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A MODIFIED DECOMPOSITION

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Abstract—The Maclaurin series is quite limited in comparison to the (Adomian) series obtained in the decomposition method. By adding procedures from the decomposition method and the expansion of nonlinearities using the Adomian polynomials, as well as a recent result of Adomian and Rach on transformation of series using the above polynomials, the Maclaurin series can be made much more useful in its applicability. However, the convergence is still slower than for Adomian's results using decomposition.

1. INTRODUCTION

Power series solutions of linear homogeneous differential equations in initial-value problems yield simple recurrence relations for the coefficients, but they are generally not adequate for nonlinear equations, although applicable to some simple cases such as the Riccati equation. In order to extend the Maclaurin method, we use results from the Adomian decomposition method [1–6] and a recent theorem [7] by Adomian and Rach on transformation of series, which we state as

$$f\left(\sum_{n=0}^{\infty} c_n x^n\right) = \sum_{n=0}^{\infty} x^n A_n\left(c_0, c_1, \dots, c_n\right),$$

where the A_n are Adomian polynomials [1–3]. For convenience, we explain this statement as follows: Adomian solves general operator equations $Fu = g$ using his decomposition $u = \sum_{n=0}^{\infty} u_n$, a special case of his expansion of $f(u) = \sum_{n=0}^{\infty} A_n(u_0, u_1, \dots, u_n)$, where the A_n are derived by convenient algorithms for $f(u)$. Thus, $f\left(\sum_{n=0}^{\infty} u_n\right) = \sum_{n=0}^{\infty} A_n(u_0, \dots, u_n)$. If u is given as a series, $u = \sum_{n=0}^{\infty} c_n x^n$, we identify each component u_n of the decomposition of u with the component $c_n x^n$ of the power series, which leads readily to

$$A_n\left(u_0, \dots, u_n\right) = x^n A_n\left(c_0, \dots, c_n\right).$$

The series $\sum_{n=0}^{\infty} x^n A_n(c_0, \dots, c_n)$ is convergent if the series $\sum_{n=0}^{\infty} c_n x^n$ is convergent. This is simply extended to Taylor series

$$f\left(\sum_{n=0}^{\infty} c_n\left(x - x_0\right)^n\right) = \sum_{n=0}^{\infty} A_n\left(c_0, \dots, c_n\right)\left(x - x_0\right)^n.$$

To clarify the procedure, before consideration of more general cases, we look at a simple linear equation in Adomian's operator form $Lu + Ru = 0$, where L will be chosen as d^2/dx^2 and $R = \rho$, a constant. We write $Lu = -Ru$ and operate with L^{-1}, a two-fold integral operator. In initial

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX condition problems we can conveniently define it as a two-fold (in this case) definite integration from 0 to t. (We can also define it as an indefinite integration leading to $\alpha_0 + \alpha_1 x + I_2(\cdot)$, where $I_2(\cdot) = \iint (\cdot)\, dx\, dx$.) Thus α_0, α_1 are determined by the initial conditions as $\alpha_0 = u(0)$ and $\alpha_1 = u'(0)$. We now have

$$u = \alpha_0 - \alpha_1 x - \rho I_2 u$$

and can substitute $u = \sum_{n=0}^{\infty} c_n x^n$:

$$\sum_{n=0}^{\infty} c_n x^n = \alpha_0 + \alpha_1 x - \rho I_2 \left(\sum_{n=0}^{\infty} c_n x^n \right) = \alpha_0 + \alpha_1 x - \rho \sum_{n=0}^{\infty} c_n \frac{x^{n+2}}{(n+1)(n+2)},$$

or

$$\sum_{n=0}^{\infty} c_n x^n = \alpha_0 + \alpha_1 x - \rho \sum_{n=2}^{\infty} c_{n-2} \frac{x^n}{(n-1)\,n}.$$

