The Twelfth East Asia-Pacific Conference on Structural Engineering and Construction

An Efficient Sensitivity Analysis Method for Optimization of Vehicle Random Vibrations

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Abstract

An efficient and accuracy sensitivity analysis method for optimal analysis of random vibration of vehicle-bridge coupled system is purposed. The pseudo-excitation method is used to transform random road surface roughness into a series of deterministic harmonic excitations, and then the precise integration method is adopted to compute vehicle/bridge system response. The pseudo-excitation method and the precise integration method are both accurate and efficient, so that the first and second order sensitivity information of the responses can be obtained very conveniently. Taking ride comfort as the objective function, an optimal analysis for a vehicle/bridge system is performed.

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Keywords: Vehicle-bridge coupled systems; Pseudo-excitation method; Precise integration method; Sensitivity analysis.

1. INTRODUCTION

In recent years, dynamic analysis of coupled vehicle-bridge systems has received much attention. The road surface roughness is well known to be a random process. It is very difficult to perform a vibration analysis concerning such random vibration (Lou and Zeng 2005, Xia 2000, Lei and Noda 2002, Zhai 1996). Furthermore, very little research concerning optimal analysis with the consideration of random response appears in the literature. There are two difficulties for the optimal analysis (Yamazaki et al. 1996).
1988, Bouazara and Richard 2001, Cassis and Schmit 1976, Kapoor and Kumarasamy 1981, Naude and Snyman 2003): One is that the conventional method for random vibration analysis require much computational effort, the other is that, generally, the objective function for optimal analysis is highly non-linear and the calculation procedure for sensitive analysis is very complicated. Beliveau et al used the least-squares iteration method to compute the first two order sensitivities of eigen values (Beliveau et al. 1996). A matrix perturbation method and a method based on Laplace transformation were developed by Chen (Chen et al. 1993) and Zimoch (Zimoch 1987 and Durbin 1974), respectively. All the above methods are very time consuming which restrict their applications. In this paper the pseudo excitation method (PEM) and precise integration method (PIM) are combined to promote computational efficiency, which have been described in detail elsewhere (Lin et al. 1994, Lin et al. 2001, Lin and Zhang 2005). Moreover, sensitivity analysis for dynamic optimization based on PEM and PIM is developed to calculate the first and second order sensitivities of random responses. In numerical examples, the proposed method is verified, and an optimal analysis for a vehicle is performed.

2. Equations of motion for vehicle systems

2.1. Vehicle stationary random vibration

The vibration is stationary random vibration for single vehicle. Consider the linear model of a double axle vehicle shown in Fig.1. For convenience, a 3-D vehicle model with eight DOF is introduced, of which the dynamic behavior is described by vertical, pitching and rolling motions. In this analysis, we do not consider yaw motion because its effect on vehicle ride comfort is negligible. The seat is modeled as a mount consisting of a linear spring and a damper. The system parameters are: mass of the seat plus driver \( m_s \); mass of the car body \( m_c \); masses of wheel axles \( m_{fl}, m_{fr}, m_{rl}, m_{rr} \) (where \( fl \) represents front-left, \( rr \) represents rear-right, etc.); the corresponding damping coefficients \( c_{fy}, c_{fc}, c_{fr}, c_{rc} \); the corresponding stiffness coefficients for the seat and wheel axles \( k_{fy}, k_{fc}, k_{fr}, k_{rc}, k_{fl}, k_{fr}, k_{rl}, k_{rr} \); the tire stiffness coefficients \( k_{wfl}, k_{wfr}, k_{wrl}, k_{wrr} \); and the tire damping coefficients \( w_{fl}, w_{fr}, w_{rl}, w_{rr} \). \( a_{fl} \) and \( a_{fr} \) are the outline dimensions of the vehicle; \( l_{f} \) and \( l_{r} \) denote the distances between the body-center and the front or rear axle; \( d_{f} \) represents the distance between the right and left tires; \( r_s \) and \( r_y \) are the location parameters of the seat; \( J_x \) and \( J_y \) are the rotational inertias of the vehicle about its \( x \) and \( y \) axes. With pitching and rolling motions denoted by small angles \( \varphi \) and \( \theta \), the equations of motion for this system can be derived as:

\[
M \ddot{z} + C \dot{z} + K z = f(t)
\]

where \( z = \begin{bmatrix} z_y & z_c & \theta & \varphi & z_{wfl} & z_{wfr} & z_{wrl} & z_{wrr} \end{bmatrix}^T \) is the displacement vector and the excitation vector \( f(t) \) is written as

\[
f(t) = G(t) \begin{bmatrix} z_{rfl} & z_{rfr} & z_{rfl} & z_{rfr} \end{bmatrix}^T + G(t) \begin{bmatrix} \dot{z}_{rfl} & \dot{z}_{rfr} & \dot{z}_{rfl} & \dot{z}_{rfr} \end{bmatrix}^T
\]

where \( G(t) \) and \( G(t) \) can be time-variant or invariant matrices, and \( z_{rfl}, z_{rfr} \) are the corresponding wheel displacement.

