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Effects of heritability on evolutionary cooperation in spatial prisoner's dilemma games

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Abstract

We study the effects of heritability on the evolution of the spatial prisoner's dilemma game. In our model, the fitness of each player is composed of the instantaneous payoff from the interactions and the inherited fitness from the last generation. Based on extensive simulations, we find that the density of cooperators is enhanced by increasing the heritability of players over a wide range of the model parameter. The mean fitness of cooperators and defectors are also studied for understanding our results.

Keywords: Heritability; Cooperation; Prisoner's dilemma game; Fitness

1. Introduction

Cooperation plays an important role in the real world, ranging from biological systems to economic and social systems. Understanding the maintenance of cooperative behavior among selfish individuals is an interesting and challenging problem as it contradicts Darwinian selection [1-3]. The Prisoner's Dilemma Game (PDG) as a general metaphor for studying cooperative behaviors, have received much attention in theoretical and experimental studies [4, 5]. The PDG seizes the characteristics of conflict between selfish individuals and the collective interests of two involved players, which has become the leading paradigm to explain cooperative behavior. In the common mathematical formulation of PDG with pairwise interaction, each of the two encountering cooperators (defectors) get a payoff *R(P)*, a defector confronting a cooperator acquires payoff *T*, while the cooperator gains *S*. The four

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parameters is required to satisfy the conditions $T>R>P>S$ and $2R > T+S$. In a well-mixed population, defectors are unbeatable and cooperators are doomed to extinction [2].

Since defecting is the dominating strategy for PDG, the cooperative behavior will disappear, which is opposite to observations in the real world. Therefore, a variety of mechanisms are proposed to understand the maintenance of cooperation [7], one of which was done by Nowak and May, who showed that the PDG on a simple spatial structure can enable cooperators to survive by forming clusters within which they benefit from mutual cooperation and protecting them from exploitation by defectors [8]. Inspired by the idea of a spatial game, much attention has been given to evolutionary games on several population structures, including regular lattices [9-18], small-world networks [19-24] and scale-free networks [25-33]. More interestingly, many realistic phenomena are also introduced into evolutionary games, such as "Tit-for-tat" [4, 34], "win stay and lose shift" [35], memory effects [36, 37], aspiration effects [38, 39], social tolerance [40, 41], social diversity [42], adaptive networks with alternative interactions [43-47], different teaching capabilities [48-53], noise in the strategy adoption [54-57] and payoff noise [58-62].

In most real biological or economic systems, the heritability is ubiquitous. A filial generation can inherit property, physical quality and other resources from their parents. In this paper, we assume that players can inherit fitness from the last generation. Meanwhile, their fitness should be correlated with their current payoffs from the interactions. In our model, we assume that the fitness of a player is determined by two components: one is the inherited fitness; the other is the current payoff. In this paper, we focus on the effect of inherited fitness on the evolution of cooperation in spatial PDG, and we find that the introduction of inherited fitness can improve the cooperation level of a system dramatically.

2. Model

We consider an evolutionary two-strategy prisoner's dilemma game with players located on a square lattice with periodic boundary conditions. Initially, either a cooperator or a defector, randomly chosen with equal probability, occupies each site. At each time step, each player interacts with its neighbors and collects the payoffs depending on the payoff-matrix elements, and self-interactions are excluded. The total payoff of a certain player is the sum over all interactions. Following the previous studies [8], the elements of the payoff matrix can be rescaled, i.e., we can choose $R = 1$, $P = S = 0$, and $T = b$ (>1) without loss of generality in the evolutionary PDG.

At the tth generation, the fitness $f_i(t)$ of the player i is defined as:

$$f_i(t) = \tau f_i(t-1) + (1-\tau) P_i(t), \tag{1}$$

where the parameter τ denotes the heritability and $P_i(t)$ denotes the payoff of player i at this time. The parameter τ sets the balance between the present and past payoff gains-the relative importance of a previous generations or strength of maternal effects decays with a factor τ per time step [63]. The values of τ are in the range [0,1). If $\tau=0$, this model reduces to the original one. By tuning the value of τ, one can investigate the effects of the heritability on evolution of cooperation. In our model, each player's initial fitness $f_i(0)$ is set to 1. We have check that the precise value of $f_i(0)$ does not affect our conclusions.