Equating coefficients,

$$c_0 = \alpha_0, \quad c_1 = \alpha_1,$$

and, for $n \geq 2$,

$$c_n = -\rho \frac{c_{n-2}}{(n-1)\,n}.$$

If $\rho = \rho(x)$, we write $\rho(x) = \sum_{n=0}^{\infty} \rho_n x^n$, and

$$\begin{aligned}\sum_{n=0}^{\infty} c_n x^n &= \alpha_0 + \alpha_1 x - I_2 \left(\sum_{n=0}^{\infty} \rho_n x^n \right) \left(\sum_{n=0}^{\infty} c_n x^n \right) \\ &= \alpha_0 + \alpha_1 x - I_2 \sum_{n=0}^{\infty} x^n \sum_{v=0}^{n} \rho_v c_{n-v} \\ &= \alpha_0 + \alpha_1 x - \sum_{n=0}^{\infty} \left\{ \frac{x^{n+2}}{(n+1)(n+2)} \right\} \sum_{v=0}^{n} \rho_v c_{n-v}\end{aligned}$$

or

$$\sum_{n=0}^{\infty} c_n x^n = \alpha_0 + \alpha_1 x - \sum_{n=2}^{\infty} \left\{ \frac{x^n}{(n-1)\,n} \right\} \sum_{v=0}^{n-2} \rho_v c_{n-2-v},$$

so that

$$c_0 = \alpha_0 \quad c_1 = \alpha_1.$$

For $n \geq 2$,

$$c_n = -\sum_{v=0}^{n-2} \rho_v \frac{c_{n-2-v}}{(n-1)\,n}.$$

2. THE NONLINEAR CASE

$Lu + Nu = 0$. We let $L = d^2/dx^2$ and N is a nonlinear operator which we write $Nu = \alpha F(u)$. Then

$$Lu = -Nu, \quad L^{-1}Lu = -L^{-1}Nu, \quad u = \alpha_0 + \alpha_1 x - \alpha I_2 \sum_{n=0}^{\infty} A_n x^n,$$

where we have replaced $f(u)$ by $f\left(\sum_{n=0}^{\infty} c_n x^n\right) = \sum_{n=0}^{\infty} A_n(c_0, \ldots, c_n)\, x^n$, as discussed in the introduction. Now

$$\begin{aligned}\sum_{n=0}^{\infty} c_n x^n &= \alpha_0 + \alpha_1 x - \alpha \sum_{n=0}^{\infty} A_n \frac{x^{n+2}}{(n+1)(n+2)} \\ &= \alpha_0 + \alpha_1 x - \alpha \sum_{n=2}^{\infty} A_{n-2} \frac{x^n}{(n-1)\,n},\end{aligned}$$

so that

$$c_0 = \alpha_0, \quad c_1 = \alpha_1,$$

and for $n \geq 2$,

$$c_n = -\alpha \frac{A_{n-2}}{(n-1)\,n},$$

where $A_m = A_m(c_0, \dots, c_m)$.

Again, if $\alpha = \alpha(x)$, we write $\alpha(x) = \sum_{n=0}^{\infty} \alpha_n x^n$. Then

$$\begin{aligned}
\sum_{n=0}^{\infty} c_n x^n &= \alpha_0 + \alpha_1 x - I_2 \left(\sum_{n=0}^{\infty} \alpha_n x^n\right)\left(\sum_{n=0}^{\infty} A_n x^n\right) \\
&= \alpha_0 + \alpha_1 x - I_2 \sum_{n=0}^{\infty} x^n \sum_{v=0}^{n} \alpha_v A_{n-v} \\
&= \alpha_0 + \alpha_1 x - \sum_{n=0}^{\infty} \left\{\frac{x^{n+2}}{(n+1)(n+2)}\right\} \sum_{v=0}^{n} \alpha_v A_{n-v} \\
&= \alpha_0 + \alpha_1 x - \sum_{n=2}^{\infty} \left\{\frac{x^n}{(n-1)\,n}\right\} \sum_{v=0}^{n-2} \alpha_v A_{n-2-v}.
\end{aligned}$$

Hence,

$$c_0 = \alpha_0, \quad c_1 = \alpha_1,$$

and, for $n \geq 2$,

$$c_n = -\sum_{v=0}^{n-2} \alpha_v \frac{A_{n-2-v}}{n\,(n-1)},$$

where $A_n = A_n(c_0, \dots, c_n)$ is derived from previously given algorithms [7].