Following the rule of GB7031 (Yu 2000), the road surface spectrum can be represented by \( 2\pi S_n(n) n^2 v / \omega^2 \). Based on the equation of motion (1) of a vehicle, the correlation matrix of the road excitation can be written as equation (3).
For vehicle stationary random vibration, the responses has been solved conveniently by PEM. The high efficiency and accuracy of this procedure has been proved (Lin et al. 1994).

\[
R_n (\tau_k \tau_l) = G_1 (\tau_k) \int_{-\infty}^{\infty} S_h (\omega) e^{j\omega t} d\omega G_1 (\tau_l)^T + G_1 (\tau_k) \int_{-\infty}^{\infty} S_h (\omega) (i\omega) e^{j\omega t} d\omega G_2 (\tau_l)^T
\]

\[
G_2 (\tau_k) \int_{-\infty}^{\infty} S_h (\omega) (-i\omega) e^{j\omega t} d\omega G_1 (\tau_l)^T + G_2 (\tau_k) \int_{-\infty}^{\infty} S_h (\omega) (-i\omega) e^{j\omega t} d\omega G_2 (\tau_l)^T
\]

(3)

2.2. The non-stationary random vibration of coupled vehicle-bridge systems

For coupled vehicle-bridge systems, the vibration can be regarded as a set of uniformly modulated, multi-point, different-phase, non-stationary random vibration. Consider a coupled vehicle-bridge model shown in Fig. 2.
Because the wheels are assumed to remain in perfect contact with the track at all times, the constraint conditions between the wheel set and bridge can be described as

\[ z_{wi} = z_{bi} + R(x_i) \quad (i = f, r) \]  

in which \( z_{bi} \) is the vertical displacement of the bridge and \( R(x_i) \) represents the road displacement roughness at this set. The equation of motion for the coupled vehicle-bridge system can be given as

\[ \mathbf{Mz} + \mathbf{Cz} + \mathbf{Kz} = \mathbf{F}(t) \]  

where: the vector \( \mathbf{z} \) is the coupled displacement vector \( \mathbf{z} = \left\{ z_c, \theta, z_f, z_r, z_{b1}, \ldots, z_{bn} \right\} \).

The coupled excitation vector \( \mathbf{F}(t) \) is composed from the deterministic loads \( \mathbf{F}_p(t) \) and the time-variable loads \( \mathbf{F}_r(t) \), given by

\[ \mathbf{F}_p(t) = \left[ \begin{array}{c} l_r m_c \{0 0 0 0 N_\theta \}^T + \left( \frac{l_f m_c}{l_f + l_r} + m_r \right) \{0 0 0 0 N_\theta \}^T \end{array} \right] \]

\[ \mathbf{F}_r(t) = -\left[ k_{w_f} R(x_f) + c_{w_f} \dot{R}(x_f) \right] \{0 0 0 1 N_\theta \}^T + \left[ k_{w_r} R(x_r) + c_{w_r} \dot{R}(x_r) \right] \{0 0 0 1 N_\theta \}^T \]

Because the system is linear, superposition enables equation (5) to be solved separately for \( \mathbf{F}_p(t) \) and \( \mathbf{F}_r(t) \). The deterministic responses are found by using PIM (Lu et al. 2006), whereas the random responses are computed using PEM, which transforms the random track excitation into a series of deterministic pseudo-excitations for which PIM gives the responses very conveniently (Zhang et al. 2009).

3. PEM/PEM-PIM based sensitivity analysis

The weighted root mean square (RMS) vertical acceleration \( z_{aw} \) of the vehicle is an important index for the comfort of passengers and security of goods, and so it is often taken as the objective function for vehicle optimization. It is expressed as

\[ z_{aw} = \left( \int_{0.5}^{j_{max}} W^2(f) S_{aw}(f) df \right)^{0.5} \]

in which \( S_{aw}(f) \) is the PSD of the vertical acceleration of the vehicle body at frequency \( f \) (Hz) and \( W(f) \) represents the frequency-weighted function.

