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Procedia Engineering 105 (2015) 287 – 293

Procedia Engineering

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6th BSME International Conference on Thermal Engineering (ICTE 2014)

Unsteady MHD Casson Fluid Flow through a Parallel Plate with Hall Current

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Abstract

Unsteady MHD Casson fluid flow through a parallel plate with hall current is investigated. The uniform magnetic field is applied perpendicular to the plates and the fluid motion is subjected to a uniform suction and injection. The lower plate is stationary and the upper plate is moving. Explicit Finite Difference technique has been used to solve the momentum and energy equations. The effect of pressure gradient, the Hall parameter and other parameters describing in the equations are shown graphically. Effect of decaying parameter with different Casson number on primary velocity, secondary velocity and temperature distributions are illustrated in the form of the graph.

*Keywords:*MHD;Heat Transfer; Hall Current; Finite Difference Techniques.

1. Introduction

In recent years, there has been considerable interest in the magnetohydrodynamic effect of viscous incompressible non-Newtonian Casson fluid flow with heat transfer with or without hall currents. The flow of an electrically conducting viscous fluid through a parallel plate in the presence of a transversely applied magnetic field has applications in many devices such as magnetohydrodynamic (MHD) power generators, MHD pumps, accelerators, aerodynamics heating, electrostatic precipitation, polymer technology, petroleum industry, pharmaceutical process, purification of crude oil, fluid droplets sprays etc. The most important non-Newtonian fluid possessing a yield value is the Casson fluid, which are carried significant applications in polymer processing

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doi:10.1016/j.proeng.2015.05.111

industries and biomechanics. Casson fluid is a shear thinning liquid which has an infinite viscosity at a zero rate of strain. Casson's constitute equation represents a nonlinear relationship between stress and rate of strain and has been found to be accurately applicable to silicon suspensions, suspensions of bentonic in water and lithographic varnishes used for printing inks. The fluid is acted upon by a constant pressure gradient and is subjected to a uniform magnetic field perpendicular to the plates. The Hall current is taken into consideration while the induced magnetic field is neglected by assuming a very small magnetic Reynolds number. The configuration is a good approximation of some practical situations such as heat exchangers, flow meters and pipes that connects system components. Walawander et al. [1] studied approximate Casson fluid model for tube flow of blood. Batra and Jena [2] showed the flow of a Casson fluid in a slightly curved tube. Attia [3] discussed unsteady MHD Couette flow and heat transfer of dusty fluid with variable physical property which is related to the Casson fluid. Attia and Sayed-Ahmed [4] analyzed Hydrodynamic impulsive Lid driven flow and heat transfer of a Casson fluid. Sayed-Ahmed et al. [5] examinedtime dependent pressure gradient effect on unsteady MHD couette flow and heat transfer of a casson fluid. Bhattacharyya et al. [6] showed analytic solution for magnetohydrodynamic boundary layer flow of Casson fluid.

Hence our main aim is to extend the work of Sayed-Ahmed et al. [5] and to investigate unsteady MHD Casson fluid flow through a parallel plate with hall current. The proposed model has been transformed into nonlinear coupled partial differential equations by usual transformation. Finally, the governing momentum and energy equations are solved numerically in case of one dimension flow and explicit finite difference method has been used to calculate the results and for stability analysis.

2. Mathematical formulation

Consider unsteady, viscous, viscous laminar and incompressible fluid flows between two infinite horizontal plates located at $y = \pm h$ planes and extended from $x = -\infty$ to ∞ and from $z = -\infty$ to ∞ .The upper plate moves with a uniform velocity U_0 while the lower plate is stationary. The upper and lower plates are kept at two constants temperature T_2 and T_1 respectively with $T_2 > T_1$. The fluid is acted upon by an exponentially decaying pressure gradient $\frac{\partial p}{\partial x}$ in the x direction and a uniform suction from above and injection from below which are applied at $t = 0$. A uniform magnetic field is applied in the positive y -direction and is assumed undistributed as the induced magnetic field is neglected by assuming a very small magnetic Reynolds number. The Hall Effect is taken into consideration and consequently a z -component for the velocity is expected to arise. The uniform suction implies that the y-component of the velocity v_0 is constant. Thus the fluid velocity vector is given by;