Consider now the operator form $Lu + Ru + Nu = 0$ where $R = \rho,\ L = d^2/dx^2,\ Nu = \alpha\, f(u)$. Write

$$\begin{aligned}
Lu &= -Ru - Nu, \\
L^{-1}Lu &= -L^{-1}Ru - L^{-1}Nu.
\end{aligned}$$

Now

$$\sum_{n=0}^{\infty} c_n x^n = \alpha_0 + \alpha_1 x - \rho\, I_2 \left(\sum_{n=0}^{\infty} c_n x^n\right) - \alpha\, I_2 \left(\sum_{n=0}^{\infty} A_n x^n\right),$$

(again using $f\left(\sum_{n=0}^{\infty} c_n x^n\right) = \sum_{n=0}^{\infty} A_n(c_0, \dots, c_n)\, x^n$),

$$\begin{aligned}
\sum_{n=0}^{\infty} c_n x^n &= \alpha_0 + \alpha_1 x - \rho \sum_{n=0}^{\infty} c_n \frac{x^{n+2}}{(n+1)(n+2)} - \alpha \sum_{n=0}^{\infty} A_n \frac{x^{n+2}}{(n+1)(n+2)} \\
&= \alpha_0 + \alpha_1 x - \rho \sum_{n=2}^{\infty} c_{n-2} \frac{x^n}{(n-1)n} - \alpha \sum_{n=2}^{\infty} A_{n-2} \frac{x^n}{(n-1)n},
\end{aligned}$$

leading to

$$c_0 = \alpha_0, \quad c_1 = \alpha_1,$$

and, for $n \geq 2$,

$$c_n = -\rho \frac{c_{n-2}}{(n-1)\,n} - \alpha \frac{A_{n-2}}{(n-1)\,n},$$

where $A_n = A_n(c_0, \dots, c_n)$.

If $\rho = \rho(x) = \sum_{n=0}^{\infty} \rho_n x^n$ and $Nu = \alpha(x) f(u) = \sum_{n=0}^{\infty} \alpha_n x^n f(u)$, we proceed as before to obtain

$$\sum_{n=0}^{\infty} c_n x^n = \alpha_0 + \alpha_1 x - I_2 \left(\sum_{n=0}^{\infty} \rho_n x^n\right) \left(\sum_{n=0}^{\infty} c_n x^n\right) - I_2 \left(\sum_{n=0}^{\infty} \alpha_n x^n\right) \left(\sum_{n=0}^{\infty} A_n x^n\right)$$
$$= \alpha_0 + \alpha_1 x - I_2 \sum_{n=0}^{\infty} x^n \sum_{v=0}^{\infty} \rho_v c_{n-v} - I_2 \sum_{n=0}^{\infty} x^n \sum_{v=0}^{n} \alpha_v A_{n-v},$$
$$\sum_{n=0}^{\infty} c_n x^n = \alpha_0 + \alpha_1 x - \sum_{n=0}^{\infty} \left\{\frac{x^{n+2}}{(n+1)(n+2)}\right\} \sum_{v=0}^{n} \rho_v c_{n-v} - \sum_{n=0}^{\infty} \left\{\frac{x^{n+2}}{(n+1)(n+2)}\right\} \sum_{v=0}^{n} \alpha_v A_{n-v},$$
$$\sum_{n=0}^{\infty} c_n x^n = \alpha_0 + \alpha_1 x - \sum_{n=2}^{\infty} x^n \sum_{v=0}^{n-2} \rho_v \frac{c_{n-2-v}}{(n-1)\,n} - \sum_{n=0}^{\infty} x^n \sum_{v=0}^{n-2} \alpha_v \frac{A_{n-2-v}}{(n-1)\,n},$$

leading to

$$c_0 = \alpha_0, \quad c_1 = \alpha_1, \quad c_n = -\sum_{v=0}^{n-2} \rho_v \frac{c_{n-2-v}}{(n-1)\,n} - \sum_{v=0}^{n-2} \alpha_v \frac{A_{n-2-v}}{(n-1)\,n},$$

for $n \geq 2$, where $A_n = A_n(c_0, \dots, c_n)$.

3. INHOMOGENEOUS CASE

Consider $Lu + Ru = g$. Let $R = \rho$ and $L = d^2/dx^2$. Let $g = \sum_{n=0}^{\infty} g_n x^n$. From $Lu = g - Ru$ and $L^{-1}Lu = L^{-1}g - L^{-1}Ru$, we have

$$u = \alpha_0 + \alpha_1 x - I_2 \sum_{n=0}^{\infty} g_n x^n - \rho I_2 u,$$

or

$$\sum_{n=0}^{\infty} c_n x^n = \alpha_0 + \alpha_1 x + \sum_{n=0}^{\infty} g_n \frac{x^{n+2}}{(n+1)(n+2)} - \rho \sum_{n=0}^{\infty} c_n \frac{x^{n+2}}{(n+1)(n+2)},$$
$$\sum_{n=0}^{\infty} c_n x^n = \alpha_0 + \alpha_1 x + \sum_{n=2}^{\infty} g_{n-2} \frac{x^n}{(n-1)\,n} - \rho \sum_{n=2}^{\infty} c_{n-2} \frac{x^n}{(n-1)\,n}.$$