After collecting payoffs, the players try to maximize their individual payoff by imitating (learning) one of the more successful neighboring strategies synchronously. Following previous studies, player i will adopt the randomly chosen neighbor j's strategy s_j with a probability depending on the fitness difference (f_i-f_j) as:

$$W_i = \frac{1}{1+\exp[(f_i - f_j)/\kappa]}, \tag{2}$$

where κ characterizes the noise introduced to permit irrational choices [9]. In this paper, we set the noise parameter is $\kappa=0.1$.

Simulations are carried out for a population of $N=100\times100$ individuals. We study the key quantity of cooperator density ρ_c in the steady state. Equilibrium frequencies of cooperators and defectors were obtained by averaging over 2000 Monte Carlo time steps from a total of 22000 steps, and each data point results from an average of over 10 realizations.

3. Simulation and analysis

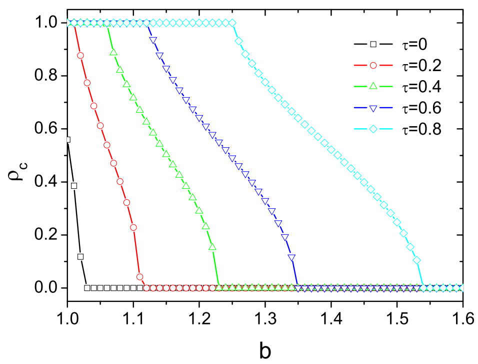

Fig. 1. (color online) Stationary fraction of cooperators ρ_c in dependence on the parameter b for different values of τ.

Figure 1 shows how ρ_c varies in dependence on the temptation to defect b for different values of τ. It displays that ρ_c decreases monotonically with the increasing of b, regardless what τ is. Moreover, for a fixed b, different values of τ can affect the final cooperation levels dramatically. To quantify the effects of varying τ on cooperation, we plot ρ_c as a function of parameter τ for different values of b, as shown in Fig. 2. From Fig. 2, we can find that the cooperator density ρ_c increases as the value of τ increases for a fixed value of b.

Notice that cooperators tend to form cluster patterns where cooperators assist each other in avoiding defectors' exploitation in spatial games. A cooperator (defector) cluster is a connected component (subgraph) fully occupied by cooperators (defectors). In order to visualize the effect of τ on cooperation, we plot some typical snapshots of the system at equilibrium for fixed b=1.34 with respect to different values of τ. Fig. 3 shows that the cooperator clusters become larger while the defector clusters become smaller as τ increases.

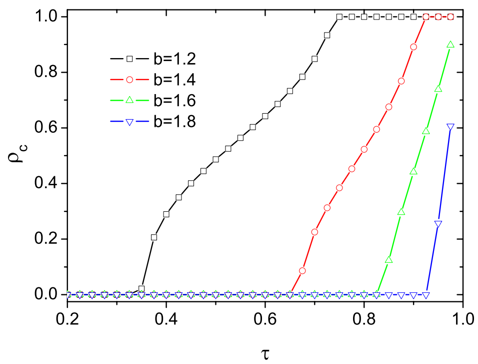

Fig. 2. (Color online) Stationary fraction of cooperators ρ_c in dependence on the heritability τ for different values of b.

Now we investigate the possible mechanisms for the promotion of cooperation. The cooperators can obtain steady payoffs by forming compact clusters (see Fig.3). Hence, the cooperators can accumulate large fitness from their history. In contrast with the cooperators, the defectors can not obtain payoff as their destructive strategy. Hence, they can not obtain fitness from their history. Fig. 4 shows the mean fitness of cooperators and defectors ($<f_c>$ and $<f_d>$) in the evolution. One can find that $<f_c>$ always exceeds $<f_d>$ during the whole evolutionary process. Fig. 5 shows that, in the equilibrium state, $<f_c>$ is no less than $<f_d>$ for most values of τ. For τ<0.35, there is no cooperator, both $<f_c>$ and $<f_d>$ equal zero. For $\tau>0.75$, there is no defector, $<f_c>$ reaches its maximum, and $<f_d>$ equals zero.