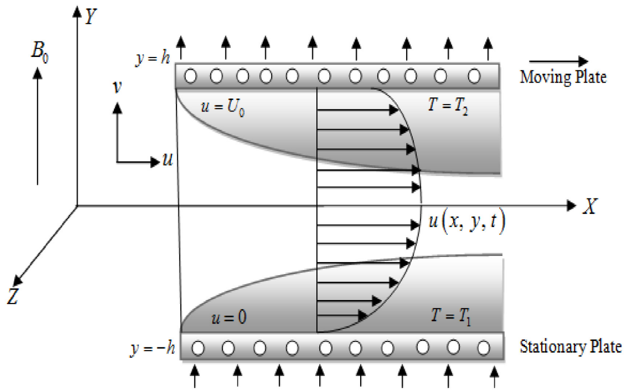


Figure 1: Geometrical configuration of thermal boundary layer.

$$v = u\mathbf{i} + v_0\mathbf{j} + w\mathbf{k}$$

The non-dimensional variables that have been used in the governing equations are

$$\bar{x} = \frac{x}{h},\ \bar{y} = \frac{y}{h},\ \bar{z} = \frac{z}{h},\ \bar{t} = \frac{tU_0}{h},\ \bar{u} = \frac{u}{U_0},\ \bar{w} = \frac{w}{U_0},\ \bar{p} = \frac{p}{\rho U^2{}_0},\ \theta = \frac{T - T_1}{T_2 - T_1},\ \bar{\mu} = \frac{\mu}{K_c{}^2}$$

Using these above dimensionless variables, the following dimensionless equations have been obtained as;

$$\frac{\partial u}{\partial t}+\frac{S}{\text{Re}}\frac{\partial u}{\partial y}=-\alpha e^{-at}+\left[\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{H_a^{\,2}}{1+m^2}(u+mw)\right] \qquad (1)$$

$$\frac{\partial w}{\partial t}+\frac{S}{\text{Re}}\frac{\partial w}{\partial y}=\frac{1}{\text{Re}}\left[\frac{\partial}{\partial y}\left(\mu\frac{\partial w}{\partial y}\right)-\frac{H_a^{\,2}}{1+m^2}(w-mu)\right] \qquad (2)$$

$$\frac{\partial \theta}{\partial t}+\frac{S}{\text{Re}}\frac{\partial \theta}{\partial y}=\frac{1}{P_r}\frac{\partial^2\theta}{\partial y^2}+E_c\mu\left[(\frac{\partial u}{\partial y})^2+(\frac{\partial w}{\partial y})^2\right]+\frac{H_a^{\,2}E_c}{(1+m^2)}(u^2+w^2) \qquad (3)$$

$$\mu=\left[1+\left(\tau_D\Big/\sqrt{(\frac{\partial u}{\partial y})^2+(\frac{\partial w}{\partial y})^2}\right)^{1/2}\right]^2 \qquad (4)$$

where α is the constant pressure gradient $(\frac{dp}{dx})$ and a is the decaying parameter.

The corresponding non-dimensional boundary conditions are;

$t>0 \quad u=0 \quad w=0 \quad T=0 \quad at\ y=-1$

$u=1 \quad w=0 \quad T=1 \quad at\ y=1$

The non-dimensional quantities are; Casson number $\tau_D=\frac{\tau_0 h}{k_c^{\,2}U_0}$,Reynolds number $R_e=\frac{\rho U_0 h}{k_c^{\,2}}$,Suction parameter $S=\frac{\rho v_0 h}{k_c^{\,2}}$, Prandtl number $P_r=\frac{\rho c_p U_0 h}{k}$, Eckert number $E_c=\frac{U_0 k_c^{\,2}}{\rho c_p h(T_2-T_1)}$,Hartmann number squared $H_a^{\,2}=\frac{\sigma B_0^{\,2}h^2}{k_c^{\,2}}$.