3.1. First-order sensitivity analysis

Suppose the suspension stiffness and damping coefficients \( k_f, k_r, c_f, \) and \( c_r \) are regarded as design variables. The first-order derivatives of specific random displacement, velocity and acceleration responses \( \mathbf{z}, \dot{\mathbf{z}}, \) and \( \ddot{\mathbf{z}} \) with respect to the \( i \)th of the \( q \) design variable \( d_i \) \( (i = 1, 2, \cdots, q) \) can be denoted as

\[ (\mathbf{z})'_i = \frac{\partial \mathbf{z}}{\partial d_i}, \quad (\dot{\mathbf{z}})'_i = \frac{\partial \dot{\mathbf{z}}}{\partial d_i}, \quad (\ddot{\mathbf{z}})'_i = \frac{\partial \ddot{\mathbf{z}}}{\partial d_i} \]  

Differentiating the vehicle equation of motion with respect to design variable \( d_i \) gives

\[ \frac{\partial \mathbf{Mz} + \mathbf{Cz} + \mathbf{Kz}}{\partial d_i} = \frac{\partial \mathbf{F}(t)}{\partial d_i} \]
\[
\begin{align*}
M \frac{\partial \ddot{z}}{\partial d_i} + C \frac{\partial \dot{z}}{\partial d_i} + K \frac{\partial z}{\partial d_i} &= \frac{\partial F(t)}{\partial d_i} - \left( \frac{\partial M}{\partial d_i} \ddot{z} + \frac{\partial C}{\partial d_i} \dot{z} + \frac{\partial K}{\partial d_i} z \right) \\
\end{align*}
\]
\(9\)

Letting the right-hand side of Equation (9) be represented by the vector \(F_i(t)\), the sensitivity equations are
\[
M(\ddot{z})' + C(\dot{z})' + K(z)' = F_i(t)
\]
\(10\)

where the vector \(F_i(t)\) is composed of two terms, namely the time-invariant vector \(\partial \left( F_p(t) \right) / \partial d_i\) (or \(\partial \left( \mathbf{f}(t) \right) / \partial d_i\)) and the time-variable vector \(\partial \left( F_p(t) \right) / \partial d_i\). Because \(F_p(t)\) is independent of the design variables, the vector \(\partial \left( F_p(t) \right) / \partial d_i\) vanishes. Clearly, Equations (10) can still be solved by using PEM (or PEM-PIM) to yield \((z)'\), i.e. the first-order sensitivity, of the random displacement \(z\) with respect to \(d_i\).

3.2. Second-order sensitivity analysis

Similarly to the above, the second-order derivatives of specific random responses with respect to the \(j\)th of the \(q\) design variable \(d_i\) (\(i = 1, 2, \cdots, q\)) can be denoted as
\[
(z)^{''}(d_i,\dot{d}_i,\ddot{d}_i, j) = \frac{\partial^2 z}{\partial d_i \partial \dot{d}_i \partial \ddot{d}_i \partial j}, \quad (z)^{''}(d_i,\dot{d}_i,\ddot{d}_i, j) = \frac{\partial^2 \dot{z}}{\partial d_i \partial \dot{d}_i \partial \ddot{d}_i \partial j}, \quad (z)^{''}(d_i,\dot{d}_i,\ddot{d}_i, j) = \frac{\partial^2 \ddot{z}}{\partial d_i \partial \dot{d}_i \partial \ddot{d}_i \partial j}
\]
\(11\)

As in the derivation of Equation (9), they must satisfy
\[
M \frac{\partial^2 \ddot{z}}{\partial d_i \partial \dot{d}_i \partial \ddot{d}_i} + C \frac{\partial^2 \dot{z}}{\partial d_i \partial \dot{d}_i \partial \ddot{d}_i} + K \frac{\partial^2 z}{\partial d_i \partial \dot{d}_i \partial \ddot{d}_i} = F_z(t)
\]
\(12\)

in which
\[
F_z(t) = \frac{\partial^2 F_p(t)}{\partial d_i \partial \dot{d}_i \partial \ddot{d}_i} - \left( \frac{\partial^2 M}{\partial d_i \partial \dot{d}_i \partial \ddot{d}_i} \ddot{z} + \frac{\partial^2 C}{\partial d_i \partial \dot{d}_i \partial \ddot{d}_i} \dot{z} + \frac{\partial^2 K}{\partial d_i \partial \dot{d}_i \partial \ddot{d}_i} z \right) - \left( \frac{\partial M}{\partial d_i} (\ddot{z})' + \frac{\partial C}{\partial d_i} (\dot{z})' + \frac{\partial K}{\partial d_i} (z)' \right)
\]
\(13\)

Here the first-order sensitivity information has already been obtained, and the matrices \(M\), \(C\) and \(K\) vary linearly with the design variables, so that the excitation vector \(F_z(t)\) of Equation (12) is deterministic. PEM-PIM (or PEM) can therefore be used to solve Equation (12) for the time-variable responses.