Now the reason why the heritability promotes cooperation can be understood: the cooperators can accumulate large fitness from their history by the heritability, while the defectors can not. Hence, the defectors can not defeat the cooperators easily.

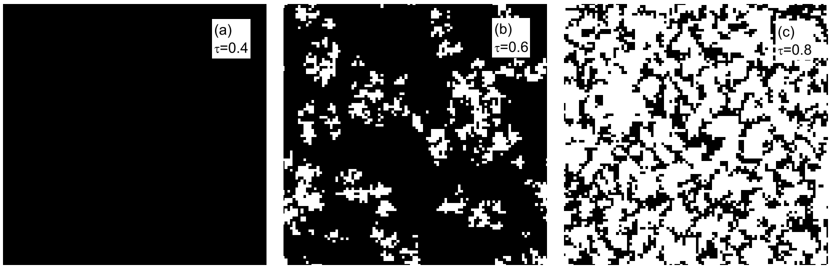

Fig.3. A series of snapshots of distribution of cooperators (white) and detectors (black) on a 100 ×100 square lattice at b = 1.34, for three values of τ: (a) τ=0.4, (b) τ=0.6, and (c) τ=0.8. Each snapshot is obtained at the time step t = 20000.

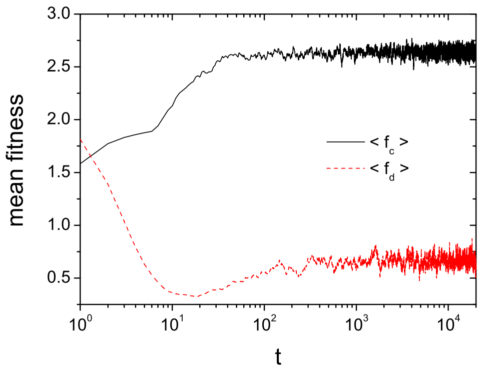

Fig. 4. (color online) The mean fitness of cooperators and defectors ($<f_c>$ and $<f_d>$) in the evolution for τ = 0.4 and b = 1.2.

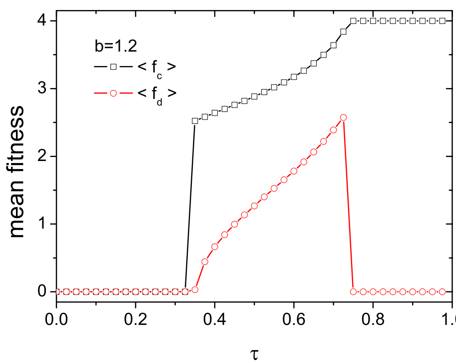

Fig. 5. (color online) The mean fitness of cooperators and defectors ($<f_c>$ and $<f_d>$) in the equilibrium state, as a function of τ for b = 1.2.

4. Conclusion

In conclusion, we have studied the effects of heritability on the prisoner's dilemma game on regular lattices. The heritability of players is controlled by a tunable parameter τ. In our model, the fitness of each player is based on the instantaneous payoff from the interactions and the inherited fitness from the past. The simulation results show that the heritability plays a positive role for the maintenance of cooperators. In order to give an intuitive account of the maintenance of cooperation, we provide some typical snapshots of the system and compare mean fitness of defectors and cooperators. It is shown that cooperators can survive by forming compact clusters, and cooperators can gain more fitness from their history than defectors can. Our work suggests that the heritability plays crucial roles in the evolution of cooperation amongst egoistic individuals. Since the heritability is common in most real biological or economic systems, we expect our work to shed some new light on the emergence of cooperation.

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