3. Numerical Solution

In this section the governing second order non-linear coupled dimensionless partial differential equations with initial and boundary conditions have been solved. The explicit finite difference method has been used to solve equations (1-4) subject to the boundary conditions. The region within the boundary layer is divided by some perpendicular line of Y-axis, where Y-axis is normal to the medium as shown in the figure. It is assumed that the maximum length of the boundary layer is $Y_{max}=2$ i.e. Y varies from -1 to $+1$ and the number of grid spacing in Y direction is $m=100$. Hence the constant mesh size along Y-axis becomes $\Delta Y=0.02(-1\le Y\le 1)$ with smaller time step $\Delta t=0.0001$.

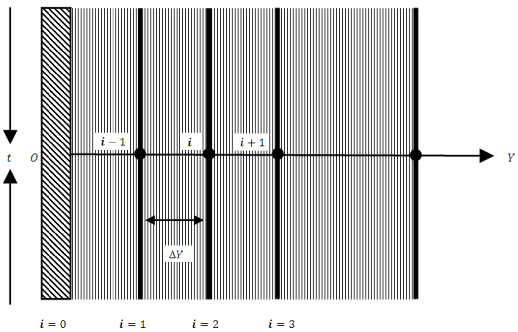


Figure 2: Finite difference space grid

Let U', W' and θ' denote the values U, W and θ at the end of a time step respectively. Using the explicit finite difference method the system of partial differential equation (1-3) is obtained an appropriate set of finite difference equations;

$$\frac{U_i^{n+1}-U_i^n}{\Delta t}+\frac{S}{R_e}\frac{U_{i+1}^n-U_i^n}{\Delta Y}=-\alpha e^{-at}$$
$$+\frac{1}{R_e}\left[\mu_i^n\frac{U_{i+1}^n-2U_i^n+U_{i-1}^n}{\Delta Y^2}+\frac{\mu_{i+1}^n-\mu_i^n}{\Delta Y}\frac{U_{i+1}^n-U_i^n}{\Delta Y}-\frac{H_a^2}{1+m^2}\left(U_i^n+mW_i^n\right)\right] \quad (5)$$

$$\frac{W_i^{n+1}-W_i^n}{\Delta t}+\frac{S}{R_e}\frac{W_{i+1}^n-W_i^n}{\Delta Y}=$$
$$+\frac{1}{R_e}\left[\mu_i^n\frac{W_{i+1}^n-2W_i^n+W_{i-1}^n}{\Delta Y^2}+\frac{\mu_{i+1}^n-\mu_i^n}{\Delta Y}\frac{W_{i+1}^n-W_i^n}{\Delta Y}-\frac{H_a^2}{1+m^2}\left(W_i^n-mU_i^n\right)\right] \quad (6)$$

$$\frac{\theta_i^{n+1}-\theta_i^n}{\Delta t}+\frac{S}{R_e}\frac{\theta_{i+1}^n-\theta_i^n}{\Delta Y}=$$
$$\frac{1}{P_r}\left[\frac{\theta_{i+1}^n-2\theta_i^n+\theta_{i-1}^n}{\Delta Y^2}\right]+E_c\mu\left[\left(\frac{U_{i+1}^n-U_i^n}{\Delta Y}\right)^2+\left(\frac{W_{i+1}^n-W_i^n}{\Delta Y}\right)^2\right]+\frac{H_a{}^2E_c}{(1+m^2)}\left[\left(U_i^n\right)^2+\left(W_i^n\right)^2\right] \quad (7)$$