Thus, $c_0 = \alpha_0$, $c_1 = \alpha_1$, and for $n \geq 2$,

$$c_n = \frac{g_{n-2} - \rho\, c_{n-2}}{(n-1)\,n}.$$

If we let $\rho = \rho(x) = \sum_{n=0}^{\infty} \rho_n x^n$ and $g = \sum_{n=0}^{\infty} g_n x^n$, we have

$$\sum_{n=0}^{\infty} c_n x^n = \alpha_0 + \alpha_1 x + I_2 \sum_{n=0}^{\infty} g_n x^n - I_2 \left(\sum_{n=0}^{\infty} \rho_n x^n\right) \left(\sum_{n=0}^{\infty} c_n x^n\right)$$
$$= \alpha_0 + \alpha_1 x + I_2 \sum_{n=0}^{\infty} g_n x^n - I_2 \sum_{n=0}^{\infty} x^n \sum_{v=0}^{n} \rho_v c_{n-v},$$

or

$$\sum_{n=0}^{\infty} c_n x^n = \alpha_0 + \alpha_1 x + \sum_{n=0}^{\infty} g_n \frac{x^{n+2}}{(n+1)(n+2)} - \sum_{n=0}^{\infty} \left\{\frac{x^{n+2}}{(n+1)(n+2)}\right\} \sum_{v=0}^{n} \rho_v c_{n-v}$$
$$= \alpha_0 + \alpha_1 x + \sum_{n=2}^{\infty} g_{n-2} \frac{x^n}{(n-1)\,n} - \sum_{n=2}^{\infty} \left\{\frac{x^n}{(n-1)\,n}\right\} \sum_{v=0}^{n-2} \rho_v c_{n-2-v},$$

leading to $c_0 = \alpha_0,\ c_1 = \alpha_1$, and for $n \geq 2$,

$$c_n = \frac{g_{n-2} - \sum_{v=0}^{n-2} \rho_v c_{n-2-v}}{(n-1)\,n}.$$

4. NONLINEAR INHOMOGENEOUS CASE

$$Lu + Nu = g = \sum_{n=0}^{\infty} g_n x^n,$$

or

$$\sum_{n=0}^{\infty} c_n x^n = \alpha_0 + \alpha_1\, x + I_2 \sum_{n=0}^{\infty} g_n x^n - I_2\, \alpha \sum_{n=0}^{\infty} A_n x^n,$$

where $f(u) = f\left(\sum_{n=0}^{\infty} c_n x^n\right)$ has been replaced by $\sum_{n=0}^{\infty} A_n\,(c_0, \dots, c_n)\, x^n$,

$$\sum_{n=0}^{\infty} c_n x^n = \alpha_0 + \alpha_1\, x + \sum_{n=0}^{\infty} g_n \frac{x^{n+2}}{(n+1)(n+2)} - \alpha \sum_{n=0}^{\infty} A_n \frac{x^{n+2}}{(n+1)(n+2)}$$
$$= \alpha_0 + \alpha_1\, x + \sum_{n=2}^{\infty} g_{n-2} \frac{x^n}{(n-1)\,n} - \alpha \sum_{n=2}^{\infty} A_{n-2} \frac{x^n}{(n-1)\,n}.$$

Thus, $c_0 = \alpha_0,\ c_1 = \alpha_1$, and for $n \geq 2$,

$$c_n = \frac{g_{n-2} - \alpha\, A_{n-2}}{(n-1)\,n},$$

where $A_n = A_n\,(c_0, \dots, c_n)$.