So that setting \(i = j\) gives the main diagonal elements of the Hessian matrix. Similarly, the second-order sensitivity of \(z_{aw}\) can be obtained as
\[
\frac{\partial^2 z_{aw}}{\partial d_i \partial \dot{d}_i \partial \ddot{d}_i} = \left[ \int^{\pi}_{-\pi} W^2(\omega) \frac{\partial^2 S_{aw}(f)}{\partial d_i \partial \dot{d}_i \partial \ddot{d}_i} d\omega - \left( \frac{\partial z_{aw}}{\partial d_i} \right)^2 - 2 \left( \frac{\partial z_{aw}}{\partial d_i} \right) \right] / (2z_{aw})
\]
\(14\)
Thus, a new method for first and second orders of flexibility analyses of structural non-stationary random responses have been developed in this paper, based on PEM or PEM-PIM. Computing them is much simpler and more efficient than using conventional methods, as will be seen in the examples which follow.

4. Illustrative example

4.1. Example 1. Ride comfort optimization for a bus

In this example, the ride comfort optimization of a bus (Fig.1) is performed based on the flexibility information and the SLP optimality method. Then the efficiency of flexibility analysis by the proposed PEM based method was compared with that of the matrix perturbation method.

Table 1: Sensitivities of RMS and CPU time

<table>
<thead>
<tr>
<th>Design variable</th>
<th>(k_f)</th>
<th>(k_r)</th>
<th>(k_r)</th>
<th>(k_r)</th>
<th>(c_r)</th>
<th>(c_r)</th>
<th>(c_r)</th>
<th>CPU time</th>
</tr>
</thead>
<tbody>
<tr>
<td>PEM based ((10^3))</td>
<td>1.329</td>
<td>1.459</td>
<td>63.06</td>
<td>57.60</td>
<td>13.42</td>
<td>-84.53</td>
<td>-908.9</td>
<td>237.8s</td>
</tr>
<tr>
<td>Perturbation((10^3))</td>
<td>1.314</td>
<td>1.449</td>
<td>63.11</td>
<td>56.92</td>
<td>13.09</td>
<td>-85.10</td>
<td>-903.9</td>
<td>956.4s</td>
</tr>
</tbody>
</table>

4.2. Example 2. Optimization based on first and second-order sensitivities

In this example, the optimization of impact coefficients of the vehicle with different velocities is performed based on first and second-order sensitivities for vehicle-bridge coupled systems (Fig.2).

Table 3: Hessian matrix of \(Z_{aw}\) and CPU times

<table>
<thead>
<tr>
<th></th>
<th>Proposed Method</th>
<th>PEM-Newmark Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Z_{aw})</td>
<td>17.81 4.22 -3.75 1.46</td>
<td>17.79 4.20 -3.73 1.46</td>
</tr>
<tr>
<td>(Hessian)</td>
<td>4.22 12.64 1.18 -1.73</td>
<td>4.20 12.65 1.18 -1.73</td>
</tr>
<tr>
<td>RMS (Z_{aw})</td>
<td>-3.75 1.18 3.17 0.45</td>
<td>-3.73 1.18 3.17 0.45</td>
</tr>
<tr>
<td>CPU times (s)</td>
<td>114.56</td>
<td>1045.93</td>
</tr>
</tbody>
</table>

It can be seen that the computed results for the Hessian matrix given by the two methods are very close, but that the CPU times required by the PEM-Newmark method is about 9.13 times that required by the proposed PEM-PIM based sensitivity analysis.
Table 4: Impact coefficients of the vehicle with different velocities

<table>
<thead>
<tr>
<th>Velocity (km/h)</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
<th>110</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial design</td>
<td>0.187</td>
<td>0.217</td>
<td>0.271</td>
<td>0.308</td>
<td>0.335</td>
<td>0.344</td>
<td>0.421</td>
<td>0.460</td>
<td>0.483</td>
<td>0.541</td>
</tr>
<tr>
<td>Optimized design</td>
<td>0.161</td>
<td>0.209</td>
<td>0.254</td>
<td>0.282</td>
<td>0.316</td>
<td>0.325</td>
<td>0.400</td>
<td>0.427</td>
<td>0.441</td>
<td>0.493</td>
</tr>
</tbody>
</table>

By combining the optimized results for vehicle ride comfort, the impact coefficients were calculated for different vehicle velocities, see Tab.4. Clearly the impact coefficient increases with the vehicle velocity, but is not proportional to it. The vehicle suspension parameters are also important factors and the optimization of vehicle performance reduces the stiffness coefficients of the suspension, increases its damping coefficients, and decreases the impact coefficients for the bridge.

5. CONCLUSIONS

A combined PEM-PIM method for optimal analysis of investigating stationary/non-stationary random vibration of vehicle-bridge systems is presented. The pseudo-excitation method (PEM) is proven to be applicable for linear time-variant system, and is adopted to transform non-stationary random excitation of the road roughness into deterministic excitations. The precise integration method (PIM) is used to compute the vehicle/bridge system responses. An optimal analysis for a vehicle is performed.

6. ACKNOWLEDGMENTS

The authors are grateful for support from State Key Laboratory of Structural Analysis and Industrial Equipment and Zhengzhou University.

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