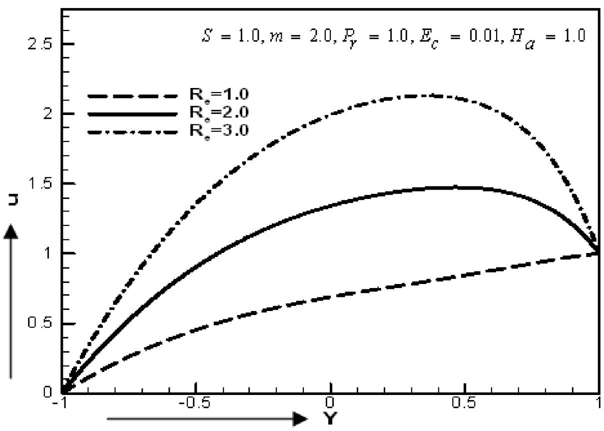
and the initial and boundary conditions with the finite difference scheme are

$t>0 \qquad U_L{}^n=0, \qquad W_L{}^n=0, \qquad \theta_L{}^n=0 \qquad where\ L=-1$

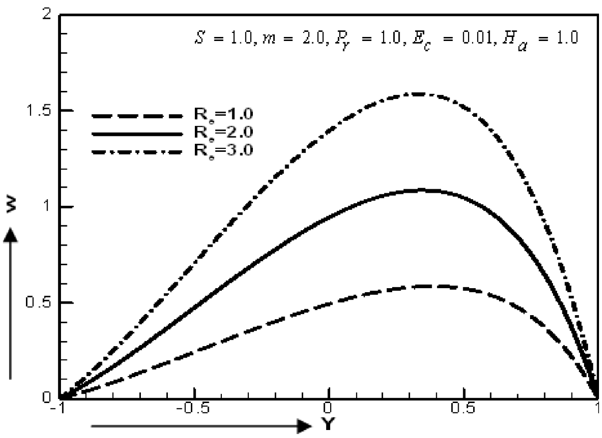
$U_L{}^n=1, \qquad W_L{}^n=0, \qquad \theta_L{}^n=1 \qquad where\ L=1$

4. Results and Discussion:

To obtain the steady-state solutions, the computations have been carried out up to dimensionless time $t=0$ to 20. The results of the computations, however, show little changes in the above mentioned quantities after dimensionless time $t=5$. Thus the solutions for dimensionless time $t=5$ are essentially steady-state solutions. To observe the physical situation of the problem, the steady-state solutions have been illustrated in figures 3-7.



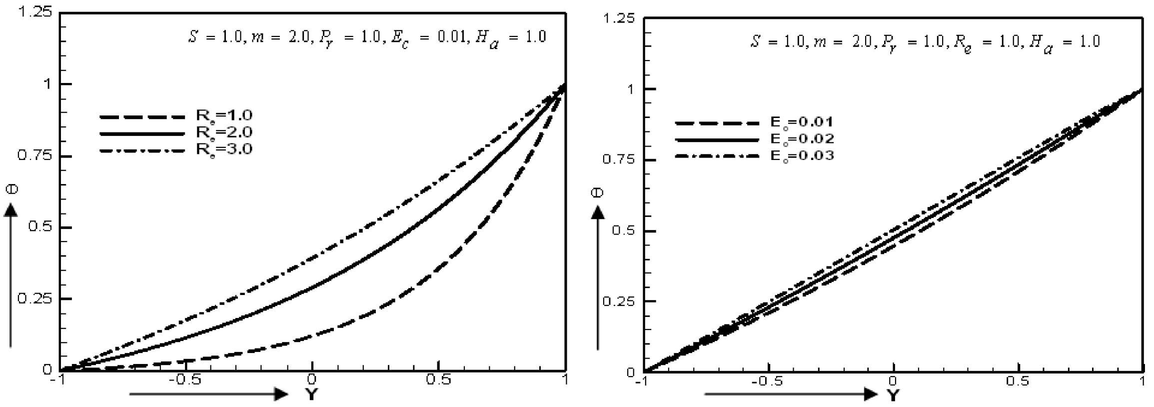
(a)



(b)

Figure 3: (a) Primary velocity distribution and (b) secondary velocity distributions for different values of Reynolds number at *t=5.0*