Now consider $Lu + Nu = g$, with $Nu = \alpha(x)\, f(x) = \sum_{n=0}^{\infty} \alpha_n x^n$ and $g = \sum_{n=0}^{\infty} g_n x^n$. Let $L = d^2/dx^2$ and substitute $\sum_{n=0}^{\infty} A_n\,(c_0, \dots, c_n)\, x^n$ for $f\left(\sum_{n=0}^{\infty} c_n x^n\right)$ as before. Then,

$$\sum_{n=0}^{\infty} c_n x^n = \alpha_0 + \alpha_1\, x + I_2 \sum_{n=0}^{\infty} g_n x^n - I_2 \left(\sum_{n=0}^{\infty} \alpha_n x^n\right) \left(\sum_{n=0}^{\infty} A_n x^n\right).$$

Since $\left(\sum_{n=0}^{\infty} \alpha_n x^n\right)\left(\sum_{n=0}^{\infty} A_n x^n\right) = \sum_{n=0}^{\infty} x^n \sum_{v=0}^{n} \alpha_v A_{n-v}$,

$$\sum_{n=0}^{\infty} c_n x^n = \alpha_0 + \alpha_1\, x + I_2 \sum_{n=0}^{\infty} g_n x^n - I_2 \sum_{n=0}^{\infty} x^n \sum_{v=0}^{n} \alpha_v A_{n-v}$$
$$= \alpha_0 + \alpha_1\, x + \sum_{n=0}^{\infty} g_n \frac{x^{n+2}}{(n+1)(n+2)} - \sum_{n=0}^{\infty} \left\{\frac{x^{n+2}}{(n+1)(n+2)}\right\} \sum_{v=0}^{n} \alpha_v A_{n-v},$$
$$\sum_{n=0}^{\infty} c_n x^n = \alpha_0 + \alpha_1\, x + \sum_{n=2}^{\infty} \left\{g_{n-2} \frac{x^n}{(n-1)\,n}\right\} - \sum_{n=2}^{\infty} \left\{\frac{x^n}{(n-1)\,n}\right\} \sum_{v=0}^{n-2} \alpha_v A_{n-2-v},$$

so that $c_0 = \alpha_0,\ c_1 = \alpha_1$ and, for $n \geq 2$,

$$c_n = \frac{g_{n-2} - \sum_{v=0}^{n-2} \alpha_v A_{n-2-v}}{(n-1)\,n},$$

where $A_n = A_n\,(c_0, \dots, c_n)$.

Now consider the general inhomogeneous nonlinear form

$$Lu + Ru + Nu = g.$$

Let $R = \rho,\ Nu = \alpha f(u),\ g = \sum_{n=0}^{\infty} g_n x^n,\ L = d^2/dx^2,$

$$Lu = g - Ru - Nu,$$
$$L^{-1}Lu = L^{-1}g - L^{-1}Ru - L^{-1}Nu,$$
$$u = \alpha_0 + \alpha_1 x + I_2 \sum_{n=0}^{\infty} g_n x^n - \rho I_2 u - \alpha I_2 \sum_{n=0}^{\infty} A_n x^n,$$

where we have replaced $f(u) = f(\sum_{n=0}^{\infty} c_n x^n)$ by $\sum_{n=0}^{\infty} A_n(c_0, \dots, c_n) x^n$. Thus,

$$\begin{aligned}\sum_{n=0}^{\infty} c_n x^n &= \alpha_0 + \alpha_1 x + I_2 \sum_{n=0}^{\infty} g_n x^n - \rho I_2 \sum_{n=0}^{\infty} c_n x^n - \alpha I_2 \sum_{n=0}^{\infty} A_n x^n \\ &= \alpha_0 + \alpha_1 x + \sum_{n=0}^{\infty} g_n \frac{x^{n+2}}{(n+1)(n+2)} - \rho \sum_{n=0}^{\infty} c_n \frac{x^{n+2}}{(n+1)(n+2)} \\ &\quad - \alpha \sum_{n=0}^{\infty} A_n \frac{x^{n+2}}{(n+1)(n+2)},\end{aligned}$$

$$\sum_{n=0}^{\infty} c_n x^n = \alpha_0 + \alpha_1 x + \sum_{n=2}^{\infty} g_{n-2} \frac{x^n}{(n-1)n} - \rho \sum_{n=2}^{\infty} c_{n-2} \frac{x^n}{(n-1)n} - \alpha \sum_{n=2}^{\infty} A_{n-2} \frac{x^n}{(n-1)n},$$

from which $c_0 = \alpha_0,\ c_1 = \alpha_1$ and, for $n \geq 2$,

$$c_n = \frac{g_{n-2} - \rho c_{n-2} - \alpha A_{n-2}}{(n-1)n},$$

where $A_n = A_n(c_0, \dots, c_n)$.

