nu;\sigma} - F_{\sigma\nu\rho}) \\ &= -F^{\mu\alpha}J_\alpha = 0. \end{aligned} \tag{22}$$

So both $T_{\mu\nu}$ and $E_{\mu\nu}$ satisfy Bianchi identity. We therefore conclude that Eq. (13) is an exact solution of the Einstein–Maxwell equations.

To show how the two parameters M and Q are related to the mass and charge of the black hole, we assume the evolution of the universe is much slower and approximately adopt the mass formula for the stationary space–time [8] (to our knowledge, we can only define the mass for the stationary and asymptotically flat space–time). We find the mass M_0 and charge Q_0 of the black hole are given by

$$\begin{aligned} M_0 &\equiv -\frac{1}{8\pi}\int_S \epsilon_{abcd}\nabla^c\xi^d = \frac{M}{a}, \\ Q_0 &\equiv \frac{1}{4\pi}\int_S \epsilon_{abcd}F^{cd} = \frac{Q}{a}. \end{aligned} \tag{23}$$

Thus for the observer in the infinity, the black hole's mass and charge will decrease with the expansion of the universe and increase with the contraction of the universe. We will return to this point in the following discussion again.

Let us now consider the two typical surfaces of the black hole in the cosmic coordinates system. We obtain the radius of the time-like limit surface (TLS) [8]

$$r_{\mathrm{TLS}} = \sqrt{\frac{M^2}{a^2} - \frac{Q^2}{a^2}}, \tag{24}$$

and the time derivative of the radius of the event horizon [8]

$$\dot{r}_{\mathrm{EH}} = \pm\frac{1-\frac{M^2}{a^2r_{\mathrm{EH}}^2}+\frac{Q^2}{a^2r_{\mathrm{EH}}^2}}{a\left[\left(1+\frac{M}{ar_{\mathrm{EH}}}\right)^2-\frac{Q^2}{a^2r_{\mathrm{EH}}^2}\right]^2}, \tag{25}$$

where two signs "+" and "−" correspond to the expanding and the contracting universe, respectively. Since the event horizon is always in the inner of TLS, we have $\dot{r}_{\mathrm{EH}} < 0$ for expanding universe and $\dot{r}_{\mathrm{EH}} > 0$ for contracting universe.

Eqs. (24), (25) tell us the typical scales of the black hole are closely related to the evolution of the universe. They shrink with the expansion of the universe and expand with the contracting of the universe. For asymptotically flat background, $a = 1$ and $\dot{r}_{\mathrm{EH}} = 0$, these two kind of surfaces coincide

$$r_{\mathrm{TLS}} = r_{\mathrm{EH}} = \sqrt{M^2 - Q^2}. \tag{26}$$

Comparing Eq. (26) with Eq. (24), we find that the black hole in the expanding universe has the mass $M_0 = M/a$ and charge Q_0/a. They are both dependent on the scale of the universe and not a constant.

From Eq. (23), one obtain the lifetime of a black hole

$$\tau \simeq \frac{1}{H}. \tag{27}$$

It is approximately the age of the universe.

4. Equation of motion of a planet

In this section, we turn to the discussion of the problem of the orbit for a planet in the expanding universe. Current measurements of the microwave background radiation show that our universe is highly likely flat in space [7]. So in the next we consider the local dynamics in the space-flat space case, i.e., $k = 0$. Eq. (13) is then reduced to

$$dl^2 = -\frac{\left(1 - \frac{M^2}{a^2x^2} + \frac{Q^2}{a^2x^2}\right)^2}{\left[\left(1 + \frac{M}{ax}\right)^2 - \frac{Q^2}{a^2x^2}\right]^2} dv^2 + a^2\left[\left(1 + \frac{M}{ax}\right)^2 - \frac{Q^2}{a^2x^2}\right]^2 \left(dx^2 + x^2\,d\theta^2 + x^2 \sin^2\theta\,d\phi^2\right). \tag{28}$$

In order to study the motion of a planet, we should rewrite Eq. (28) in the Schwarzschild or solar coordinates system. Similar to Eq. (2), make variables transformation as follows

$$T = 2v, \qquad s = 2l, \qquad 2r = ax\left(1 + \frac{M}{ax}\right)^2 - \frac{Q^2}{a^2x^2}. \tag{29}$$