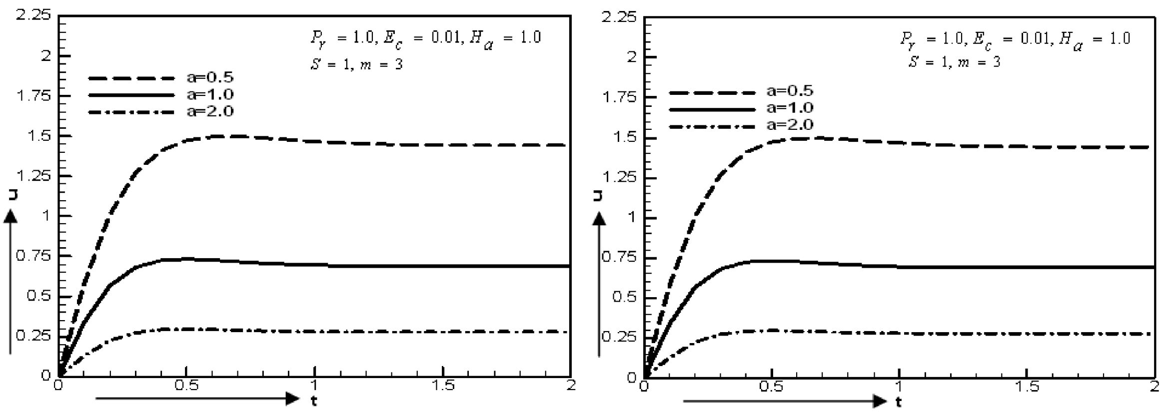
The primary and secondary velocity distributions have been shown in figures 3(a) and 3(b) for different values of Reynolds number. Both the primary and secondary velocity distributions have been increased with the increase of R_e.



(a) (b)

Figure 4:(a) Temperature distribution for different values of Reynolds number and (b) Temperature distributions for different values of Eckert number at *t*=5.0

The temperature distributions have been shown in figure 4(a) and 4(b) for different values of Reynolds number R_e and Eckert number E_c respectively. In both cases the temperature distribution increases with the increase of R_e and E_c .It is shown from figure 5(a) and 5(b) the primary velocity component decreases with increasing decaying parameter for all values of τ_D.It is observed that the time at which primary velocity reaches its steady state value decreases with increasing a for $a > 0$. Increasing τ_D increases primary velocity for all values of decaying parameter but with small difference.



(a) (b)

Figure 5: (a) Effect of decaying parameter a on primary velocity at $y = 0$ for $\tau_D = 0.05$ and (b) Effect of decaying parameter a on primary velocity at $y = 0$ for $\tau_D = 0.1$