Finally, consider $Lu + Ru + Nu = g$ with

$$\begin{aligned}R = \rho(x) = \sum_{n=0}^{\infty} \rho_n x^n,\ g = \sum_{n=0}^{\infty} g_n x^n,\ Nu = \alpha(x) f(u) &= \sum_{n=0}^{\infty} \alpha_n x^n f(u) \\ = \sum_{n=0}^{\infty} \alpha_n x^n f(\sum_{n=0}^{\infty} c_n x^n) &= \sum_{n=0}^{\infty} \alpha_n x^n \sum_{n=0}^{\infty} A_n(c_0, \dots, c_n) x^n.\end{aligned}$$

Then,

$$\sum_{n=0}^{\infty} c_n x^n = \alpha_0 + \alpha_1 x + I_2 \sum_{n=0}^{\infty} g_n x^n - I_2 \left(\sum_{n=0}^{\infty} \rho_n x^n\right)\left(\sum_{n=0}^{\infty} c_n x^n\right) - I_2 (\sum_{n=0}^{\infty} \alpha_n x^n)(\sum_{n=0}^{\infty} A_n x^n).$$

Since

$$\sum_{n=0}^{\infty} \rho_n x^n \sum_{n=0}^{\infty} c_n x^n = \sum_{n=0}^{\infty} x^n \sum_{v=0}^{n} \rho_v c_{n-v} \quad \text{and}$$
$$\sum_{n=0}^{\infty} \alpha_n x^n \sum_{n=0}^{\infty} A_n x^n = \sum_{n=0}^{\infty} x^n \sum_{v=0}^{n} \alpha_v A_{n-v},$$

we have

$$\begin{aligned}\sum_{n=0}^{\infty} c_n x^n &= \alpha_0 + \alpha_1 x + I_2 \sum_{n=0}^{\infty} g_n x^n - I_2 \sum_{n=0}^{\infty} x^n \sum_{v=0}^{n} \rho_v c_{n-v} - I_2 \sum_{n=0}^{\infty} x^n \sum_{v=0}^{n} \alpha_v A_{n-v} \\ &= \alpha_0 + \alpha_1 x + \sum_{n=0}^{\infty} g_n \frac{x^{n+2}}{(n+1)(n+2)} - \sum_{n=0}^{\infty} \left\{\frac{x^{n+2}}{(n+1)(n+2)}\right\} \sum_{v=0}^{n} \rho_v c_{n-v} \\ &\quad - \sum_{n=0}^{\infty} \left\{\frac{x^{n+2}}{(n+1)(n+2)}\right\} \sum_{v=0}^{n} \alpha_v A_{n-v},\end{aligned}$$

or

$$\sum_{n=0}^{\infty} c_n x^n = \alpha_0 + \alpha_1 x + \sum_{n=2}^{\infty} g_{n-2} \frac{x^n}{(n-1)\,n}$$
$$= \sum_{n=2}^{\infty} x^n \sum_{v=0}^{n-2} \rho_v \frac{c_{n-2-v}}{(n-1)\,n} - \sum_{n=2}^{\infty} x^n \sum_{v=0}^{n-2} \alpha_v \frac{A_{n-2-v}}{(n-1)\,n}.$$

Thus $c_0 = \alpha_0$, $c_1 = \alpha_1$ and, for $n \geq 2$,

$$c_n = \frac{g_{n-2} - \sum_{v=0}^{n-2} \rho_v\, c_{n-2-v} - \sum_{v=0}^{n-2} \alpha_v\, A_{n-2-v}}{(n-1)\,n},$$

where $A_n = A_n(c_0, \dots, c_n)$.

Thus the Adomian series is not the Maclaurin series. It is actually (as stated by Adomian [1,2]) a generalized Taylor series about a function rather than a point, and can reduce, in a linear case, to the well-known series. Further, by combining Adomian polynomials and decomposition with Maclaurin series, we can make a Maclaurin series more useful. Finally, we state and will show in Part II and Part III (to appear) that the extended Maclaurin series can be used for coupled differential equations and partial differential equations and that, despite the improved power of the Maclaurin series with the use of the Adomian polynomials and decomposition techniques, the Adomian series resulting from the decomposition method is still superior in convergence properties. In the (extended) Maclaurin solution, the u_0 term only incorporates the first term of the g expansion while the decomposition series uses all of it. Also, it is evident that in collecting terms, the power series solution becomes complicated, while the decomposition series is always simple in this respect. Of course, to get convergent solution series, the series assumed for R or for g must also be convergent. Further work to appear will show applicability also to linear or nonlinear partial differential equations and coupled equations, and also show that the convergence is faster using (Adomian) decomposition series than the extended or improved power series results.

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