Then Eq. (28) becomes

$$\begin{aligned} ds^2 = {} & -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - H^2r^2\right) dT^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 \\ & - 2rH\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1/2} dT\,dr + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right), \end{aligned} \tag{30}$$

where $H \equiv \frac{1}{a}\frac{da}{dT}$ is the Hubble parameter. The coordinate system of (T, r, θ, ϕ) is not orthogonal. We can eliminate the coefficient of $dT\,dr$ by introducing a new time coordinate, t. The form of Eq. (30) suggests we set

$$dt = F(T,r)\,dT + F(T,r)rH\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1/2}\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - H^2r^2\right)^{-1} dr, \tag{31}$$

where $F(T, r)$ is a perfect differential factor and it always exists. Then Eq. (29) can be written as

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - H^2r^2\right)F^2\,dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - H^2r^2\right)^{-1} dr^2 + r^2\,d\Omega^2, \tag{32}$$

where H and F are both the functions of variables t and r. If $\rho = p = 0$, we have $H = 0$. Eqs. (30) and (32) both turn into the static Reissner–Nordström solution. If $2M/r \to 0$ and $Q^2/r^2 \to 0$, Eq. (32) represents the solution for the Friedman–Robertson–Walker universe in Schwarzschild or solar coordinate system. For the de Sitter universe,

H is a constant. Eq. (31) tells us we may choose $F(t,r)=1$. Then Eq. (32) is just the well-known Reissner–Nordström–de Sitter space–time. It is a static space–time. Thus the geodesic is time-independent. In other words, the orbit of the planet does not vary in the background of de Sitter universe.

In general, H is the function of t and r. So Eq. (32) is a non-stationary space–time and the geodesic in this space–time is generally time-dependent. Thus we conclude that universe expansion would influence the orbit of planet. However, the effect term H^2r^2 is very small. Since

$$\varepsilon \equiv \frac{H^2r^2}{2M/r} = \frac{\rho}{M/(4\pi r^3/3)} = \frac{\rho}{\tilde{\rho}}, \tag{33}$$

where ρ is the universe energy density and $\tilde{\rho}$ is the energy density within the region r where the energy M is distributed. The ratio of ρ to $\tilde{\rho}$ varies from 4×10^{-34} for Mercury–Sun system (In detail, r represents the distance from the Sun to Mercury and M is the mass of the Sun. The mass of the planet is much smaller than the Sun.), 1.8×10^{-28} for Neptune–Sun system to 10^{-7} for Galaxy.

Keep this in mind or regard H as a constant, we obtain the motion equation of a planet

$$u'' + u = \frac{M}{L^2} - \frac{Q^2u}{L^2} + 3Mu^2 - 2Q^2u^3 - \frac{H^2}{L^2u^3}, \tag{34}$$

where $u\equiv 1/r$ and the prime denotes differentiation with respect to ϕ. L is the angular momentum of the planet. The term M/L^2 is the most significant one. The two terms $3Mu^2$ and $H^2/(L^2u^3)$ come from the GR (general relativity) effect and CE (cosmic expansion) effect, respectively. Q^2u/L^2 and $-2Q^2u^2$ are related to the charge of the source. Eq. (34) tells us the orbit of the test body will be influenced by the expansion of the universe. Since

$$\frac{H^2/(L^2u^3)}{M/L^2} = 2\varepsilon = \begin{cases} 8\times10^{-34}, & \text{for Mercury,} \\ 3.6\times10^{-28}, & \text{for Neptune,} \end{cases} \tag{35}$$

so the influence is related to the ratio of ρ to $\tilde{\rho}$. It is very small and negligible.

5. Conclusion and discussion

In conclusion, we have presented the metric for the Reissner–Nordström black hole in the background of FRW universe. It extends McVittie's solution, Reissner–Nordström–de Sitter solution and the special case for a single black hole of the Kastor and Traschen solution. Assume the evolution of the universe is much slower and adopt the formulas for computing the mass and charge of stationary space–time, we find that both the mass and charge of the black hole decrease with the expansion of the universe and increase with the contraction of the universe. We also find that the two typical scales, the time-like surface and the event horizon, of the black hole both shrink with the expansion of the universe and expand with the contraction of the universe. This is due to the fact that the mass and charge of the black hole are both varying with the evolution of the universe.

To obtain the equation of motion of a planet, we rewrote the metric from the cosmic coordinate system to the Schwarzschild or solar coordinate system and deduced the geodesic equation. The equation shows that the orbit of the planet in the Reissner–Nordström field embedded in the FRW universe will be influenced by the evolution of the universe. The magnitude of influence depends on the ratio ε between the energy density of the system and the energy density of the universe. Since the ratio ε is extremely small, varying from 4×10^{-34} for Mercury–Sun system, 1.8×10^{-28} for Neptune–Sun system to 10^{-7} for Galaxy, the influence of the expansion of the universe is very small and negligible [9].

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