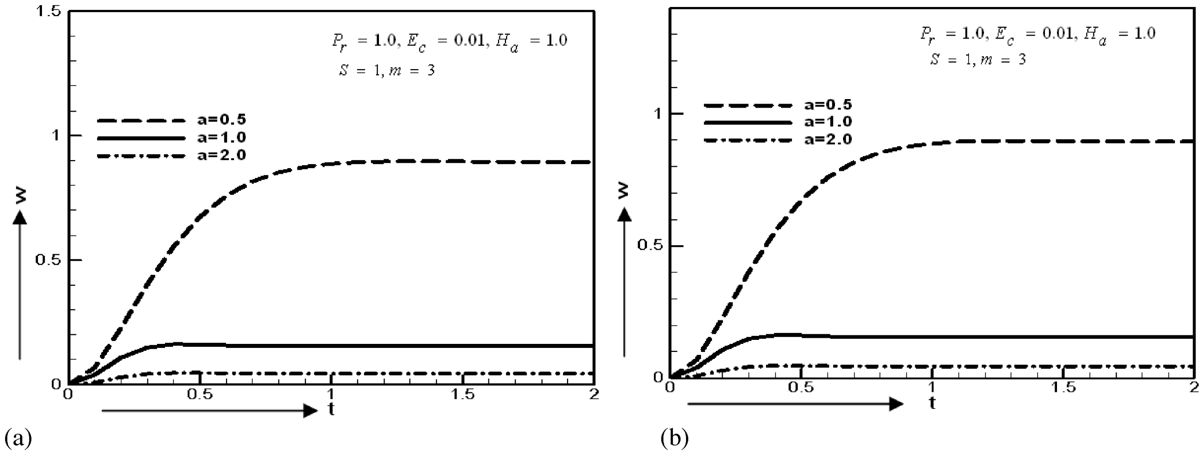


Figure 6: (a) Effect of decaying parameter a on secondary velocity at $y = 0$ for $\tau_D = 0.05$ and (b) Effect of decaying parameter a on secondary velocity at $y = 0$ for $\tau_D = 0.1$

The secondary velocity distributions have been shown in figure 6(a) and 6(b) for different values of τ_D with different decaying parameter. It is observed that secondary velocity component decreases with increasing decaying parameter. These figures indicate that the influence of τ_D on secondary velocity depends on t and become more clear when decaying parameter near to zero but this influence is small for large a. From the figure 7(a) and 7(b), it have been shown that influence of decaying parameter on temperature distributions depend on t. It is also shown that increasing a decreases θ while it is not greatly affected by changing τ_D.

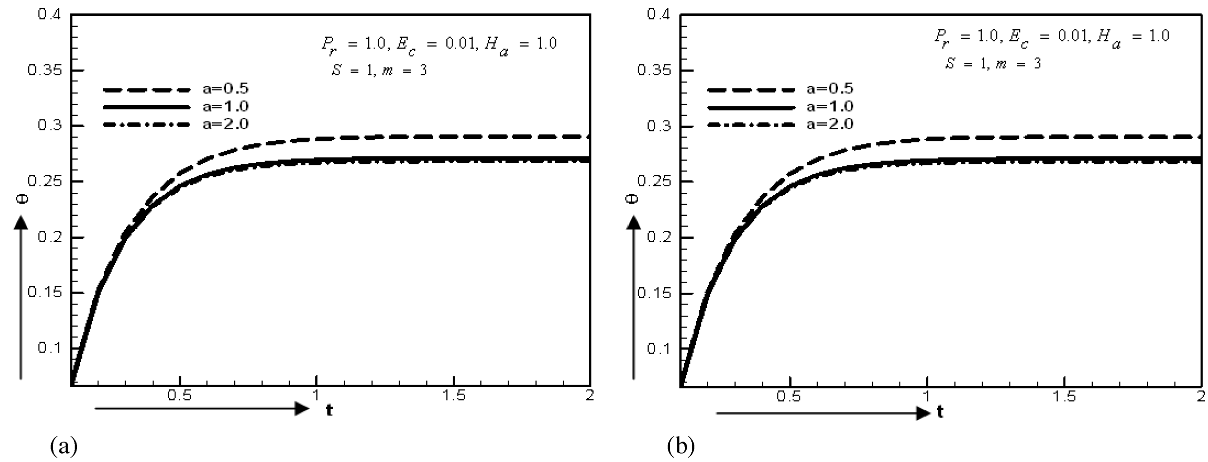


Figure 7: (a) Effect of decaying parameter a on temperature at $y = 0$ for $\tau_D = 0.05$ and (b) Effect of decaying parameter a on temperature at $y = 0$ for $\tau_D = 0.1$

Conclusions:

In this research work, the explicit finite difference method of unsteady one dimensional Casson non-Newtonian fluid flow through a parallel plate with a hall current is investigated. The physical properties are illustrated graphically for different values of parameter. The primary velocity, secondary velocity and temperature distributions have been increased with the increase of Reynolds number. The effect of decaying parameter a , Casson fluid yield stress τ_D , and the Hall parameter m , on the velocity and temperature distributions are studied. The decaying parameter a affects the main velocity components and the temperature. The result shows that the influence of the parameters a and τ_D on velocity components and the temperature depends on Hall parameter m and suction parameter S . The time at which two velocity component reach the steady state increases with increasing m , but decreases when τ_D increases. The time at which θ reaches its steady state increases with increasing m while it is not greatly affected by changing τ_D .

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